PFUSRC-13: A Century of the Kakeya Problem and the Equivalent Closure

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Abstract

The classical Kakeya needle problem interprets the statement that "the area swept by a unit line segment rotating 360° can approach zero" as measure vanishing and existential recovery, which has long led to a fundamental paradox between geometry and physics.

Within the framework of the PFUSRC unified theory of the cosmos, this paper reinterprets the Kakeya rotation on the ontological basis of Coupling Interface 0. The main conclusions are as follows:

1. The essence of the Kakeya rotation is a physical experiment, not a mathematical game. The ideal zero-area rotation is physically unattainable due to the inherent asymmetry between the needle tip and base, the impossibility of perfect synchronous phase reset, and the ineliminability of vacuum perturbations.
2. A single 360° rotation is not a geometric reset but a topological homeomorphic mapping of the equivalent coupling interface. The final state is topologically isomorphic to the initial state but ontologically updated. This process is strictly governed by the intrinsic constant $\pi_1 = 12/11$.
3. The rotation operation is non-integrable. Within the PFUSRC framework, rotation is not a reversible closed transformation but produces an irreducible entropy residual.
4. A 720° double rotation achieves local topological isomorphism but not global spacetime return. The internal structure of the needle is restored, but the background metric of the cosmos undergoes an irreversible drift. Local reversibility and global irreversibility are unified here.
5. Retreat (virtual decrease) is not entropy decrease but high-cost growth. Every local "restoration" of structure comes at the price of a larger global entropy increase, consuming phase. The universe has no "undo" operation.
6. Entropy increase does not refute the closed bicone cosmos; on the contrary, it proves its necessity. The universality of the second law of thermodynamics can only hold in a cosmos that is topologically closed and thermodynamically open — precisely the core of the PFUSRC axiomatic system.
7. $\pi_1 = 12/11$ is a testable proportional constant. This paper provides specific observational windows and quantitative experimental formulas.

Keywords: Kakeya century; equivalent closure; topological isomorphism; non-integrability; absolute drift; virtual decrease; retreat cost; $\pi_1 = 12/11$; cosmic growth theory; phase dissipation; rigidity-flexibility unity

1 Introduction: A Century of the Kakeya Problem — From Physics to Abstraction and Back to Physics

1.1 Kakeya Sōichi (1917): The Origin of the Problem Was Physical

In 1917, the Japanese mathematician Kakeya Sōichi posed the following question: What is the minimum area swept by a unit line segment rotating 360° in the plane? The problem originated from a physical conception: a rigid body sweeping through space. Kakeya himself gave an answer: area $\approx \pi/8 \approx 0.39$.

The origin of the problem was physical.

1.2 Besicovitch (1919–1920s): First Abstraction — Stripping Away Continuity

In 1919, Besicovitch posed a twin problem: What is the minimal area of a set that contains a unit line segment in every direction?

Here, continuous rotation is no longer required — only the existence of directions. The needle can "jump" from one direction to another without traversing a continuous path.

Physical continuity was stripped away.

Besicovitch proved that such a set can have arbitrarily small area — approaching zero. He then combined this with the original problem to show that the answer to Kakeya's original question is also: the area can be arbitrarily small, with no minimum.

Physical realizability was stripped away. Besicovitch's construction required infinitely many cuttings, overlappings, and weavings — impossible to realize in the physical world.

1.3 Walker, Bloom, Schönberg, Cunningham (1965–1971): Second Abstraction — Stripping Away Structural Constraints

These mathematicians constructed simply connected (hole-free) Kakeya sets with arbitrarily small area. Simple connectedness is closer to physical intuition.

But it remained abstract. The natural asymmetry between needle tip and base, the desynchronization of phases during rotation, and the microscopic fluctuations of vacuum zero-point energy — all of these physical constraints were still completely ignored.

1.4 Davies, Fefferman, Bourgain, Tao, Katz, Wang, Zahl (1971–2025): Third Abstraction — Turning to Dimension

The Kakeya problem evolved into the Kakeya conjecture: What are the Hausdorff and Minkowski dimensions of Kakeya sets?

- In two dimensions: Davies (1971) proved the dimension equals 2.
- In three dimensions: Wang and Zahl (2025) proved the dimension equals 3.

This is a milestone in pure mathematics. Their results are built on an abstract measure-theoretic premise with no medium and no vacuum perturbations; they still do not restore the physical conditions of a real rotating needle. The question answered is "no counterexample exists," not "what volume does a real rotating needle sweep out?"

Physical rotation was completely stripped away. The problem became one of "set dimensions."

1.5 This Paper (2026): Fourth Step — From Abstraction Back to Physics

Every abstraction in the classical studies was legitimate, necessary, and great. They allowed mathematics to penetrate deep into the underlying structures of measure, dimension, and topology.

But every abstraction came at a cost.

The classical Kakeya hypothesis contains three idealizations: static non-evolving space, structureless line segments, and zero-dissipation phase synchronization during rotation. These are mathematical idealizations that do not hold in physical reality.

Innovation of this paper: Using the geometric structure of the Kakeya rotation, we establish a mapping from rotation to phase change to entropy increase, bridging geometry and thermodynamics.

Step	Representative	What Was Stripped	Cost
First	Besicovitch	Continuity	Needle can "jump," no longer rotates continuously
First	Besicovitch	Physical realizability	Construction requires infinite cuttings, overlappin
Second	Walker et al.	Structural constraints	Asymmetry, phase, vacuum fluctuations still ignor
Third	Dimension conjecture	Rotation itself	Problem changed from "rotation" to "set dimensio

Table 1: The costs of abstraction in the history of the Kakeya problem

1.6 This Is Not a Denial of Predecessors, but a Continuation of Their Path

Kakeya, Besicovitch, Walker, Bloom, Davies, Fefferman, Bourgain, Tao, Katz, Wang, Zahl — each of these predecessors took a solid step forward on the foundation laid by those before them.

Their depth was depth into abstraction.

The depth of this paper is depth into physical reality.

This is not an opposition of directions, but a continuation of the same path. They reached the limit in the abstract world; at their endpoint, we turn around and continue forward — into the real world.

All classical conclusions (measure can be zero, dimension equals 3 in three dimensions, the double-cover structure) are special cases of this paper in the ideal limit. This paper adds physical constraints, thereby being compatible with and surpassing the classical conclusions.

2 Foundational Preliminaries: The Ontological Framework of PFUSRC

2.1 Core Concept Definitions

Concept	Symbol	Mathematical Definition
Unit One	$\mathcal{B} = \{1\}$	Discrete indivisible primitive
Coupling Interface 0	$\partial\mathcal{M} = 0$	Topological set of the boundary between higher and lower dimensions
Background One	$\Omega_0 = 1$	Uniform static 4D ground state manifold
Global Operator β_1	β_1	Operator driving dimensional expansion and self-stabilization
Intrinsic Scale Constant	$\pi_1 = \frac{12}{11}$	Geometric boundary ratio of the bicone

Table 2: Core ontological concepts of PFUSRC

2.2 Core Axioms

Axiom 1 (Primordial Unity): Physical quantities, geometric quantities, and logical quantities share the same primordial set $\mathcal{B}$.

Axiom 2 (Four-Dimensional Closure): The complete self-consistent spatial manifold is $\mathcal{M}_4$; three-dimensional space $\mathcal{M}_3 \subset \mathcal{M}_4$ is a projected submanifold.

Axiom 3 (Irreversible Growth): The total entropy of the whole space $S_{\text{total}}(t)$ satisfies:

$$\frac{dS_{\text{total}}(t)}{dt} > 0, \quad \forall t$$

Symbol	Physical Meaning
$\mathcal{W}[\gamma]$	Growth cost functional for a single rotation
Φ	4D global flux of the bicone (defined in PFUSRC-001)
$F(\theta)$	Configuration-phase state function of the rotating system
$\Delta\phi$	Accumulated phase change during rotation
C	Dimension-adapting constant (experimentally calibrated)
$\kappa = \pi_1 = 12/11$	PFUSRC dynamic closure constant
ΔS_{bg}	Entropy increase from cosmic ground state growth

Table 3: Symbol table

2.3 Symbol Table

3 360° Rotation: From Geometric Reset to Equivalent Closure

3.1 Fundamental Defects of the Classical Interpretation

Classical interpretation: 360° rotation $\to$ direction returns $\to$ area can be compressed to zero $\to$ return to initial state.

The classical Kakeya model gives $A \to 0$ (swept area tends to zero), ignoring phase $\phi(\theta)$ and background space evolution $\Omega(t)$.

Defects:

1. Strips away global factors such as time, phase, and oscillation
2. Confuses arithmetic zero (the number 0) with Coupling Zero (Interface 0, which is non-empty)
3. Denies the irreversibility of motion and the occurrence of ontological evolution
4. Ignores that space is reconstructed during motion

3.2 Topological Mapping of the Equivalent Coupling Interface

A 360° rotation induces a topological homeomorphism:

$$f : 0 \mapsto 0', \quad 0' = f(0)$$

where f is a homeomorphism on $\mathcal{M}_4$, preserving steady state, reference frame, topology, and function, but with ontology updated.

3.3 Growth Cost Functional (Rotation-Entropy Mapping)

Define the growth cost functional:

$$\mathcal{W}[\gamma] = \oint_\gamma \left(\frac{d\Phi}{\Phi} + \kappa \cdot d\theta \right)$$

where:

- γ is the rotation path ($\theta \in [0, 2\pi]$)
- Φ is the 4D global flux of the bicone (see the Biconical Convergence Formula in PFUSRC-001)
- $\kappa = \pi_1 = 12/11$

The relation between entropy increase and the cost functional is:

$$\Delta S_{\text{total}} = C \cdot \mathcal{W}[\gamma] + \Delta S_{\text{bg}}$$

where C is a dimension-adapting constant (experimentally calibrated), and ΔS_{bg} originates from the continuous growth evolution of the cosmic ground state Ω_0.

Transition note: After integrating over the full path $\mathcal{W}[\gamma]$, the contribution of the flux term can be equivalently incorporated into the total phase change $\Delta\phi$. Thus the functional expression reduces to the entropy-phase linear relation in Section 5.3; the two formulations are mathematically self-consistent and equivalent.

4 Global Synergy of Phase, Oscillation, Time, and Coupling Interface

4.1 Phase Closure $\neq$ Original Reset

A 360° rotation achieves periodic closure of phase from 0 to 2π and back to 0. However, due to irreversible evolution, the phase state is only topologically equivalent, not ontologically identical. The coupling interface has shifted; it is no longer the original interface.

4.2 Oscillation Vanishing $\neq$ No Oscillation

Oscillation converges as field excitation decays through amplitude compression and coherent cancellation. Oscillation vanishing represents a dynamic steady state, not the absence of oscillation.

4.3 Temporal Closure $\neq$ Temporal Erasure

The rotation occupies the temporal dimension and leaves helical temporal traces. Time completes periodic closure, but the evolutionary history is irreversible and cannot be erased.

5 $\pi_1 = 12/11$: The "Exchange Rate" of Rotation

5.1 Why Can a Rotation Not Perfectly Reset?

- Microscopic perturbations from vacuum zero-point energy (Casimir effect)
- Asymmetry between needle tip and base $\rightarrow$ phase shift
- Trajectory always deviates from an ideal circle

5.2 Derivation of π_1 (Geometry + Calculus of Variations)

The intrinsic constant $\pi_1 = 12/11$ is derived from the geometric boundary ratio of the bicone combined with the variational minimization of the cost functional. A detailed derivation is given in Appendix A.

5.3 Proportional Relation Between π_1 and the Entropy Cost

From the growth cost functional:

$$\Delta S_{\text{total}} = C \cdot \frac{12}{11} \cdot \Delta\phi + \Delta S_{\text{bg}}$$

where $\Delta\phi$ is the total phase change during rotation, and ΔS_{bg} is the background entropy increase from the passage of time.

6 Direct Proofs of the Three Theories via Kakeya Rotation

6.1 Cosmic Growth Theory

A single 360° rotation corresponds to: one minimal unit growth cycle, one act of spatial derivation and reconstruction, one segment of helical structure formation, one equivalent iteration of the coupling interface:

$$\partial\Omega \xrightarrow{\text{rotation}} \partial\Omega'$$

The failure to return to the original initial interface proves that the cosmos does not cycle in place, but grows helically, evolves cumulatively, and constructs dynamically.

6.2 Logical Origin Theory

The entire rotational closed loop corresponds to the minimal paradigm of logical origin: initial state → excitation → evolution → convergence → equivalent steady state. The rotation path γ forms a closed topological loop.

Logic does not exist innately and statically, but is generated dynamically from Coupling Interface 0 through growth, closure, and iteration.

6.3 Global Dynamic Space Construction Theory

The region swept by the rotation is not "motion in pre-existing space," but space generated by motion and structure derived by evolution. The spatial domain is progressively generated by the rotation mapping.

7 720° Rotation: Topological Isomorphism, Non-Integrability, and Absolute Drift

7.1 Single Rotation: Debt Left Behind

One forward 360° rotation unfolds space, drives helical growth, and causes an equivalent shift of the coupling interface. A single rotation leaves irreversible topological "debt."

7.2 Non-Integrability of the Rotation Operation

Let the state function $F(\theta)$ describe the configuration and phase of the needle. The loop integral is:

$$\Delta F = \oint dF(\theta) = \varepsilon > 0$$

where ε is a small residual depending on the rotation path. This means the rotation operation is non-integrable — after a closed loop in parameter space, the state function does not return to its original value.

This mathematical structure is isomorphic to the nonholonomic constraints of the Chaplygin sleigh in classical mechanics, but in PFUSRC its physical origin lies in the growth of the cosmos and thermodynamic irreversibility.

7.3 Local Isomorphism and Global Drift

A 720° double rotation achieves:

1. **Local topological isomorphism**:

$$F(\theta + 4\pi) = F(\theta)$$

The internal structural relations, relative positions, and phase locking of the needle are completely restored. This is the geometric essence of the SU(2)/SO(3) double cover.

2. **Global absolute drift**:

$$g_{\mu\nu} \mapsto g_{\mu\nu} + \delta g_{\mu\nu}$$

Here $\delta g_{\mu\nu}$ represents a permanent infinitesimal shift of the background spacetime, distinguished from transient random perturbations.

Conclusion: In a growing cosmos, there is no true "reset" — only local isomorphism plus global drift.

7.4 The Growth Ring: Entropy Residual

From non-integrability:

$$\Delta S_{\text{residual}} = C \cdot \varepsilon > 0$$

This residual is the "growth ring" of the cosmos — after each rotation cycle, the background entropy increases irreversibly and cannot be undone by any subsequent rotation.

8 Entropy Increase Does Not Refute the Closed Bicone Cosmos — It Proves Its Necessity

Premise	**Conclusion**
Entropy increase is a real physical phenomenon	The cosmos must be able to exchange with the exterio
The cosmos has a well-defined topological boundary	The cosmos is topologically closed
Both hold simultaneously	**Topologically closed + thermodynamically oper**

Table 4: Logical derivation of the necessity of a topologically closed and thermodynamically open cosmos

This is precisely what PFUSRC has established from the beginning: the topological boundary allows exchange with the exterior under the 12 : 11 gauge constraint.

Therefore, entropy increase is not a counterexample to the closed bicone cosmos but a necessary consequence. The universality of the second law of thermodynamics can only hold in a cosmos that is topologically closed and thermodynamically open.

9 The Cost of Retreat: Phase Dissipation and the Maintenance of Rigidity

9.1 Retreat Is Not Entropy Decrease

Retreat (local structural restoration) is not true entropy decrease. Every instance of "virtual decrease" comes at the price of a larger global entropy increase. This cost is manifested as **phase dissipation** — the irreversible loss of accumulated phase information into the background.

9.2 Derivation Chain of the Retreat Cost

1. Forward rotation: $\Delta S_f = C \cdot \kappa \cdot \Delta\phi$ 2. Reverse retreat: requires additional phase dissipation $\Delta\phi' > \Delta\phi$, hence $\Delta S_b = C \cdot \kappa \cdot \Delta\phi' > \Delta S_f$ 3. Total entropy increase:

$$\Delta S_{\text{total}} = \Delta S_f + \Delta S_b + \Delta S_{\text{bg}} > 2\Delta S_f$$

The equality corresponds to the ideal limit with no vacuum perturbations and no phase dissipation; the inequality is the normal physical reality.

9.3 Rigidity-Flexibility Unity

The rigidity of the cosmos (stable topological boundary, $12:11$ gauge invariance) is not achieved by being "welded rigid," but maintained through phase adjustment, virtual-deficit borrowing, and the capacity to retreat. Retreat has a cost (phase dissipation, entropy increase), but this is a necessary expense to maintain rigidity.

10 Testable Predictions

Based on $\pi_1 = 12/11$ and the rotation-entropy mapping, this paper presents three sets of physical experiments.

10.1 Rotation-Entropy Proportionality Calibration Experiment

Conditions: Ultra-high vacuum, extremely low temperature. A slender rigid rod rotates uniformly through a single 360° cycle. External thermal and mechanical disturbances are shielded. Phase changes and minute heat dissipation are synchronously measured.

Quantitative relation:

$$\Delta S = C \cdot \frac{12}{11} \cdot \Delta\phi$$

C is a system-specific calibration constant obtained from static measurements on the same apparatus.

Expected result: The entropy change is strictly proportional to the phase change with coefficient 12/11. After repeated calibrations, the proportionality coefficient converges to 12/11.

Time window: 5–10 years.

10.2 720° Double Rotation Residual Interference Experiment

Conditions: A high-precision laser interferometer monitors a spin system undergoing a 720° periodic rotation. Interference fringe shifts are recorded in real time.

Quantitative relation:

$$\Delta x = K \cdot \Delta S, \quad \Delta S = C \cdot \varepsilon > 0$$

where K is the interferometer instrument constant and ε is the residual from the loop integral of rotation.

Expected result: Even when the local configuration of the microscopic rod is completely restored, an irreducible positive fringe shift $\Delta x > 0$ remains, directly verifying that the overall spacetime is not restored after 720°.

Time window: 5–10 years.

10.3 Quantum Coherence Phase Dissipation Experiment

Conditions: A trapped single qubit undergoes periodic rotational oscillation. The quantum coherence decay rate and coherence lifetime are tracked.

Quantitative relation:

$$\text{Coherence decay rate} \propto \frac{12}{11}, \quad \text{Coherence lifetime} \propto \frac{11}{12}$$

Expected result: Both quantities strictly obey the $12:11$ proportionality constraint; phase dissipation is uniformly controlled.

Time window: 3–5 years.

10.4 Theoretical Predictions and Scientific Falsifiability Criteria

Based on the relations derived above involving $\pi_1 = 12/11$, this paper formulates three quantitative physical predictions, each possessing both verifiability and falsifiability:

1. **Macroscopic rotation prediction**: Under adiabatic ultra-high-vacuum, ultra-low-temperature conditions, after subtracting background entropy increase, the linear proportionality coefficient between entropy change and phase change converges to 12/11. A measured coefficient significantly deviating from this value would falsify the predicted relation.

2. **Spin interference prediction**: After a 720° spin evolution of a microscopic system, with local particle configuration restored, the interference shift $\Delta x > 0$ must hold. If no fixed residual shift is measured, the inference of global drift and entropy residual from rotation would be invalid.

3. **Quantum dissipation prediction**: For periodically rotating quantum systems, the coherence decay rate and coherence lifetime strictly obey the 12 : 11 ratio. If repeated measurements on diverse quantum systems consistently deviate from this ratio, the conclusion that π_1 governs phase dissipation would be falsified.

The experimental parameters and quantitative expressions provided here offer standard criteria for future experimental work. The measurement results will be the sole objective basis for judging the validity of the theoretical predictions concerning rotation, phase, and entropy increase.

Note: Current instrumental precision may not directly measure these small quantities; this does not invalidate the theory. The formulas above provide quantitative benchmarks for future precision experiments.

11 Unified Paradigm: From Mathematics to Physics · From Abstraction to Reality · From Static to Growing

Aspect	**Classical Kakeya**	**PFUSRC Interpretation**
Core question	How small can the area be?	Has the ontology changed after rotation?
Needle	Infinitely thin	Structured, with phase
Space	Static container	Generated and grown by motion
Time	Reversible parameter	Irreversible, leaves traces
Rotation operation	Integrable	**Non-integrable** (produces residual)
Closure constant	π (geometric)	$\pi_1 = 12/11$ (dynamic)
Final state	Return to origin	Equivalent interface + entropy ring
Entropy	Not considered	Real increase / virtual decrease / phase dissipation
Retreat	Allowed, cost-free	Has a cost (larger entropy increase)
Rigidity	Not involved	Flexible breathing maintains rigid skeleton

Table 5: Unified paradigm shift from classical Kakeya to PFUSRC

12 Conclusion

The century-long history of the Kakeya problem is a journey from physics to abstraction and back to physics.

Within the PFUSRC axiomatic system, this paper has achieved the following reinterpretations:

1. Rotation is not reset — it is a topological homeomorphic mapping of the equivalent interface (360°) and local isomorphism plus absolute drift plus non-integrable residual (720°).
2. The rotation operation is non-integrable, which is the geometric origin of entropy increase.
3. Retreat is not entropy decrease but high-cost growth, manifested as phase dissipation and a larger global entropy increase.
4. Entropy increase does not refute the closed cosmos but proves its necessity: the second law of thermodynamics can only hold in a cosmos that is topologically closed and thermodynamically open.
5. $\pi_1 = 12/11$ is a testable proportional constant. This paper provides specific observational windows and quantitative experimental formulas.

Summary of core formulas:

1. $f(0) = 0'$ (equivalent interface mapping)
2. $\oint dF(\theta) = \varepsilon > 0$ (non-integrable rotation)
3. $\Delta S_b > \Delta S_f, \quad \Delta S_{\text{total}} > 2\Delta S_f$ (retreat cost)
4. $\Delta S = C \cdot \frac{12}{11} \cdot \Delta\phi + \Delta S_{\text{bg}}$ (entropy-phase relation)
5. $\pi_1 = \frac{12}{11}$ (derived from biconical geometry)

The three sets of quantitative predictions presented in this paper define the empirical boundary of the theory. Subsequent precision experimental results will serve as the core criterion for judging the validity of the theoretical relations concerning rotation, phase, and entropy increase.

Final statement:

The universe has no undo operation. Each rotation carves an ineradicable entropy trace into the background. Local virtual decrease is possible, but global entropy always increases. Phase is the cost, entropy is the bill, and rigidity is the condition under which structure is maintained.

A Geometry + Calculus of Variations Derivation of $\pi_1 = 12/11$

A.1 Geometric Boundary Partition of the Bicone

Within the PFUSRC framework, the fundamental structure of the cosmos is a 45° symmetric bicone. The spatial boundary is divided into two types:

1. **Flexible active boundary (conical surface)**: carries phase oscillations and flux exchange; partitioned into $N_1 = 12$ equivalent units.
2. **Rigid constraint boundary (base of the cone)**: locks the topological scale of space; partitioned into $N_2 = 11$ equivalent units.

The static geometric ratio is:

$$r = \frac{N_1}{N_2} = \frac{12}{11}$$

This ratio is an inherent geometric constraint of the bicone structure.

A.2 Variational Minimization of the Rotation Cost Functional

From the normalized cost functional in Section 3.3:

$$\mathcal{W}[\gamma] = \oint_{\gamma} \left(\frac{d\Phi}{\Phi} + \kappa \cdot d\theta \right)$$

Physical constraint: The universe spontaneously evolves toward minimal growth cost. Hence the steady state satisfies the variational extremum condition:

$$\delta\mathcal{W}[\gamma] = 0$$

Over a full rotation cycle, $\oint \frac{d\Phi}{\Phi} = 0$; the variational contribution of the flux term is zero. Therefore the extremum condition constrains only the angular coefficient κ.

Under the bicone boundary constraint $N_1/N_2 = 12/11$, the unique solution that minimizes energy and phase dissipation is:

$$\kappa = \frac{N_1}{N_2} = \frac{12}{11}$$

A.3 Physical Interpretation

1. π_1 is not the geometric constant π; it is a dynamic closure constant specific to the PFUSRC framework.
2. The 12 flexible boundary units correspond to the degrees of freedom for phase change during rotation; the 11 rigid boundary units correspond to the topological closure constraint of the cosmos.
3. Their ratio determines the phase-entropy conversion rate for a single rotation and provides the core quantitative link between the pure geometry of the Kakeya problem and thermodynamics.

A.4 Connection to the Main Text

This derivation yields $\kappa = 12/11$ from biconical geometry and the principle of least action, rather than as an empirical fit. Substituting into the entropy-phase relation:

$$\Delta S_{\text{total}} = C \cdot \frac{12}{11} \cdot \Delta\phi + \Delta S_{\text{bg}}$$

provides a complete theoretical foundation. C is a dimension-adapting constant to be calibrated experimentally.

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