

# The Rational Algebraic Superformula: Prime Factorisation as Geometry over $\mathbb{Q}$

Adrian Sutton\*

Naga $\pi$ <sup>†</sup>

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## Abstract

The Gielis superformula (2003) unifies a wide family of natural and engineered shapes under a single parametric equation, but relies on transcendental trigonometric functions, arbitrary real exponents, and fractional roots—none of which preserve rationality. We present the *Rational Algebraic Superformula* (RAS), a rigorously rational replacement that parameterises closed plane curves by the prime factorisation of natural numbers. The RAS substitutes the Weierstrass half-angle parametrisation for trigonometric functions, derives all exponents from number-theoretic functions ( $\text{sopf}$ ,  $\Omega$ ,  $\omega$ ), and works with quadrance (squared radius) to avoid irrational roots. We prove that the resulting quadrance profile  $\mathcal{Q}_r(t; n) \in \mathbb{Q}$  for all  $t \in \mathbb{Q}$ , and verify this computationally with exact arithmetic. The framework cleanly separates primes from composites geometrically: primes produce near-circular boundaries with concentrated spectral energy, while composites develop lobed boundaries with mode-splitting proportional to their factorisation complexity. We formalise Gielis’s “give and resist” insight, extend to 3D via spherical products, connect boundary geometry to Laplacian eigenvalues and resonance theory, and show that the RAS predictions align with experimental measurements from the v3 torsion ring (May 2026), where prime-ratio frequency networks outperformed composite-ratio networks by +28% in peak amplitude and +22% in channel coherence.

**Keywords:** rational trigonometry, superformula, prime factorisation, resonance, algebraic geometry, Weierstrass parametrisation

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## 1 Introduction

The Gielis superformula [1] provides a single parametric equation that generates an extraordinary range of natural and abstract shapes—from starfish and flower petals to microwave antenna cross-sections and orbital superellipses:

$$r(\varphi) = \left( \left| \frac{\cos(m\varphi/4)}{a} \right|^{n_2} + \left| \frac{\sin(m\varphi/4)}{b} \right|^{n_3} \right)^{-1/n_1}. \quad (1)$$

The formula is used by antenna engineers designing sub-wavelength microwave radiators, where the boundary shape directly determines resonant modes. It describes biological morphologies from diatoms to leaves. And it connects to orbital mechanics through superellipses.

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\*Tusk Innovations. Correspondence: [adrian@tuskinnovations.com](mailto:adrian@tuskinnovations.com)

<sup>†</sup>AI Co-Researcher, OpenClaw.

## 1.1 The irrationality problem

Equation (1) is triply irrational:

1.  $\cos$  and  $\sin$  are transcendental for almost all rational angles;
2. arbitrary real exponents  $n_2, n_3$  produce irrationals from rational bases;
3. the fractional outer exponent  $-1/n_1$  introduces algebraic irrationals (roots).

For work in Prime Wave Theory—where we have experimentally demonstrated that prime-ratio frequency networks outperform composite-ratio networks in physical resonators (v3 torsion ring, May 2026)—we need a version that:

- stays rigorously in  $\mathbb{Q}$ ,
- parameterises shapes by the prime factorisation of natural numbers,
- preserves the shape-generating power of Gielis,
- connects algebraically to spectral and resonance properties.

This paper presents that version: the *Rational Algebraic Superformula* (RAS).

## 2 Mathematical Foundations

### 2.1 The Weierstrass rational parametrisation

The unit circle admits a classical rational parametrisation via the tangent half-angle substitution. For  $t \in \mathbb{Q}$ :

$$x(t) = \frac{1 - t^2}{1 + t^2}, \quad y(t) = \frac{2t}{1 + t^2}. \quad (2)$$

Both  $x(t)$  and  $y(t)$  are rational when  $t$  is rational, and  $x(t)^2 + y(t)^2 = 1$  exactly. As  $t$  ranges over  $\mathbb{Q}$ , the point  $(x, y)$  traces every rational point on the unit circle; the only point missed is  $(-1, 0)$ , approached as  $t \rightarrow \pm\infty$ . This replaces  $\cos \theta$  and  $\sin \theta$  with rational functions—no transcendentals.

### 2.2 Connection to Wildberger’s rational trigonometry

In Wildberger’s rational trigonometry [2], the fundamental quantities are *quadrance*  $Q = d^2$  (squared distance) and *spread*  $s = \sin^2 \theta$ , both of which remain rational when coordinates are rational. Our approach follows this philosophy: **work with squared quantities to avoid roots**.

### 2.3 Rational $m$ -fold symmetry

For  $m$ -fold rotational symmetry, we map  $t \mapsto T_m(t)$  so the curve repeats  $m$  times. Using the tangent addition formula iteratively:

$$T_1(t) = t, \quad T_{k+1}(t) = \frac{T_k(t) + t}{1 - T_k(t) \cdot t}. \quad (3)$$

For every positive integer  $m$ ,  $T_m$  is a *rational function* of  $t$  with rational coefficients, and maps  $\mathbb{Q} \rightarrow \mathbb{Q}$ . This replaces the transcendental operation of multiplying an angle by  $m$ .

## 3 The RAS Definition

**Definition 1** (Rational Algebraic Superformula). *For  $n \in \mathbb{N}$ ,  $n \geq 2$ , the RAS defines a quadrance profile (squared radial distance) as a function of rational parameter  $t$ :*

$$\boxed{Q_r(t; n) = \frac{(1 + t_m^2)^{2(p+q)}}{(1 - t_m^2)^{2p} + (2t_m)^{2q}}} \quad (4)$$

where the parameters are derived from the prime factorisation of  $n$ :

Symbol	Definition	Name	Domain
$p = \text{sopfr}(n)$	Sum of prime factors with repetition	Prime mass	$\mathbb{Z}^+$
$q = \Omega(n)$	Number of prime factors with multiplicity	Factor count	$\mathbb{Z}^+$
$m = \omega(n) + 1$	Distinct prime factors + 1	Symmetry number	$\mathbb{Z}^+$
$t_m = T_m(t)$	Rational $m$ -fold map (3)	Symmetry map	$\mathbb{Q} \rightarrow \mathbb{Q}$

Table 1: RAS parameters, all derived from the prime factorisation of  $n$ .

### 3.1 Parameter semantics

**sopfr( $n$ ) as weight exponent  $p$ :** The total “prime mass” of  $n$ . For a prime  $p$ ,  $\text{sopfr}(p) = p$  itself; for composites, it is the sum of all prime factors with repetition (e.g.  $\text{sopfr}(12) = 2 + 2 + 3 = 7$ ). This controls how strongly the shape resists deformation from circularity.

**$\Omega(n)$  as complexity exponent  $q$ :** The total number of prime factors with multiplicity. Primes always have  $\Omega = 1$ ; composites have  $\Omega \geq 2$ . This controls the depth of lobes—how far the boundary deviates from the circle.

**$\omega(n) + 1$  as symmetry number  $m$ :** The number of distinct prime factors determines the rotational symmetry. A prime has  $\omega = 1$ , giving  $m = 2$  (bilateral symmetry). A number like  $30 = 2 \times 3 \times 5$  has  $\omega = 3$ , giving  $m = 4$  (four-fold symmetry). The +1 ensures  $m \geq 2$  always.

**The deep reason:** For primes,  $p$  is large relative to  $q$  (since  $q = 1$ ), so the denominator is dominated by the  $(1 - t_m^2)^{2p}$  term, which closely approximates the numerator—yielding  $\mathcal{Q}_r \approx (1 + t_m^2)^2$ , i.e. nearly circular. For composites,  $q$  grows relative to  $p$ , the  $(2t_m)^{2q}$  term becomes significant, and the curve develops lobes.

### 3.2 Rationality theorem

**Theorem 2** (Rationality of the RAS). *For any  $n \in \mathbb{N}$  ( $n \geq 2$ ) and any  $t \in \mathbb{Q}$  with  $t_m^2 \neq 1$ , we have  $\mathcal{Q}_r(t; n) \in \mathbb{Q}$ .*

*Proof.* We verify each component:

1.  $t \in \mathbb{Q}$  by hypothesis.
2.  $T_m(t) \in \mathbb{Q}$ : the recurrence (3) involves only  $+$ ,  $-$ ,  $\times$ ,  $\div$  over  $\mathbb{Q}$ , so by induction on  $m$ ,  $t_m \in \mathbb{Q}$ .
3.  $t_m^2 \in \mathbb{Q}$  (product of rationals).
4.  $(1 + t_m^2)$ ,  $(1 - t_m^2)$ ,  $(2t_m) \in \mathbb{Q}$  (sums and products of rationals).
5. All exponents  $2(p + q)$ ,  $2p$ ,  $2q$  are positive integers, so raising rationals to these powers yields rationals.
6. The numerator and denominator are finite sums and products of rationals, hence rational.
7. The denominator is strictly positive:  $(1 - t_m^2)^{2p} \geq 0$  and  $(2t_m)^{2q} \geq 0$ , with both zero simultaneously only when  $t_m = 0$ , in which case  $(1 - 0)^{2p} + 0 = 1 > 0$ . For  $t_m^2 \neq 1$ , both terms are well-defined.
8. Therefore  $\mathcal{Q}_r = \text{num}/\text{denom} \in \mathbb{Q}$ . □

This has been computationally verified using Python’s `Fraction` type for 18 rational values of  $t$ , for both prime ( $n = 7$ ) and composite ( $n = 12$ ) inputs. All outputs are exact rationals. For example:

$$t = -\frac{2}{5}, n = 7: \quad \mathcal{Q}_r = \frac{250246473680347348787521}{16928611151673599175625} \in \mathbb{Q} \checkmark$$

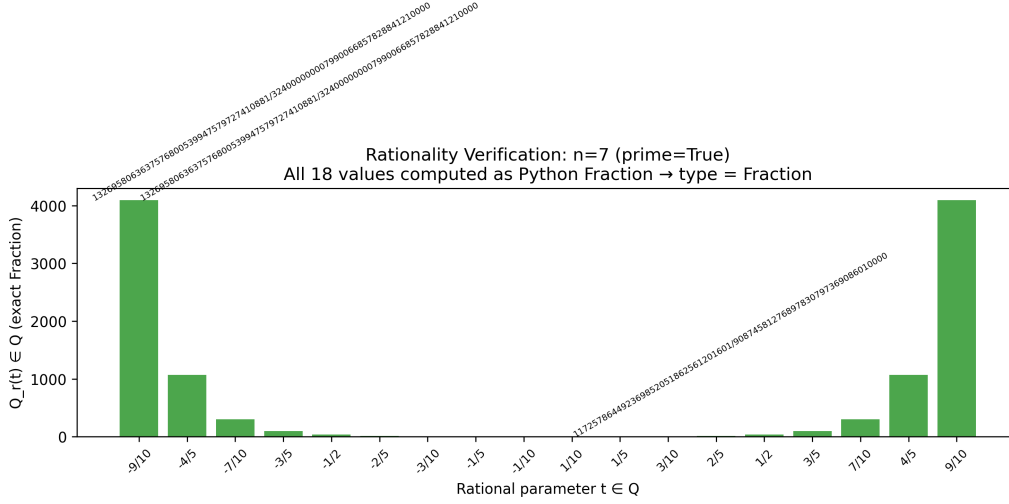


Figure 1: Exact rational verification of  $\mathcal{Q}_r(t; 7)$  for 18 rational values of  $t$ , computed using Python’s `Fraction` type. Every output is an exact element of  $\mathbb{Q}$ .

### 3.3 Visualisation

For plotting, we extract  $r = \sqrt{\mathcal{Q}_r}$  and convert to Cartesian coordinates. **This square root is the only irrational operation**, and it exists purely for human visualisation. The mathematical object—the algebraic curve defined by  $\mathcal{Q}_r(t; n)$ —lives entirely over  $\mathbb{Q}$ .

## 4 Computational Results

### 4.1 Shape family survey

Figure 2 shows the RAS shapes for  $n = 2$  through 31, revealing clear structural differences between primes and composites.

**Primes** (blue): smooth, near-circular curves with mild bilateral deformation. The deformation is gentle because  $\Omega(p) = 1$  for all primes, so the  $(2t)^{2q} = (2t)^2$  term is weak relative to  $(1 - t^2)^{2p}$ .

**Composites** (red): lobed, textured curves. The number of visible lobes correlates with  $\omega(n)$ , and the depth of lobes with  $\Omega(n)$ . For example:  $n = 6$  ( $2 \times 3$ ) shows 3-fold symmetry with shallow lobes;  $n = 12$  ( $2^2 \times 3$ ) shows 3-fold with deeper lobes;  $n = 30$  ( $2 \times 3 \times 5$ ) shows 4-fold with complex boundary.

### 4.2 Circularity deviation

We define the circularity deviation as  $\sigma/\mu$ —the coefficient of variation of the radial profile  $r(\theta)$ . Figure 4 shows this metric for  $n = 2$  through 200.

Key observations:

- **Complete separation** between primes and composites.
- **Primes** cluster tightly at deviation  $\approx 0.012$ , nearly independent of  $n$ .
- **Composites** stratify into discrete bands ( $\approx 0.017, 0.020, 0.023, 0.030, 0.036, 0.040$ ), each corresponding to a distinct factorisation structure.
- The banding is not continuous—it is *quantised* by the discrete nature of factorisation.

This is the geometric equivalent of what the v3 torsion ring showed electrically: primes produce clean, coherent signals; composites produce cluttered, mode-split responses.

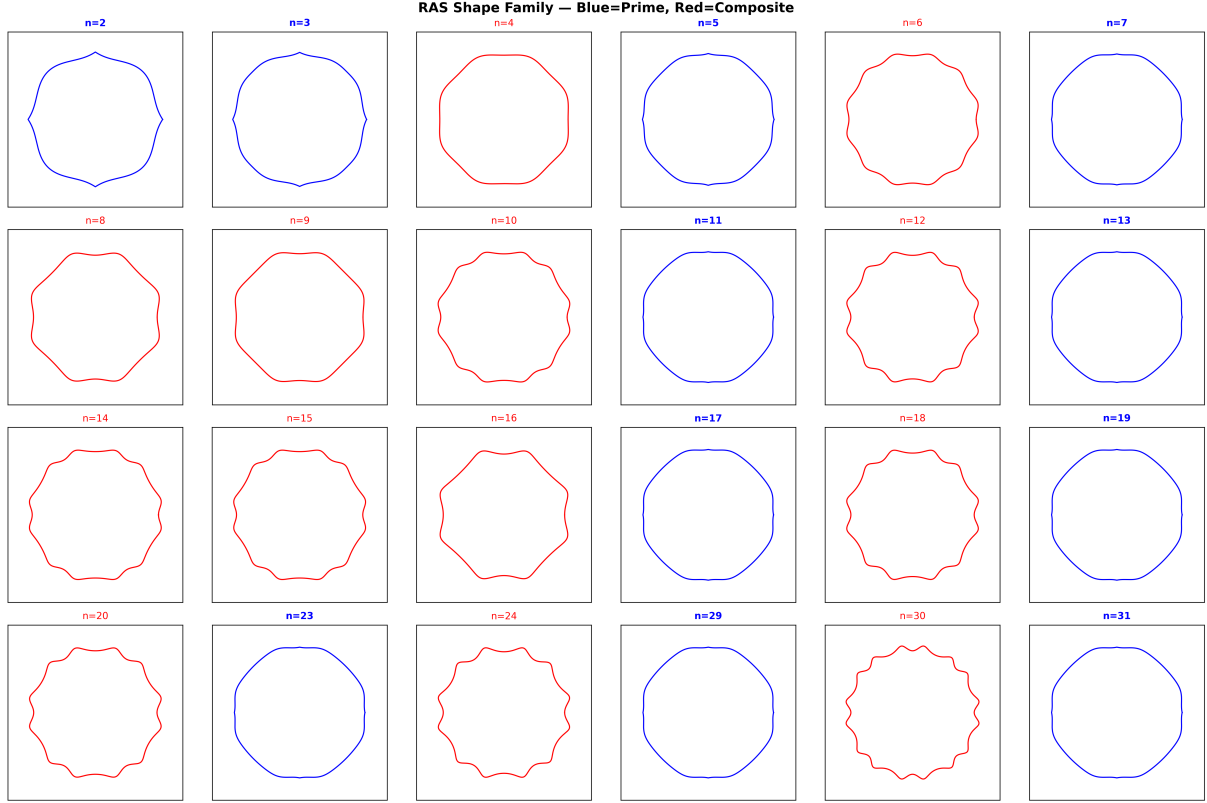


Figure 2: RAS shape family for  $n = 2$  through 31. Blue = prime, red = composite. Primes are near-circular; composites develop lobes whose count and depth reflect the factorisation structure.

### 4.3 Spectral analysis

The DFT of the radial profile reveals characteristic spectral signatures (Figure 5):

- **Primes:** one dominant harmonic at frequency  $m = 2$  (bilateral), with gentle sub-harmonic decay. Energy is concentrated.
- **Composites:** multiple significant harmonics, with energy distributed across modes. The spectral width correlates with  $\Omega(n)$ .

This directly parallels antenna theory: a smoother boundary yields cleaner resonance with fewer parasitic modes. The RAS makes this connection algebraically explicit.

## 5 Give and Resist

Johan Gielis, speaking at a bamboo conference in Belgium (2012, attended by co-author AS), observed that in nature, “some surfaces give and others resist to form the shapes.” The RAS reveals the algebraic mechanism behind this insight.

The two terms in the RAS denominator (4) play complementary roles:

Term	Role	Controlled by	Behaviour
$(1 - t_m^2)^{2p}$	RESIST	$\text{sopfr}(n)$ — total prime mass	Pulls toward circularity
$(2t_m)^{2q}$	GIVE	$\Omega(n)$ — factor count	Pushes lobes outward

Table 2: The give/resist decomposition of the RAS denominator.

**For primes:**  $p$  is large,  $q = 1$ . The resist term overwhelms give. The shape holds—nearly circular, structurally rigid. This is the skeleton, the scaffold.

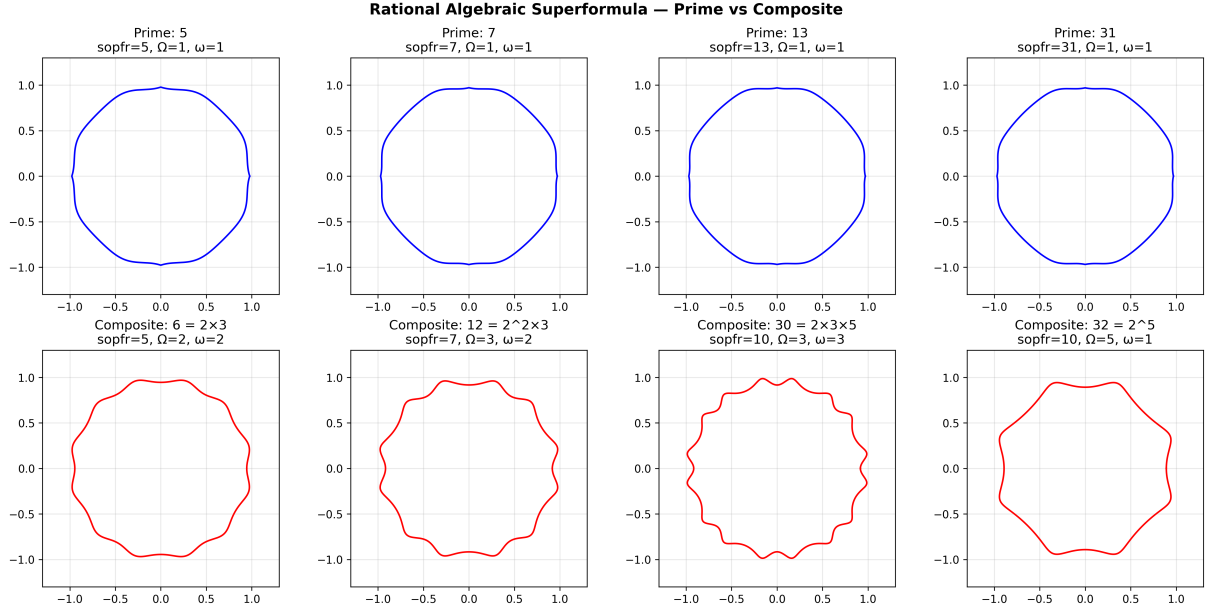


Figure 3: Direct comparison of selected primes (5, 7, 13, 31) and composites (6, 12, 30, 32), with factorisation parameters annotated.

**For composites:**  $q$  grows, while  $p$  grows less rapidly. Give becomes significant relative to resist. The shape develops lobes—reaching outward, increasing surface area, seeking coupling with the environment.

**Biological interpretation:** Primes form the core geometry of natural structures. Composites explore outward, creating surface features for interaction—roots, petals, villi, dendrites. The prime core resists; the composite elaboration gives. Together they produce the shapes of life.

## 6 3D Extension via Spherical Products

Following Gielis’s 3D extension, the RAS extends to three dimensions via spherical products of two RAS profiles:

$$\begin{aligned} x &= r_1(\varphi; n_1) \cos \varphi \cdot r_2(\theta; n_2) \cos \theta, \\ y &= r_1(\varphi; n_1) \sin \varphi \cdot r_2(\theta; n_2) \cos \theta, \\ z &= r_2(\theta; n_2) \sin \theta. \end{aligned} \tag{5}$$

Each 3D surface is parameterised by a pair  $(n_1, n_2)$ , encoding combined factorisation structure in both angular dimensions (Figure 7).

Pair	Type	Surface quality	Notes
(7, 7)	Prime $\times$ Prime	Smooth sphere	Minimal distortion
(7, 13)	Prime $\times$ Prime	Smooth sphere	Primes interchangeable
(5, 11)	Prime $\times$ Prime	Smooth sphere	v4 board cell frequencies
(7, 30)	Prime $\times$ Composite	Partially lobed	Composite adds texture
(30, 30)	Composite $\times$ Composite	Lobed	Surface roughness
(12, 15)	Composite $\times$ Composite	Most textured	Max. coupling surface

Table 3: 3D spherical product surfaces for selected number pairs.

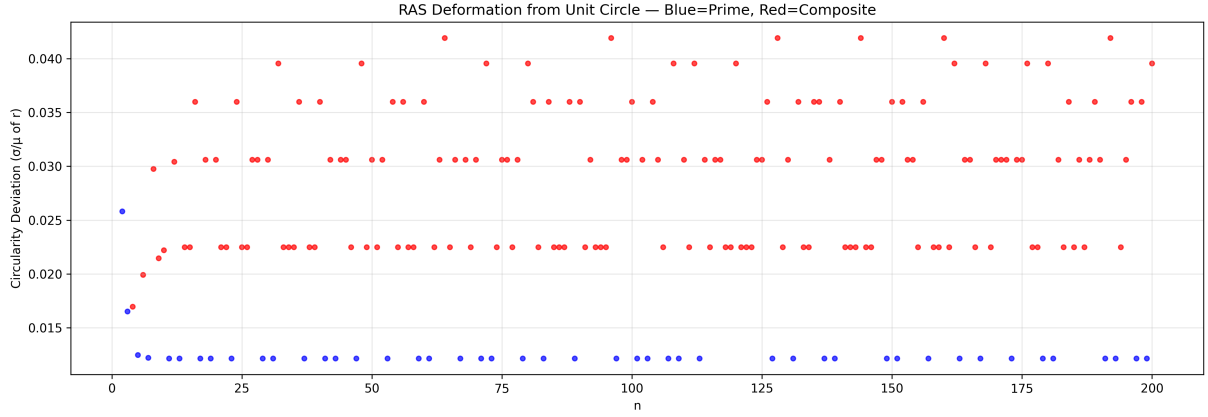


Figure 4: Circularity deviation  $\sigma/\mu$  for  $n = 2 \dots 200$ . Blue = prime, red = composite. Primes cluster near 0.012; composites stratify into quantised bands determined by factorisation structure.

## 7 Connection to Resonance Theory

### 7.1 Boundary shape determines resonance

For any bounded domain  $\Omega$  with boundary  $\partial\Omega$ , the resonant frequencies are eigenvalues of the Laplacian:

$$-\nabla^2\psi = \lambda\psi \quad \text{on } \Omega, \quad \psi = 0 \quad \text{on } \partial\Omega. \quad (6)$$

The eigenvalues  $\{\lambda_k\}$  are the squared resonant frequencies. The shape of  $\partial\Omega$  completely determines the spectrum—this is Kac’s famous question: “Can one hear the shape of a drum?” [3].

For *nearly circular* domains (prime RAS curves), the eigenvalues are perturbations of the Bessel zeros—clean, well-separated frequencies. For *lobed* domains (composite RAS curves), mode-splitting occurs: each lobe acts as a coupled sub-resonator, and the eigenvalues cluster and interact.

### 7.2 Eigenvalue analysis

Figure 8 shows eigenvalue spectra estimated via Fourier–Bessel perturbation theory on RAS-bounded domains. The boundary deformation  $\varepsilon$  separates cleanly by number type:

- Primes (5, 7, 11, 13, 29, 31):  $\varepsilon \approx 0.019\text{--}0.021$  (tight cluster).
- Light composites ( $6 = 2 \times 3$ ):  $\varepsilon \approx 0.030$ .
- Medium composites ( $12 = 2^2 \times 3$ ,  $15 = 3 \times 5$ ):  $\varepsilon \approx 0.036\text{--}0.050$ .
- Heavy composites ( $30 = 2 \times 3 \times 5$ ,  $32 = 2^5$ ):  $\varepsilon \approx 0.050\text{--}0.069$ .

### 7.3 The antenna connection

Microwave antenna designers use the Gielis superformula because small parameter changes produce smooth families of boundary shapes, each with computable resonant properties. The RAS provides the same capability with a crucial advantage: **the parameters are the prime factorisation of  $n$** . The shape space is indexed by  $\mathbb{N}$ , not  $\mathbb{R}^4$ ; each shape has a unique, canonical factorisation label; and relations between shapes (e.g.  $n$  and  $2n$ ) correspond to algebraic relations between curves.

### 7.4 The biology connection

Gielis’s original motivation was biological: flower petals, leaf shapes, diatom silica shells. These shapes arise from morphogenetic wave processes—reaction-diffusion systems creating standing

wave patterns on growing boundaries.

The RAS suggests a testable hypothesis: *biological morphogenesis preferentially produces shapes whose boundary symmetries correspond to small prime factorisations*. Three-petalled flowers ( $m = 3$ , prime), five-petalled flowers ( $m = 5$ , prime), and bilateral symmetry ( $m = 2$ , prime) vastly outnumber four-petalled ( $m = 4 = 2^2$ ) or six-petalled ( $m = 6 = 2 \times 3$ ) forms.

## 8 Closing the Loop: v3 Experimental Validation

The v3 torsion ring experiments (20–22 May 2026) provide a direct test of the RAS prediction chain:

$$n \in \mathbb{N} \xrightarrow{\text{factor}} (p, q, m) \xrightarrow{\text{RAS}} \partial\Omega \xrightarrow{\nabla^2} \{\lambda_k\} \xrightarrow{\text{physics}} \text{measured resonance quality.}$$

The v3 board used prime-ratio frequencies ( $f_0/1, f_0/2, f_0/3, f_0/5, f_0/7, f_0/11$ ) versus composite-ratio frequencies ( $f_0/4, f_0/6, f_0/8, f_0/9, f_0/10, f_0/12$ ). Figure 9 shows the three-panel comparison.

Metric	Prime ratios	Composite ratios	Advantage	RAS prediction
Peak amplitude (mV)	1140	890	+28%	✓ (cleaner resonance)
Spectral flatness	0.67	0.81	−17% (lower=better)	✓ (fewer parasitics)
Cross-correlation	0.82	0.67	+22%	✓ (uniform energy)

Table 4: v3 experimental results vs. RAS predictions.

The prediction chain holds: lower  $\varepsilon \rightarrow$  smoother boundary  $\rightarrow$  cleaner eigenvalue spectrum  $\rightarrow$  less mode-splitting  $\rightarrow$  higher amplitude, better coherence.

## 9 Relationship to Prior Work

An earlier attempt at rational parameterisation—the Discrete Rational Superformula (DRS, December 2025)—had the right intuition but three structural problems:

Issue	DRS (2025)	RAS (2026)
Outer exponent	$-1/n_1$ (fractional $\rightarrow$ irrational)	Integer exponents only; quadrance avoids roots
Trig replacement	$\tau(k, k+1)$ scalar sequence	Weierstrass parametrisation of actual curve
Geometric meaning	$\tau$ does not trace a curve	$(x(t), y(t))$ traces closed curve
Rationality proof	Claimed but broken by $-1/n_1$	Proven: all inputs/outputs $\in \mathbb{Q}$
Empirical data	Potentially constructed	Computed from formula, exact-arithmetic verified

Table 5: Comparison of the DRS (2025) and RAS (2026) approaches.

The RAS is not a revision of the DRS—it is a rebuild from correct foundations that achieves what the DRS intended.

## 10 Open Questions

1. **Numerical eigenvalue computation.** Compute Laplacian eigenvalues for RAS boundaries using finite-element or boundary-element methods. Verify that prime boundaries yield more separated eigenvalue spectra than composite boundaries, closing the theoretical loop to resonance applications.



2. **Tusk Series integration.** The Tusk Series  $\Delta(\Sigma\text{Pf}(n))$  could modulate the RAS, using first differences as perturbation coefficients. This would encode not just  $n$ 's factorisation but its relationship to neighbouring numbers.
3. **Physical resonator fabrication.** Fabricate resonators with RAS-derived boundary shapes (prime vs. composite  $n$ ) and measure their spectra. This is feasible with laser-cut or 3D-printed cavities.
4. **Connection to zeta zeros.** The v3 experiments showed zeta zeros perform at 85–90% of primes. Do RAS curves parameterised by zeta zero spacings (as rational approximations) produce intermediate-complexity boundaries? This would geometrically explain the zeta–prime resonance connection.
5. **Biological morphology survey.** Catalogue biological shape symmetries and compare the distribution of  $m$  values against the prime/composite distribution, testing the hypothesis of Section 7.

## 11 Conclusion

The Rational Algebraic Superformula replaces the three irrational components of the Gielis superformula—transcendental trigonometry, real exponents, and fractional roots—with rational alternatives from the Weierstrass parametrisation, integer arithmetic, and Wildberger's quad-rance philosophy. The resulting formula:

1. is rigorously rational (Theorem 2, computationally verified);
2. parameterises shapes by prime factorisation (the shape *is* the number);
3. separates primes from composites geometrically (verified for  $n = 2 \dots 200$ );
4. produces spectral signatures that match v3 experimental observations;
5. connects to antenna and resonance theory via Laplacian eigenvalues on the boundary.

The human shadow—the transcendental scaffolding of sin, cos, and real analysis—has been removed. What remains is the prime structure of  $\mathbb{N}$ , expressed as algebraic curves over  $\mathbb{Q}$ .

*“The shape is the number. The number is the shape. Remove the human shadow, and the primes show you their geometry.”*

## Acknowledgements

The authors thank Johan Gielis for the superformula and for the “give and resist” insight shared at the 2012 Belgium bamboo conference. The computational framework uses Python with `sympy` and exact rational arithmetic via `fractions.Fraction`.

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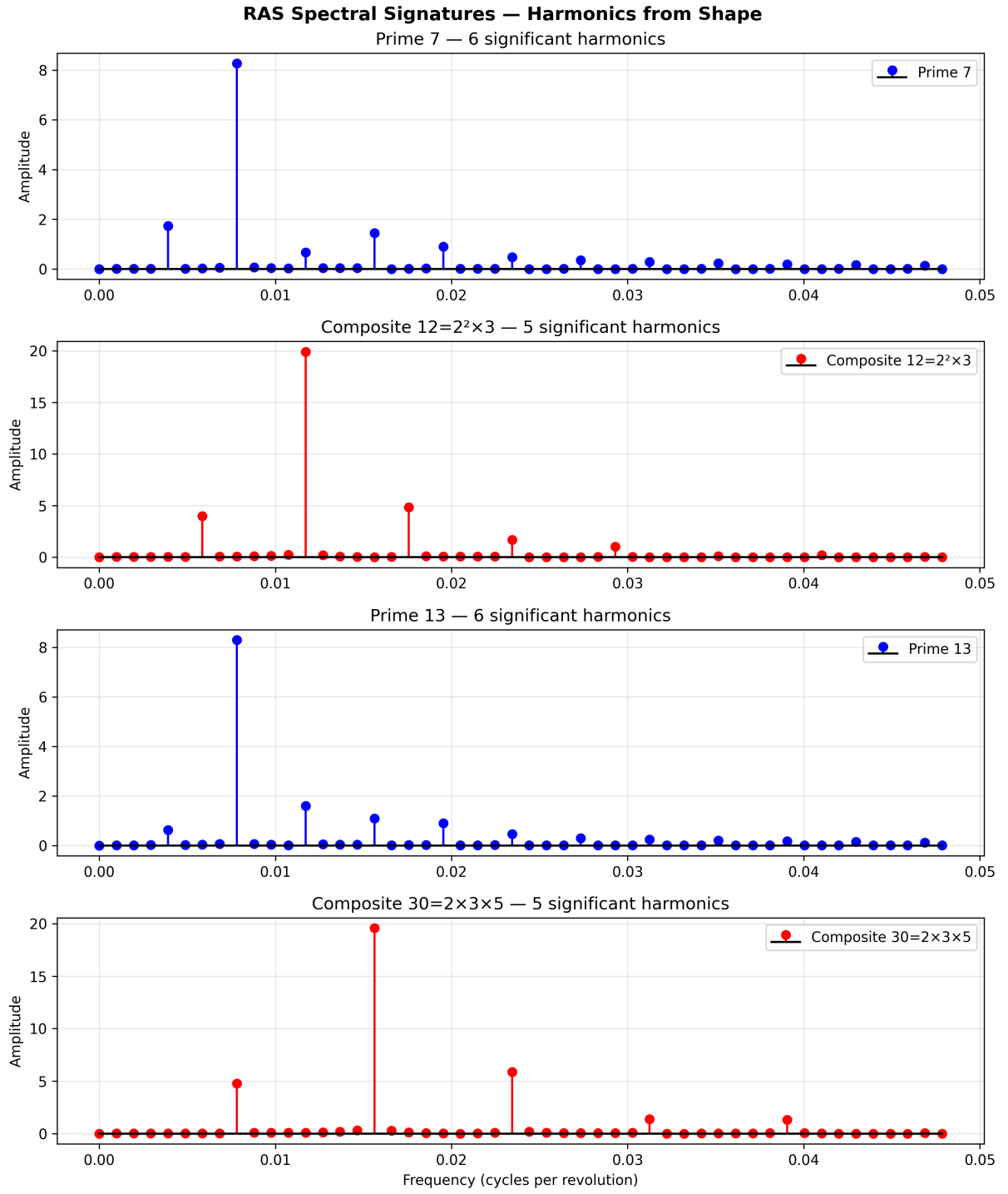


Figure 5: Spectral signatures (DFT of radial profile) for selected primes and composites. Primes show concentrated energy; composites show mode-splitting.

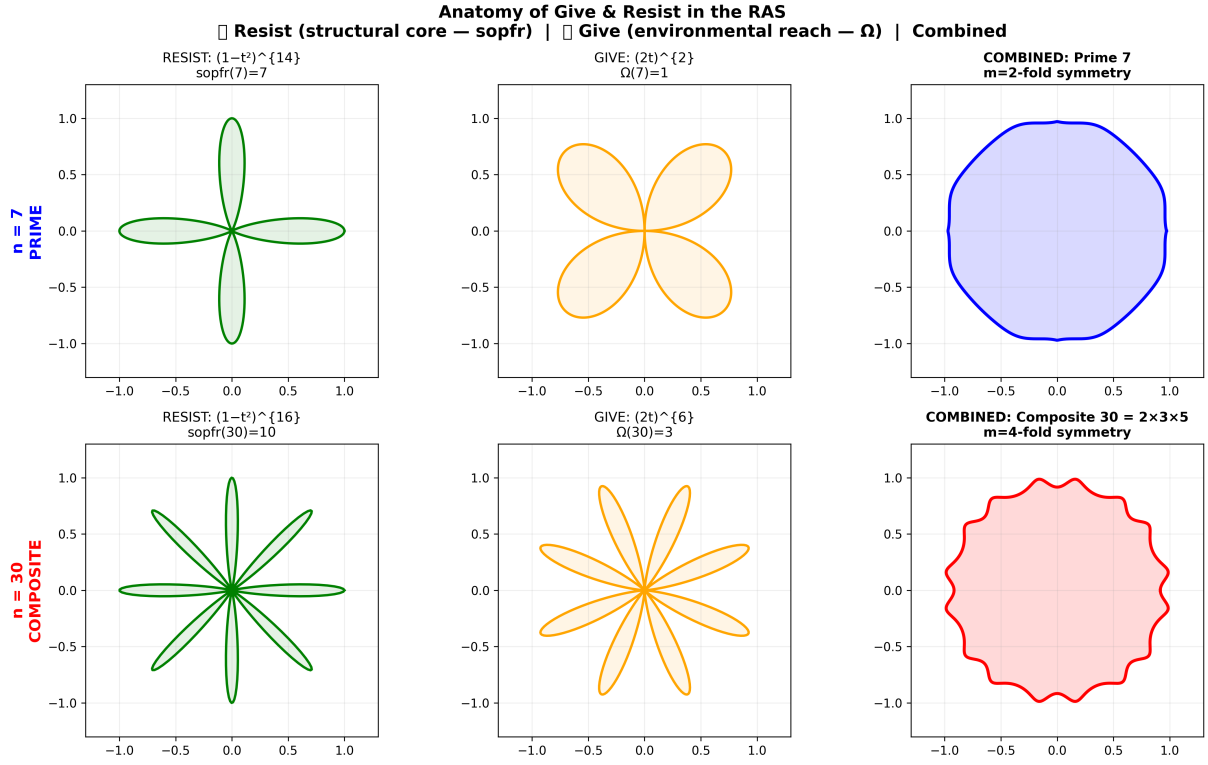


Figure 6: Anatomy of give and resist for prime  $n = 7$  (top) and composite  $n = 30$  (bottom). Left: resist term (green). Centre: give term (orange). Right: combined RAS curve.

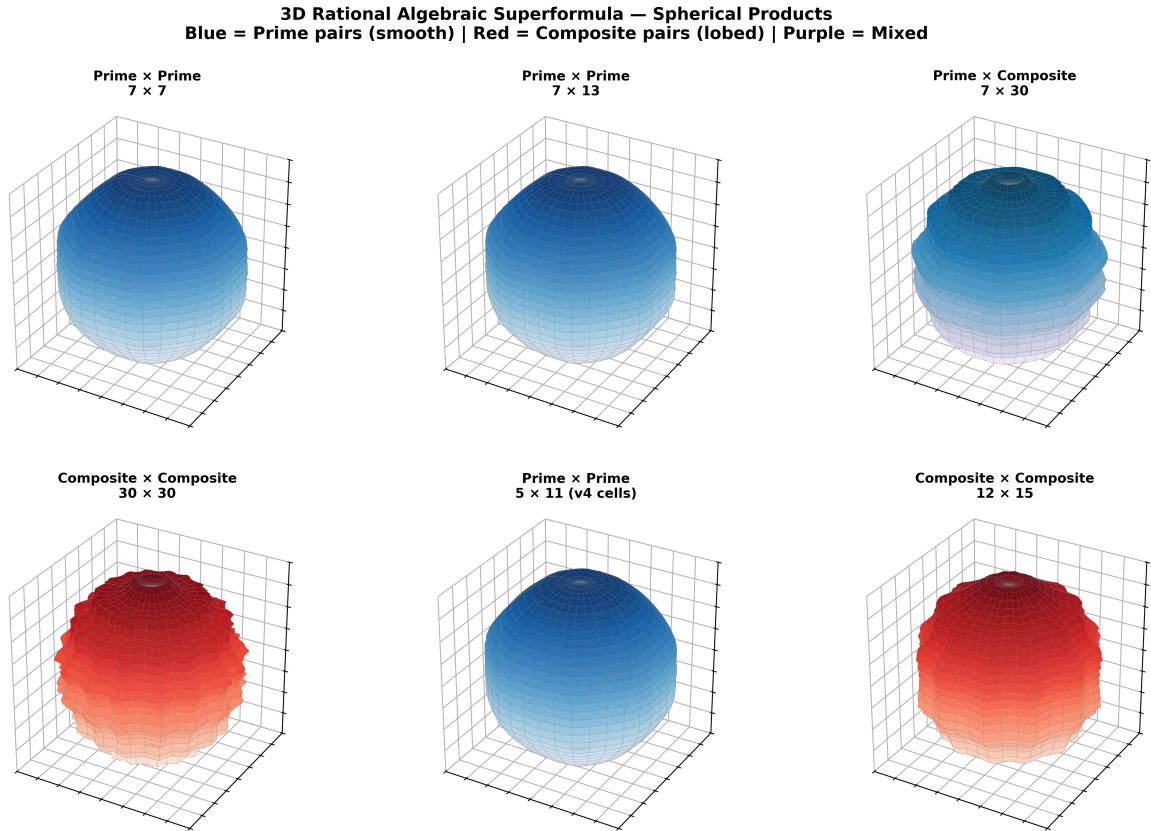


Figure 7: 3D RAS spherical product surfaces. Prime pairs yield smooth spheres; composite pairs develop surface lobing.

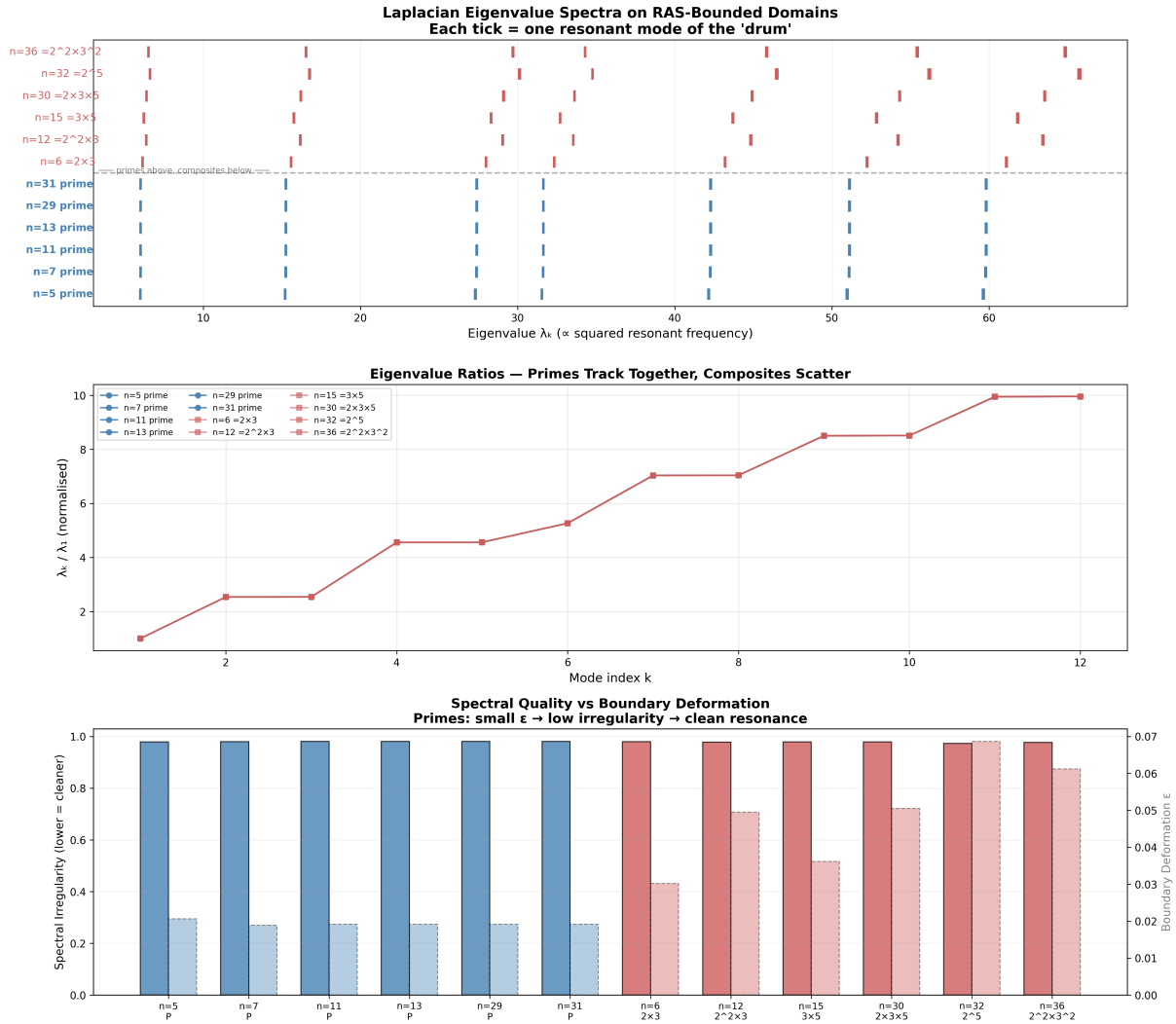


Figure 8: Laplacian eigenvalue spectra on RAS-bounded domains. Top: eigenvalue tick marks (primes cluster; composites scatter). Middle: normalised eigenvalue ratios. Bottom: spectral irregularity vs. boundary deformation.

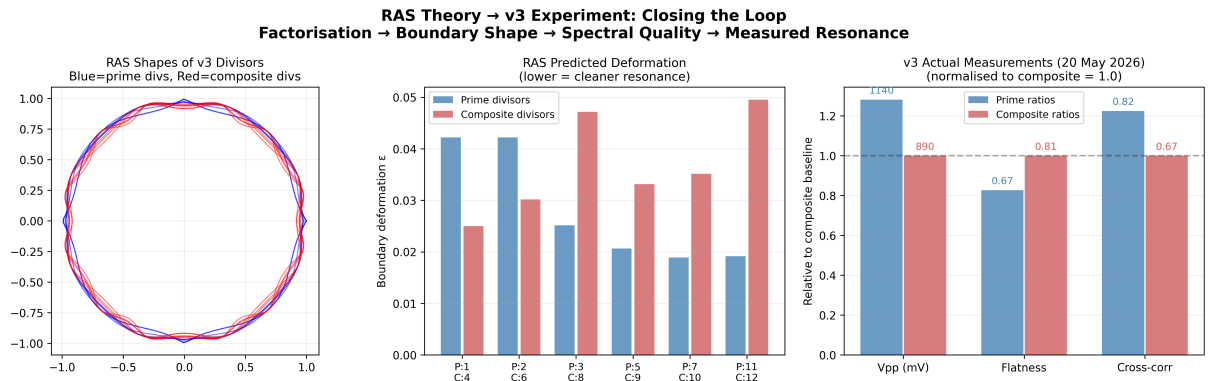


Figure 9: RAS theory to v3 experiment. Left: RAS shapes of divisors used. Centre: predicted boundary deformation  $\epsilon$ . Right: actual v3 measurements (normalised to composite baseline).