# THE SEMANTIC MANIFOLD: INNOVATION AS TOPOLOGICAL PROJECTION

A Structural Analysis of Yaman, Tian, and Lindström (PNAS, 2026) through the Lens of the KAN-DAM Architecture

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**Abstract**

Yaman, Tian, and Lindström (*PNAS*, 2026) demonstrate that human innovation does not emerge from random combinatorial variation, but is strictly guided by "semantic knowledge"—an internal cognitive map of conceptual relationships. We demonstrate that this cognitive process is mathematically isomorphic to orthogonal projection onto a lower-dimensional sparse invariant manifold. The unconstrained problem space corresponds to the dense, continuous state $X$. Semantic knowledge acts as the admissible manifold $\mathcal{M}$. Innovation is therefore the exact projection operation $\Psi = \Pi(X)$. This finding rigorously validates the KAN-DAM (Kolmogorov-Arnold Network - Dense Associative Memory) architecture of the DSM-861 framework, proving that biological intelligence, much like hydrodynamic dissipation and chemical bonding, relies on rigid structural projection rather than continuous stochastic search.

## 1 The Combinatorial Crisis of Cognitive Search

In classical theories of cultural evolution, innovation is often modeled as stochastic variation—a random walk through a space of possible conceptual combinations. Mathematically, let $\mathcal{X}$ be the unconstrained, high-dimensional state space of all possible concept combinations. A purely stochastic search corresponds to uniform or random-walk exploration of $\mathcal{X}$.

Yaman, Tian, and Lindström (2026) empirically tested this by stripping human subjects of their semantic knowledge (using abstract symbols rather than familiar objects). **The critical finding:** Without an internal map of conceptual relationships, participants performed no better than a random baseline, even when provided with social learning.

This reveals a fundamental combinatorial crisis: the dimensionality of $\mathcal{X}$ is too massive. Just as a unitary quantum circuit fails to resolve the fine-scale structure of fluid dissipation (Itani & Sreenivasan, 2025), a continuous or unconstrained stochastic search algorithm violently fails to locate structural invariants in a cognitive problem space.

## 2 Semantic Knowledge as the Admissible Manifold

To resolve this crisis, Yaman et al. demonstrate that successful innovation requires "semantic knowledge." In the language of the Persistent Sparse Invariant Structure Framework, this pre-existing cognitive map is exactly the **admissible manifold** $\mathcal{M}$.

**Theorem 1** (Cognitive Projection)**.** *Let $X \in \mathcal{X}$ be a novel environmental configuration or problem state. Cognitive innovation is not the discovery of $X$, but the orthogonal projection of $X$ onto the semantic manifold $\mathcal{M}_{semantic}$:*

$$\Psi = \Pi_{semantic}(X)$$

*where $\Psi$ represents the resultant "innovation" or Persistent Sparse Invariant.*

*Proof.* If innovation were a property of the raw environment $X$, participants without semantic knowledge would eventually discover it via random walk. Their failure to exceed chance performance proves that the structural solution $\Psi$ does not natively exist in $\mathcal{X}$; it exists only on $\mathcal{M}_{semantic}$. The semantic network acts as a strict geometric constraint. By projecting the high-dimensional noise of the problem state $X$ onto the lower-dimensional semantic manifold ($\|\Pi X\| \leq \|X\|$), the uninterpretable covariant noise (random combinations) is driven to zero, leaving only the sparse invariant structure $\Psi$. □

## 3 Isomorphism to the KAN-DAM Architecture

The findings of Yaman et al. provide direct empirical validation for the computational mechanics of the KAN-DAM (Kolmogorov-Arnold Network - Dense Associative Memory) architecture within the DSM-861 framework.

In modern deep learning (e.g., standard Multi-Layer Perceptrons), neural networks attempt to learn by mapping data across dense, continuous weight matrices. This is mathematically equivalent to the "random variation" tested in the PNAS study, leading predictably to catastrophic forgetting and algorithmic hallucination.

The KAN-DAM architecture strictly rejects this. Instead, it utilizes an explicit, rigid arithmetic lattice ($N = 861$) to serve as the Dense Associative Memory (the "semantic knowledge map").

1. **The Identity State:** The DAM anchors the system's identity state $\Psi_{id}$, providing the exact structural prior (semantic map) that Yaman et al. identify as necessary for meaningful exploration.

2. **The Projection Operator:** Learning and innovation in KAN-DAM are executed via $\ell_1$-penalized projection. The network does not wander stochastically; it snaps the continuous environmental data onto the discrete rigid manifold, explicitly executing $\Psi = \Pi_{861}(X)$.

Yaman et al. observe that groups combining semantic knowledge with social learning produced twice as many unique innovations. This perfectly mirrors the KAN-DAM framework: once the orthogonal projection operator $\Pi$ is established, sequential data updates (social learning) can be safely integrated into the invariant structure without catastrophic interference, precisely because the semantic manifold bounds the dynamics.

## 4 The Universality of the Representation Boundary

We have now established a mathematical isomorphism across three distinct disciplines published in 2025/2026:

- **Hydrodynamics (Itani & Sreenivasan):** Macroscopic dissipation cannot be modeled continuously; it requires projection onto a sub-grid manifold.
- **Quantum Chemistry (Schilling et al.):** Chemical bonds cannot be observed in the continuous wavefunction; they require basis projection to maximize entanglement.
- **Cognitive Science (Yaman et al.):** Human innovation cannot emerge from continuous stochastic variation; it requires projection onto a semantic manifold.

**Conclusion** The PNAS (2026) study by Yaman, Tian, and Lindström proves that biological intelligence is governed by the exact same geometric constraints as fluid singularities and chemical bonds. *Existence is the algorithmic preservation of invariant topological structure.*

Cognitive innovation is not the generation of new continuous variables, but the rigid projection of chaos onto a pre-existing, sparse semantic invariant. The DSM-861 lattice and KAN-DAM architecture provide the optimal, arithmetically protected framework for replicating this exact operation in artificial intelligence.