Quantum Future-Trial Computing as a Geometric Primitive Resource Model

Gate-Reducible Equivalence, Hamiltonian Rollback, and Resource-Separation Hypotheses

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Abstract

Quantum Future-Trial Computing (QFTC) was originally introduced as a reversible, geometry-controlled computational framework in which candidate futures are explored coherently, selectively phase-marked, and rolled back without measurement-induced collapse. A central ambiguity in such a framework is its relationship to standard quantum complexity theory. If every component of a QFTC macro-step, including the branch-conditioned trial unitaries, predicate circuit, phase-commit operation, and rollback transformation, is uniformly decomposable into polynomial-size quantum circuits, then the entire macro-step is itself an ordinary polynomial-depth quantum circuit. Under this gate-reducible assumption, QFTC cannot be claimed as a strict extension beyond BQP.

This paper reformulates QFTC by separating two regimes: $\mathsf{QFTC}_{\text{gate}}$, the gate-reducible regime, and $\mathsf{QFTC}_{\text{geom}}$, the geometric-primitive physical resource regime. In the first regime, we prove that $\mathsf{QFTC}_{\text{gate}} = \mathsf{BQP}$ under standard uniformity and bounded-error assumptions. This result absorbs the strongest complexity-theoretic criticism of QFTC rather than evading it. In the second regime, QFTC is not treated as an automatically stronger logical complexity class, but as a physical resource model in which reversible Hamiltonian rollback, Fubini–Study geometric action, phase-memory retention, decoherence leakage, and rollback capacity are explicit resources.

The resulting theory shifts the main question from "Does QFTC unconditionally exceed BQP?" to "Can physically primitive geometric rollback implement effective branch operators at lower action, coherence, or control cost than their compiled gate-circuit simulations?" We define a resource-separation hypothesis between standard gate depth and geometric rollback control, introduce operational cost measures, formulate stability bounds under open-system dynamics, and identify falsifiable experimental signatures. This reformulation preserves the conceptual core of QFTC while making its complexity-theoretic status precise, conservative, and testable.

1 Introduction

Standard quantum computation is usually formulated as a sequence of unitary gates followed by measurement. A computation begins with an initial state $|\psi_0\rangle$, evolves through a circuit

$$|\psi_T\rangle = U_T U_{T-1} \cdots U_1 |\psi_0\rangle, \tag{1}$$

and ends with a readout in a chosen basis. Time, in this picture, is represented primarily by circuit depth.

Quantum mechanics itself, however, is reversible at the level of closed-system unitary dynamics. For an ideal unitary U, one has

$$U^\dagger U = I. \tag{2}$$

In ordinary algorithmic descriptions, this reversibility is used implicitly in techniques such as uncomputation, phase kickback, amplitude amplification, and coherent oracle construction.

QFTC begins from the premise that this reversibility can be elevated from a passive background property into an explicit computational organizing principle.

The original QFTC intuition is simple: explore multiple future computational branches coherently, mark their validity without measurement, roll back the work register, and retain a branch-level phase imprint that can be converted into interference. In ideal notation, one prepares

$$|\Psi_0\rangle = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} |k\rangle_B \, |\psi_0\rangle_W \,, \tag{3}$$

applies branch-conditioned trial unitaries,

$$|\Psi_1\rangle = \frac{1}{\sqrt{N}} \sum_{k=1}^{N} |k\rangle_B \, U_k \, |\psi_0\rangle_W \,, \tag{4}$$

performs a coherent phase-commit operation, and then applies rollback $R_k = U_k^\dagger$. In the ideal limit, the work register returns to $|\psi_0\rangle$, while the branch register retains relative phase information.

This picture is attractive, but it creates a serious complexity-theoretic question. If all of the operations just described are ordinary polynomial-size quantum circuits, then the entire QFTC macro-step is also an ordinary polynomial-size quantum circuit. In that case, QFTC cannot be treated as a strict computational class beyond BQP merely because it uses rollback language. This criticism is not peripheral. It is the central consistency constraint that any mature QFTC theory must face.

The purpose of this paper is to rebuild QFTC around that constraint.

We separate QFTC into two regimes:

(i) **Gate-reducible QFTC**, denoted $\mathsf{QFTC}_{\mathrm{gate}}$, where all QFTC components are uniformly decomposable into ordinary polynomial-size quantum circuits.

(ii) **Geometric-primitive QFTC**, denoted $\mathsf{QFTC}_{\mathrm{geom}}$, where reversible Hamiltonian rollback and geometric evolution are treated as physical primitive resources whose cost is not measured only by circuit depth.

The first regime gives a clean result:

$$\mathsf{QFTC}_{\mathrm{gate}} = \mathsf{BQP}. \tag{5}$$

This equality is not a defeat. It is a necessary normalization. It clarifies that QFTC does not become stronger than standard quantum computation simply by describing a circuit as "future trial, commit, and rollback."

The second regime is where the genuine QFTC question resides. In $\mathsf{QFTC}_{\mathrm{geom}}$, the relevant cost is not only the number of elementary gates, but also geometric action, Hamiltonian reversibility, decoherence leakage, control precision, predicate construction cost, and rollback capacity. The central conjecture becomes a resource-separation conjecture, not an immediate complexity-class separation:

> There may exist physically implementable geometric rollback primitives whose effective branch transformations require substantially larger gate depth, action, or coherence cost when compiled into a standard gate model.

This is a more conservative and more powerful position. It avoids claiming an unproven separation from BQP, while preserving the possibility that geometric rollback control may define a distinct physical resource structure.

2 Motivation and Conceptual Reframing

2.1 The Original QFTC Intuition

The motivating idea behind QFTC is that quantum reversibility allows a computational system to explore a candidate future without irreversibly committing to it. A future trial is not a classical simulation of a possible outcome; it is a coherent unitary excursion in Hilbert space. Rollback is not time travel; it is controlled inversion of the trial evolution.

Let $\mathcal{H}_B$ be a branch register and $\mathcal{H}_W$ be a work register. A branch-conditioned future-trial unitary has the form

$$U = \sum_{k=1}^{N} |k\rangle\langle k|_B \otimes U_k. \tag{6}$$

A rollback unitary is ideally

$$R = \sum_{k=1}^{N} |k\rangle\langle k|_B \otimes U_k^\dagger. \tag{7}$$

Thus

$$RU = I \tag{8}$$

on the branch-work space.

A coherent predicate circuit V evaluates whether a trial branch satisfies a specified condition:

$$V : |k\rangle_B |\psi\rangle_W |0\rangle_A \mapsto |k\rangle_B |\psi\rangle_W |g(k,\psi)\rangle_A , \tag{9}$$

where $g(k,\psi) \in \{0,1\}$ is a reversible predicate. A phase-commit operator may then be implemented by phase kickback:

$$S(\phi) = V^\dagger \left[I_{BW} \otimes \Big(|1\rangle\langle 1| + e^{i\phi} |0\rangle\langle 0| \Big) \right] V. \tag{10}$$

This operator is unitary and measurement-free. It does not select a good branch by postselection; it imposes a relative phase between predicate sectors.

The QFTC macro-step is then

$$\mathcal{T} = R\, S(\phi)\, U. \tag{11}$$

2.2 The Critical Objection

The critical objection is immediate. If U, $S(\phi)$, and R are all polynomial-size quantum circuits, then $\mathcal{T}$ is also a polynomial-size quantum circuit. A polynomial number of such macro-steps remains a polynomial-size quantum circuit. Therefore, within the standard uniform circuit model, QFTC is contained in BQP.

Furthermore, any ordinary BQP circuit can be regarded as a degenerate QFTC circuit with no nontrivial rollback and $S(\phi) = I$. Therefore, under gate reducibility, BQP is contained in QFTC. The natural conclusion is equality.

This criticism is correct. A strong QFTC theory must not try to bypass it by hiding complexity inside $S(\phi)$ or Π_{good}. If the predicate projector secretly contains the answer, the model collapses into an oracle with the solution encoded in it. If the rollback operator is merely the inverse of a known polynomial circuit, it remains within ordinary circuit complexity.

Therefore, the theory must distinguish between two different claims:

(a) QFTC as a logical gate model exceeds BQP.

(b) QFTC as a physical resource model may possess cost structures not captured by gate depth alone.

The first claim is not justified under standard assumptions. The second claim is the defensible and scientifically interesting version.

2.3 From Complexity-Class Supremacy to Resource Separation

The revised QFTC thesis is not:

$$\mathsf{QFTC} > \mathsf{BQP}. \tag{12}$$

The revised thesis is:

$$\mathsf{QFTC}_{\text{gate}} = \mathsf{BQP}, \tag{13}$$

while

$$\mathsf{QFTC}_{\text{geom}} \tag{14}$$

defines a physical resource model whose advantage depends on whether geometric rollback primitives can realize certain effective branch transformations more efficiently than their standard gate decompositions.

In other words, QFTC should be compared to standard quantum computation not only by language recognition classes, but by resource measures such as:

- elementary gate depth,
- Fubini–Study geometric action,
- Hamiltonian control action,
- rollback fidelity,
- decoherence leakage,
- predicate construction cost,
- calibration overhead,
- physical reversibility window.

This distinction is analogous in spirit to the difference between an abstract circuit class and a physical architecture. Two architectures may compute the same class of languages in polynomial time while having radically different resource profiles for physically relevant problem families.

3 Geometric Time and State Displacement

3.1 Fubini–Study Computational Time

QFTC treats computational time as geometric displacement in projective Hilbert space. For normalized pure states $|\psi\rangle$ and $|\phi\rangle$, the Fubini–Study angle is

$$\theta_{\text{FS}}(\psi, \phi) = \arccos |\langle\psi|\phi\rangle|. \tag{15}$$

The QFTC computational time differential is defined as

$$dt_{\text{QFTC}} = \alpha\, d\theta_{\text{FS}}, \tag{16}$$

where $\alpha > 0$ is a scale factor converting geometric displacement into computational time units.

This does not imply that external laboratory time is identical to Fubini–Study distance. Rather, it defines an internal computational clock associated with distinguishable state motion. In this interpretation:

$$d\theta_{\text{FS}} > 0 \quad \text{forward trial}, \tag{17}$$

$$d\theta_{\text{FS}} < 0 \quad \text{rollback direction}, \tag{18}$$

$$d\theta_{\text{FS}} = 0 \quad \text{geometric freeze}. \tag{19}$$

A rollback is not a reversal of the external universe. It is a controlled inverse trajectory of the computational state.

3.2 Hamiltonian Realization of Rollback

Let a branch trial unitary be generated by a branch-dependent Hamiltonian:

$$U_k(T) = \mathcal{T} \exp\left(-i \int_0^T H_k(t)\, dt\right). \tag{20}$$

An ideal rollback may be realized by the time-reversed Hamiltonian

$$H_k^{\text{rev}}(t) = -H_k(T - t), \tag{21}$$

which generates

$$R_k(T) = U_k(T)^\dagger. \tag{22}$$

This representation is crucial for $\mathsf{QFTC}_{\text{geom}}$. In the gate-reducible regime, R_k is simply the inverse circuit. In the geometric-primitive regime, R_k is a physically implemented Hamiltonian reversal primitive whose cost is described by action, calibration precision, and decoherence constraints.

Define the Hamiltonian control action for branch k:

$$A_k = \int_0^T \|H_k(t)\|\, dt. \tag{23}$$

For m macro-cycles, define total control action:

$$A_{\text{total}} = \sum_{i=1}^{m} A_{k_i}. \tag{24}$$

A QFTC advantage is physically meaningful only if any reduction in circuit depth or query count is not offset by an uncontrolled growth in A_{total} or calibration overhead.

3.3 Geometric Freeze

A geometric freeze is a regime in which the computational state experiences negligible projective displacement:

$$d\theta_{\text{FS}} \approx 0. \tag{25}$$

In a real open system, this requires both suppressed coherent evolution and bounded environmental leakage. If $H_{\text{eff}}(t)$ is the effective controlled Hamiltonian, then an approximate freeze requires

$$\int_0^T \|H_{\text{eff}}(t)\| dt \ll 1, \tag{26}$$

together with a small decoherence functional δ defined later. Thus QFTC does not claim literal suspension of physical time. It defines a controllable low-displacement region in computational state space.

4 The Gate-Reducible Regime

4.1 Uniform Circuit Assumption

We first define the conservative regime in which QFTC components are ordinary quantum circuits.

Definition 1 (Gate-reducible QFTC component)**.** *A QFTC component is gate-reducible if it is generated by a uniform family of quantum circuits of size* $\text{poly}(n)$ *over a fixed universal gate set, where* n *is the input length.*

Definition 2 (Gate-reducible QFTC machine)**.** *A gate-reducible QFTC machine is a tuple*

$$Q_{\text{gate}} = (\mathcal{H}_B, \mathcal{H}_W, \mathcal{H}_A, |\psi_0\rangle, U, V, S(\phi), R, M) \tag{27}$$

such that:

(i) U is a branch-conditioned polynomial-size quantum circuit,

(ii) V is a reversible predicate circuit of polynomial size,

(iii) $S(\phi)$ is implemented by polynomial-size phase kickback through V,

(iv) R is a polynomial-size rollback circuit,

(v) M is a final measurement,

(vi) the machine uses polynomially many macro-steps.

The class of languages decided with bounded error by such machines is denoted $\mathsf{QFTC}_{\text{gate}}$.

This definition intentionally prevents hidden computational power from being inserted into Π_{good} or R. If V is allowed to be arbitrary, the model becomes uninformative. If R is allowed as a black-box inverse of exponential structure, the model is no longer a standard circuit model.

4.2 QFTC Macro-Step Compilation

Let

$$U = \sum_k |k\rangle\langle k|_B \otimes U_k \tag{28}$$

be the branch-conditioned trial operation, and let

$$R = \sum_k |k\rangle\langle k|_B \otimes U_k^\dagger \tag{29}$$

be the rollback. The commit operation is

$$S(\phi) = V^\dagger \left[I_{BW} \otimes \Big(|1\rangle\langle 1| + e^{i\phi} |0\rangle\langle 0| \Big) \right] V. \tag{30}$$

Then a macro-step is

$$\mathcal{T} = RS(\phi)U. \tag{31}$$

If U, V, the controlled phase, and R are each polynomial-size circuits, their composition is polynomial-size. Therefore, no new language-recognition power is obtained solely by grouping them into a macro-step.

Theorem 1 (Gate-reducible containment)**.** *Under the uniform polynomial-size circuit assumption,*

$$\mathsf{QFTC}_{\text{gate}} \subseteq \mathsf{BQP}. \tag{32}$$

Proof. Let $L \in \mathsf{QFTC}_{\text{gate}}$. By definition, there exists a uniform family of QFTC machines deciding L with bounded error using polynomially many macro-steps. Each macro-step consists of a composition of gate-reducible circuits U, V, a controlled phase operation, $V^\dagger$, and R. Therefore each macro-step has polynomial circuit size. A polynomial number of polynomial-size macro-steps yields an ordinary polynomial-size quantum circuit. The final measurement is a standard quantum measurement with bounded error. Hence $L \in \mathsf{BQP}$. □

Theorem 2 (BQP embedding)**.**

$$\mathsf{BQP} \subseteq \mathsf{QFTC}_{\text{gate}}. \tag{33}$$

Proof. Let $L \in \mathsf{BQP}$. Then there exists a uniform polynomial-size quantum circuit family $\{C_n\}$ deciding L with bounded error. Construct a QFTC machine with a trivial branch register, set $U = C_n$, set $S(\phi) = I$, and either omit rollback or set $R = I$ after the final computational evolution. This is a valid gate-reducible QFTC machine. Therefore $L \in \mathsf{QFTC}_{\text{gate}}$. □

Theorem 3 (Gate-reducible equivalence)**.** *Under the standard uniform circuit model,*

$$\mathsf{QFTC}_{\text{gate}} = \mathsf{BQP}. \tag{34}$$

Proof. This follows immediately from the two previous theorems. □

Remark 1. *This theorem is a normalization result. It states that QFTC language does not by itself generate computational power beyond* BQP. *Any claim of advantage must identify a resource not already counted as an ordinary polynomial-size gate sequence.*

5 Avoiding the Hidden Oracle Trap

5.1 Predicate Construction

The strongest failure mode of QFTC is hiding the answer inside the commit operator. To prevent this, the good-branch projector must be constructible from an explicit reversible predicate circuit.

Let V act as

$$V : |k\rangle_B |\psi\rangle_W |0\rangle_A \mapsto |k\rangle_B |\psi\rangle_W |g(k,\psi)\rangle_A . \tag{35}$$

Define

$$\Pi_1^A = |1\rangle\langle 1|_A . \tag{36}$$

Then the good projector is

$$\Pi_{\text{good}} = V^\dagger \left(I_{BW} \otimes \Pi_1^A \right) V. \tag{37}$$

The bad projector is

$$\Pi_{\text{bad}} = I - \Pi_{\text{good}}. \tag{38}$$

The commit operator is

$$S(\phi) = \Pi_{\text{good}} + e^{i\phi}\Pi_{\text{bad}}. \tag{39}$$

5.2 Nontriviality Constraint

For a QFTC advantage to be meaningful, the predicate circuit must not encode the solution in a form as hard as the original search. We impose:

Assumption 1 (Predicate nontriviality)**.** *The predicate circuit V satisfies*

$$\text{Cost}(V) \leq \text{poly}(n, \log N), \tag{40}$$

where n is the input length and N is the number of candidate branches.

This assumption does not guarantee advantage. It only prevents a false advantage caused by inserting an exponentially hard predicate into $S(\phi)$.

5.3 Commit Is Not Postselection

The commit operator is unitary. It does not condition on a measurement outcome. Therefore it should not be identified with postselection.

A postselected computation may discard all branches except those satisfying a rare event. QFTC does not do this. Instead, it applies a relative phase:

$$|\Psi\rangle \mapsto S(\phi)\,|\Psi\rangle\,. \tag{41}$$

All branches remain present. Any probability advantage must arise later through interference.

This distinction is essential. If QFTC used actual postselection as a primitive, its complexity-theoretic status would change dramatically. The present theory avoids that assumption.

6 The Geometric-Primitive Regime

6.1 Why a Second Regime Is Needed

The equality $\mathsf{QFTC}_{\text{gate}} = \mathsf{BQP}$ settles the standard circuit interpretation. However, it does not settle whether physically implemented rollback control may have a different resource profile from its compiled gate simulation.

In a physical quantum processor, a transformation may be available as a native analog or Hamiltonian primitive even if its decomposition into elementary gates is costly. Conversely, a transformation that is short in gate notation may be physically expensive due to calibration, decoherence, or control amplitude. Therefore, language recognition classes do not exhaust the resource question.

QFTC becomes nontrivial when rollback is treated not merely as a symbolic inverse circuit, but as a physically primitive operation:

$$H_k(t) \longmapsto -H_k(T-t). \tag{42}$$

The resource question is then:

> Can this Hamiltonian reversal and the associated phase-memory retention implement an effective branch transformation with lower physical cost than a standard compiled circuit?

6.2 Definition of Geometric-Primitive QFTC

Definition 3 (Geometric-primitive QFTC machine)**.** *A geometric-primitive QFTC machine is a tuple*

$$Q_{\text{geom}} = (\mathcal{H}_B, \mathcal{H}_W, \mathcal{H}_A, |\psi_0\rangle\,, \{H_k(t)\}, \{H_k^{\text{rev}}(t)\}, V, S(\phi), \mathcal{E}, \mathcal{R}) \tag{43}$$

where:

(i) $\{H_k(t)\}$ *are branch-conditioned physical trial Hamiltonians,*

(ii) $H_k^{\text{rev}}(t) = -H_k(T-t)$ *implements rollback up to bounded error,*

(iii) V *is an explicit reversible predicate mechanism,*

(iv) $S(\phi)$ *is a coherent phase-commit operation,*

(v) $\mathcal{E}$ *is an open-system noise model,*

(vi) $\mathcal{R}$ *is a specified resource accounting scheme including action, coherence time, rollback fidelity, and predicate cost.*

The class $\mathsf{QFTC}_{\text{geom}}$ is not immediately a standard language class. It is a physical resource model. To compare it with BQP, one must specify a translation between physical resources and circuit resources.

6.3 Resource Vector

For a QFTC computation, define the resource vector

$$\mathbf{R}_{\mathrm{QFTC}} = (D_{\mathrm{gate}}, A_{\mathrm{FS}}, A_{\mathrm{H}}, \epsilon_{\mathrm{eff}}, C_{\mathrm{phys}}, \mathrm{Cost}(V), \delta, T_2, \Omega_{\mathrm{cal}}) \,, \tag{44}$$

where:

- D_{gate} is the compiled gate depth,
- A_{FS} is Fubini–Study geometric action,
- A_{H} is Hamiltonian control action,
- ϵ_{eff} is effective rollback error,
- $C_{\mathrm{phys}} = 1/\epsilon_{\mathrm{eff}}$ is physical rollback capacity,
- $\mathrm{Cost}(V)$ is predicate construction cost,
- δ is environmental leakage,
- T_2 is coherence time,
- Ω_{cal} is calibration overhead.

A standard gate-model comparison primarily emphasizes D_{gate}. QFTC emphasizes that physical feasibility is governed by the full resource vector.

6.4 Resource-Separation Question

Let F_{geom} be an effective branch transformation induced by a QFTC macro-process:

$$F_{\mathrm{geom}} = \mathrm{Tr}_{W,A}\left[R\, S(\phi)\, U\,(\cdot)\, U^{\dagger} S(\phi)^{\dagger} R^{\dagger}\right], \tag{45}$$

where the trace is taken after successful approximate restoration of the work and ancilla registers.

Let $\mathrm{Compile}(F_{\mathrm{geom}})$ be the best known standard gate-circuit implementation of the same effective branch channel to accuracy η.

The geometric resource-separation question is whether there exist families for which

$$\mathrm{Cost}_{\mathrm{phys}}(F_{\mathrm{geom}}) \ll \mathrm{Cost}_{\mathrm{gate}}\left(\mathrm{Compile}(F_{\mathrm{geom}})\right) \tag{46}$$

under a physically meaningful cost measure.

Conjecture 1 (Geometric rollback resource separation)**.** *There exist physically realizable families of QFTC macro-processes whose rollback-induced effective branch transformations can be implemented with polynomially bounded geometric action and rollback error, while any standard compiled gate implementation achieving the same transformation to comparable accuracy requires asymptotically larger gate depth, action, or coherence cost.*

This conjecture is intentionally not stated as $\mathsf{QFTC}_{\mathrm{geom}} \supsetneq \mathsf{BQP}$. It is a resource-separation conjecture. Its proof would require lower bounds comparing physical primitive cost and compiled circuit cost, not merely a new notation for known circuits.

7 Rollback Stability Under Open-System Dynamics

7.1 CPTP Rollback Model

Real QFTC systems are open quantum systems. Let ρ_0 be the initial density operator of the work register. For branch k, a noisy forward trial is modeled as

$$\rho_f^{(k)} = \mathcal{E}_f^{(k)} \left(U_k \rho_0 U_k^\dagger \right), \tag{47}$$

and rollback as

$$\rho_{rb}^{(k)} = \mathcal{E}_r^{(k)} \left(R_k \rho_f^{(k)} R_k^\dagger \right). \tag{48}$$

The effective rollback channel is

$$\mathcal{R}_k(\rho) = \mathcal{E}_r^{(k)} \left[R_k \mathcal{E}_f^{(k)} \left(U_k \rho U_k^\dagger \right) R_k^\dagger \right]. \tag{49}$$

Definition 4 (Rollback error)**.** *The branch-k rollback error on state ρ is*

$$\epsilon_k(\rho) = D\left(\rho, \mathcal{R}_k(\rho)\right), \tag{50}$$

where

$$D(\rho, \sigma) = \frac{1}{2} \|\rho - \sigma\|_1 \tag{51}$$

is trace distance.

Assumption 2 (Uniform rollback stability)**.** *For all reachable states ρ and all active branches k,*

$$D\left(\rho, \mathcal{R}_k(\rho)\right) \leq \epsilon_{\text{eff}}. \tag{52}$$

Theorem 4 (Linear rollback error accumulation)**.** *If each rollback cycle satisfies the uniform rollback stability condition, then after m cycles,*

$$D(\rho_0, \rho_m) \leq m\epsilon_{\text{eff}}. \tag{53}$$

Proof. By the triangle inequality for trace distance,

$$D(\rho_0, \rho_m) \leq \sum_{i=1}^{m} D(\rho_{i-1}, \rho_i). \tag{54}$$

Each term is at most ϵ_{eff}, hence

$$D(\rho_0, \rho_m) \leq m\epsilon_{\text{eff}}. \tag{55}$$

□

The operational stability condition is therefore

$$m\epsilon_{\text{eff}} \ll 1. \tag{56}$$

7.2 Lindblad Leakage

A continuous open-system model is given by

$$\frac{d\rho}{dt} = -i[H(t), \rho] + \sum_j \gamma_j \left(L_j \rho L_j^\dagger - \frac{1}{2}\{L_j^\dagger L_j, \rho\} \right). \tag{57}$$

Define environmental leakage over a trial of duration T as

$$\delta = \int_0^T \sum_j \gamma_j \mathrm{Tr}\left(L_j^\dagger L_j \rho(t) \right) dt. \tag{58}$$

Let ϵ_{ctrl} denote coherent control error. Then

$$\epsilon_{\text{eff}} = \epsilon_{\text{ctrl}} + \delta + O(\epsilon_{\text{ctrl}}\delta). \tag{59}$$

At leading order,

$$\epsilon_{\text{eff}} \approx \epsilon_{\text{ctrl}} + \delta. \tag{60}$$

Define physical rollback capacity:

$$C_{\text{phys}} = \frac{1}{\epsilon_{\text{eff}}}. \tag{61}$$

The maximum stable number of useful rollback cycles scales as

$$m_{\max} = O(C_{\text{phys}}). \tag{62}$$

7.3 Feasibility Window

Let T be the duration of one trial-commit-rollback macro-cycle. Let T_2 be the relevant coherence time. A basic feasibility window is

$$mT \ll T_2, \tag{63}$$

and

$$m\epsilon_{\text{eff}} \ll 1. \tag{64}$$

In addition, the total control action must remain bounded:

$$A_{\text{total}} = \sum_{i=1}^{m} A_{k_i} \leq A_{\max}. \tag{65}$$

These inequalities make QFTC physically conservative. Any claimed advantage must survive coherence, rollback error, and control-action accounting.

8 Branch-Level Phase Memory

8.1 Macro-Step Phase Retention

The defining operational feature of QFTC is not that the work register is merely restored. Ordinary uncomputation already restores auxiliary registers. The distinctive QFTC claim is that a reversible future-trial process can return the work register close to its initial state while retaining a coherent phase imprint in the branch register.

Let the initial branch-work state be

$$|\Psi_0\rangle = \sum_{k=1}^{N} c_k \, |k\rangle_B \, |\psi_0\rangle_W \, . \tag{66}$$

The branch-conditioned trial operation gives

$$|\Psi_1\rangle = \sum_{k=1}^{N} c_k \, |k\rangle_B \, U_k \, |\psi_0\rangle_W \, . \tag{67}$$

Assume that a reversible predicate and phase-commit operation induce a branch-dependent phase ϕ_k on the relevant trial sector:

$$S(\phi) \, |k\rangle_B \, U_k \, |\psi_0\rangle_W = e^{i\phi_k} \, |k\rangle_B \, U_k \, |\psi_0\rangle_W \tag{68}$$

for the idealized eigen-branch case. After rollback,

$$|\Psi_2\rangle = \sum_{k=1}^{N} c_k e^{i\phi_k} |k\rangle_B \, U_k^\dagger U_k |\psi_0\rangle_W \,. \tag{69}$$

In the ideal limit,

$$|\Psi_2\rangle = \left(\sum_{k=1}^{N} c_k e^{i\phi_k} |k\rangle_B \right) \otimes |\psi_0\rangle_W \,. \tag{70}$$

Proposition 1 (Work restoration with branch phase retention)**.** *In the ideal closed-system regime, a QFTC macro-step can restore the work register to $|\psi_0\rangle$ while transforming the branch amplitudes by a diagonal phase operation.*

Proof. The proof follows directly from $U_k^\dagger U_k = I$ for each branch. The phase-commit operation acts before rollback and is not removed by rollback if it is stored in the branch or predicate-relative phase sector. Thus the work register returns to its initial state while the branch register undergoes

$$|k\rangle_B \mapsto e^{i\phi_k} |k\rangle_B \,. \tag{71}$$

□

Remark 2. *By itself, a diagonal phase operation is not beyond standard quantum computation. In the gate-reducible regime, it is simply a unitary circuit. Its importance in* $\mathsf{QFTC}_{\mathrm{geom}}$ *is not logical superpower, but the possibility that the phase profile arises natively from physical rollback geometry at a lower physical cost than a compiled gate implementation.*

8.2 Beyond Pure Phase: Effective Branch Channels

In realistic QFTC, the phase imprint need not be a perfect diagonal phase. Entanglement, imperfect rollback, predicate uncomputation, and open-system effects induce an effective channel on the branch register. Let ρ_B be the reduced branch state. A macro-process induces

$$\rho_B \mapsto \Phi_B(\rho_B), \tag{72}$$

where

$$\Phi_B(\rho_B) = \mathrm{Tr}_{W,A} \left[\mathcal{M} \left(\rho_B \otimes \rho_W \otimes \rho_A \right) \right], \tag{73}$$

and $\mathcal{M}$ is the full trial-commit-rollback open-system macro-channel.

The idealized coherent case gives

$$\Phi_B(\rho_B) = D_\phi \rho_B D_\phi^\dagger, \tag{74}$$

where

$$D_\phi = \sum_{k=1}^{N} e^{i\phi_k} |k\rangle\langle k|_B \,. \tag{75}$$

However, a useful QFTC steering process requires more than diagonal phase marking. It requires a mixing or interference layer that converts relative phase into probability redistribution. Let W_B be a branch-space mixing unitary, such as a Fourier transform, diffusion operator, or physically native mode-mixing operation. Then one effective steering macro-step may be represented as

$$T_B = W_B D_\phi. \tag{76}$$

After m cycles,

$$\rho_{B,m} = T_B^m \rho_{B,0} (T_B^\dagger)^m \tag{77}$$

in the ideal unitary branch model.

8.3 Good-Branch Probability

Let Π_G^B denote the projector onto good branch indices. The good-branch probability after m macro-cycles is

$$p_G(m) = \operatorname{Tr}\left(\Pi_G^B \rho_{B,m}\right). \tag{78}$$

A QFTC steering advantage requires that $p_G(m)$ increase faster, under the relevant physical resource measure, than it would under the best available compiled standard circuit with comparable oracle or predicate access.

This requirement is stronger than simply showing that $p_G(m)$ can increase. Grover-style amplitude amplification already increases the marked probability. QFTC must either reproduce Grover within its language or identify a resource regime where its physical implementation has a better cost profile.

9 Compatibility with Grover Lower Bounds

9.1 The Unstructured Search Constraint

Any mature QFTC theory must be compatible with known lower bounds for unstructured quantum search. For a search space of size N with a black-box oracle marking one item, standard quantum query complexity requires

$$\Omega(\sqrt{N}) \tag{79}$$

queries in the black-box model. Grover search achieves

$$O(\sqrt{N}) \tag{80}$$

queries and is therefore optimal in that model.

Therefore, QFTC must not claim constant-time unstructured search using only the same black-box oracle resources. If QFTC appears to produce

$$O(1) \tag{81}$$

search for arbitrary unstructured databases, then one of the following must be true:

(i) an oracle stronger than the standard search oracle has been introduced,

(ii) the predicate circuit hides the solution,

(iii) the physical primitive is not being counted as a query-equivalent resource,

(iv) the model leaves the standard black-box setting,

(v) the claim is wrong.

9.2 Grover-Compatible QFTC Principle

We state the compatibility principle explicitly.

Assumption 3 (Grover compatibility)**.** *For unstructured black-box search with standard oracle access, QFTC macro-steps are charged for every effective oracle or predicate interaction. Under this accounting, QFTC does not achieve $o(\sqrt{N})$ query complexity.*

This assumption expresses the Grover-compatible QFTC principle: QFTC cannot beat Grover's unstructured search lower bound under the same black-box oracle model and the same query accounting.

This assumption protects the theory from a false super-Grover claim. It also clarifies that any real advantage must be structural or physical-resource-based rather than a violation of the standard query lower bound.

9.3 Effective Rotation Angle

In a two-dimensional good-bad subspace, amplitude amplification can be represented as a rotation. Let $|G\rangle$ be the normalized good subspace state and $|B\rangle$ the normalized bad subspace state. A branch state may be written as

$$|\psi_B\rangle = \sin\theta\ |G\rangle + \cos\theta\ |B\rangle\,. \tag{82}$$

Grover iteration increases θ by an amount of order

$$\Delta\theta_{\text{Grover}} = O\left(\frac{1}{\sqrt{N}}\right) \tag{83}$$

when there is a single marked item.

A QFTC steering macro-step may be described by an effective rotation

$$\Delta\theta_{\text{QFTC}} = \theta_{\text{eff}}. \tag{84}$$

If the problem is unstructured and the oracle access is standard, then consistency with Grover lower bounds requires

$$\theta_{\text{eff}} = O\left(\frac{1}{\sqrt{N}}\right) \tag{85}$$

under query-equivalent accounting.

Therefore, a claim that

$$\theta_{\text{eff}} = O(1) \tag{86}$$

must be interpreted as one of the following:

(a) the problem family contains additional exploitable structure,

(b) the physical QFTC primitive supplies a nonstandard resource,

(c) the cost of generating θ_{eff} has been hidden in control action, predicate cost, or Hamiltonian preparation,

(d) the model is not solving black-box unstructured search.

9.4 No-Free-Rotation Lemma

Lemma 1 (No-free-rotation condition)**.** *In a QFTC implementation of unstructured search, a constant effective branch rotation $\theta_{\text{eff}} = O(1)$ cannot be counted as a unit-cost macro-step unless the physical or oracle resources required to generate it are also counted.*

Proof. If a constant rotation toward a marked item could be generated at unit cost using only standard black-box oracle access, then the marked item could be identified in $O(1)$ iterations, contradicting the standard quantum lower bound for unstructured search. Therefore, the resource generating such a rotation must either exceed standard oracle access, encode additional structure, or carry a cost that scales with N under proper accounting. □

10 Resource Accounting and Advantage Conditions

10.1 Compiled-Gate Cost Versus Geometric-Primitive Cost

Let F denote a desired effective branch transformation. Define:

$$C_{\text{gate}}(F, \eta) \tag{87}$$

as the minimum cost of implementing F to error η using a standard gate-circuit model. This cost may include gate depth, gate count, query count, and required coherence time.

Define:

$$C_{\mathrm{geom}}(F, \eta) \tag{88}$$

as the cost of implementing the same effective transformation using geometric QFTC primitives, including Hamiltonian action, rollback error, predicate cost, leakage, and calibration overhead.

A genuine QFTC resource advantage requires

$$C_{\mathrm{geom}}(F, \eta) < C_{\mathrm{gate}}(F, \eta) \tag{89}$$

by an asymptotically or practically significant margin.

10.2 Geometric Cost Functional

We define a composite geometric cost functional:

$$C_{\mathrm{geom}} = \lambda_A A_H + \lambda_{\mathrm{FS}} A_{\mathrm{FS}} + \lambda_\epsilon m \epsilon_{\mathrm{eff}} + \lambda_V \mathrm{Cost}(V) + \lambda_\delta m \delta + \lambda_{\mathrm{cal}} \Omega_{\mathrm{cal}}, \tag{90}$$

where all λ coefficients are positive architecture-dependent conversion weights. This functional does not claim universal units. Instead, it provides an explicit accounting structure so that advantage claims cannot ignore physical costs.

The Hamiltonian action term is

$$A_H = \sum_{i=1}^{m} \int_0^{T_i} \|H_i(t)\| dt. \tag{91}$$

The Fubini–Study action is

$$A_{\mathrm{FS}} = \sum_{i=1}^{m} \theta_{\mathrm{FS}}(\psi_i, \psi_{i+1}). \tag{92}$$

The stability penalty is

$$m\epsilon_{\mathrm{eff}}, \tag{93}$$

and the leakage penalty is

$$m\delta. \tag{94}$$

10.3 Advantage Index

Define the QFTC advantage index:

$$\mathcal{A}_{\mathrm{QFTC}}(F, \eta) = \frac{C_{\mathrm{gate}}(F, \eta)}{C_{\mathrm{geom}}(F, \eta)}. \tag{95}$$

A resource advantage exists when

$$\mathcal{A}_{\mathrm{QFTC}}(F, \eta) > 1. \tag{96}$$

A strong asymptotic resource separation would require a family F_n such that

$$\mathcal{A}_{\mathrm{QFTC}}(F_n, \eta_n) \to \infty \tag{97}$$

as $n \to \infty$.

10.4 Rollback Capacity Constraint

Even if the geometric implementation appears efficient, it is useful only if the number of required cycles is below the physical rollback capacity:

$$m \ll C_{\text{phys}} = \frac{1}{\epsilon_{\text{eff}}}. \tag{98}$$

Thus the advantage condition becomes

$$C_{\text{geom}}(F, \eta) < C_{\text{gate}}(F, \eta) \quad \text{and} \quad m\epsilon_{\text{eff}} \ll 1. \tag{99}$$

This prevents formal speedups from being claimed in regimes where rollback fidelity is too low to preserve interference.

10.5 Predicate Cost Constraint

The predicate circuit must also satisfy

$$\text{Cost}(V) \leq \text{poly}(n, \log N) \tag{100}$$

for the advantage to be nontrivial. More precisely, if C_{base} denotes the cost of solving the original problem by the best known non-QFTC method, then a QFTC advantage requires

$$\text{Cost}(V) + C_{\text{geom}}(F, \eta) < C_{\text{base}} \tag{101}$$

under the selected resource metric.

If Cost(V) is comparable to C_{base}, QFTC has not solved the problem faster. It has moved the difficulty into the predicate.

11 Structured Problems and Candidate Advantage Regimes

11.1 Why Structure Matters

QFTC is not naturally suited to defeating black-box lower bounds. Its plausible advantage lies in structured systems where the physical trial dynamics and rollback dynamics are native to the hardware or the problem Hamiltonian.

Examples of potentially relevant structured regimes include:

- reversible simulation of Hamiltonian families with natural sign reversal,
- phase-sensitive variational landscapes,
- analog quantum control problems,
- coherent spectroscopy with reversible probe dynamics,
- structured constraint systems where predicates are cheap but search paths are dynamically expensive,
- quantum control tasks in which rollback fidelity is naturally high.

In these regimes, QFTC should not be described as a generic search accelerator. It should be described as a rollback-based interference architecture for structured quantum processes.

11.2 Hamiltonian-Native Trial Families

A promising candidate regime is a family of branch Hamiltonians

$$\{H_k(t)\}_{k=1}^{N} \tag{102}$$

that are physically native to the device. Suppose that applying $H_k(t)$ and reversing it via $-H_k(T-t)$ can be done with low control overhead, while compiling the same family into a universal gate sequence requires large overhead.

Then QFTC may possess an architectural resource advantage even if it does not define a new logical complexity class. The advantage is not:

$$\mathsf{QFTC}_{\text{geom}} > \mathsf{BQP} \tag{103}$$

as a language claim. The advantage is:

$$C_{\text{geom}}(F,\eta) \ll C_{\text{gate}}(F,\eta) \tag{104}$$

for physically relevant transformations F.

11.3 Rollback-Friendly Predicate Regime

A second candidate regime occurs when V is inexpensive but direct forward search is costly. Let the predicate be a low-depth reversible verifier:

$$\text{Cost}(V) = \text{poly}(n, \log N), \tag{105}$$

while the space of possible trajectories is large. If trial dynamics naturally generate trajectory-dependent phases, QFTC can use rollback to erase trajectory workload while retaining phase information. This may reduce physical memory and reset costs, even if it does not reduce black-box query complexity.

11.4 Phase-Landscape Amplification

Let a structured problem define a smooth phase landscape

$$\phi : k \mapsto \phi_k. \tag{106}$$

A QFTC macro-step produces

$$D_\phi = \sum_k e^{i\phi_k} |k\rangle\langle k|. \tag{107}$$

If ϕ_k contains global information about the solution structure and can be generated natively by trial dynamics, then a mixing operation W_B can convert this phase landscape into amplitude concentration.

This is closer to phase estimation, quantum signal processing, or analog interference than to unstructured Grover search. Therefore, the appropriate benchmark is not always Grover. It may be the best compiled circuit for generating the same phase landscape with the same precision.

12 Branch Steering Dynamics

12.1 Effective Two-Subspace Model

For analysis, decompose the branch Hilbert space into good and bad subspaces:

$$\mathcal{H}_B = \mathcal{G} \oplus \mathcal{B}. \tag{108}$$

Let

$$|G\rangle \tag{109}$$

be the normalized projection of the branch state into $\mathcal{G}$, and

$$|B\rangle \tag{110}$$

the normalized projection into $\mathcal{B}$. A branch state can be approximated as

$$|\psi_B(m)\rangle = \sin\theta_m |G\rangle + \cos\theta_m |B\rangle . \tag{111}$$

A branch steering macro-step changes

$$\theta_{m+1} = \theta_m + \Delta\theta_{\mathrm{eff}}(m). \tag{112}$$

In Grover-like unstructured search,

$$\Delta\theta_{\mathrm{eff}}(m) = O(1/\sqrt{N}). \tag{113}$$

In structured QFTC, the effective rotation may depend on phase gradients, native Hamiltonian structure, and branch mixing:

$$\Delta\theta_{\mathrm{eff}} = f\left(\nabla_k\phi, W_B, A_H, \epsilon_{\mathrm{eff}}, \mathrm{Cost}(V)\right). \tag{114}$$

12.2 Noisy Steering Bound

Let $\rho_{B,m}^{\mathrm{ideal}}$ be the ideal branch state after m macro-cycles and $\rho_{B,m}^{\mathrm{noisy}}$ the noisy branch state. If each cycle has effective error at most ϵ_{eff}, then

$$D\left(\rho_{B,m}^{\mathrm{ideal}}, \rho_{B,m}^{\mathrm{noisy}}\right) \le m\epsilon_{\mathrm{eff}}. \tag{115}$$

Therefore, for any branch measurement projector Π_G^B,

$$\left|p_G^{\mathrm{ideal}}(m) - p_G^{\mathrm{noisy}}(m)\right| \le m\epsilon_{\mathrm{eff}}. \tag{116}$$

Proposition 2 (Probability stability under rollback noise)**.** *If a QFTC steering process has rollback error ϵ_{eff} per macro-cycle, then its good-branch probability deviates from the ideal value by at most $m\epsilon_{\mathrm{eff}}$ after m cycles.*

Proof. For any POVM element $0 \le M \le I$, trace distance bounds distinguishability:

$$|\mathrm{Tr}[M\rho] - \mathrm{Tr}[M\sigma]| \le D(\rho, \sigma). \tag{117}$$

Set $M = \Pi_G^B$ and use the accumulated trace-distance bound. □

12.3 Coherence-Limited Steering

If a desired branch amplification requires m_* macro-cycles, then a necessary feasibility condition is

$$m_* < \min\left(\frac{T_2}{T}, \frac{1}{\epsilon_{\mathrm{eff}}}, \frac{A_{\max}}{\bar{A}}\right), \tag{118}$$

where $\bar{A}$ is the average control action per macro-cycle.

This can be summarized as

$$m_* < M_{\mathrm{phys}}, \tag{119}$$

with

$$M_{\mathrm{phys}} = \min\left(\frac{T_2}{T}, C_{\mathrm{phys}}, \frac{A_{\max}}{\bar{A}}\right). \tag{120}$$

A QFTC procedure that requires $m_* > M_{\mathrm{phys}}$ is formally defined but physically ineffective.

13 Thermodynamic Resource Reinterpretation

13.1 Rollback Does Not Remove Physical Cost

QFTC reduces logical erasure during trial exploration because the work register is restored by inverse evolution rather than reset. However, reversible computation does not imply zero energy cost in real hardware. Physical cost appears through control precision, isolation, calibration, and entropy leakage into the environment.

Let ρ_S be the system state and ρ_E the environment state. Total entropy accounting gives

$$\Delta S_{\text{total}} = \Delta S_S + \Delta S_E. \tag{121}$$

In ideal closed-system rollback,

$$\Delta S_S = 0. \tag{122}$$

In an open system,

$$\Delta S_E \geq 0 \tag{123}$$

in ordinary thermodynamic operation.

Thus QFTC does not violate thermodynamics. It shifts cost from logical erasure to reversible control.

13.2 Entropy Deviation Bound

If

$$D(\rho_0, \rho_m) \leq m\epsilon_{\text{eff}}, \tag{124}$$

then continuity bounds for von Neumann entropy imply, for Hilbert-space dimension d,

$$|S(\rho_m) - S(\rho_0)| \leq O\left(m\epsilon_{\text{eff}} \log d\right) \tag{125}$$

when $m\epsilon_{\text{eff}}$ is small.

Therefore, stable rollback also bounds entropy deviation in the computational subsystem.

13.3 Energy-Action Tradeoff

The energy-action tradeoff of QFTC can be expressed as

$$\text{Benefit} \sim \frac{\text{interference gain}}{\text{control action} + \text{stabilization cost} + \text{predicate cost}}. \tag{126}$$

This motivates the normalized gain functional

$$G_{\text{QFTC}} = \frac{\Delta p_G}{A_H + \mu_\epsilon m\epsilon_{\text{eff}} + \mu_V \text{Cost}(V) + \mu_\delta m\delta}, \tag{127}$$

where $\Delta p_G = p_G(m) - p_G(0)$.

This functional is not universal, but it gives experimentalists an explicit way to compare QFTC-like rollback interference against ordinary compiled protocols.

14 Experimental Signatures

14.1 Minimal Demonstration Target

The minimal experimental target is not a large-scale search speedup. The minimal target is to demonstrate branch-level phase retention after work-register rollback.

A minimal experiment requires:

(i) a small branch register,

(ii) a work register with controllable branch-conditioned dynamics,

(iii) a reversible predicate or phase marker,

(iv) calibrated inverse dynamics,

(v) final branch interference readout.

The key experimental sequence is:

$$\text{prepare} \to \text{trial} \to \text{phase commit} \to \text{rollback} \to \text{branch interference}. \tag{128}$$

The signature is:

$$F_W \approx 1 \tag{129}$$

for work-register restoration, while the branch register exhibits a measurable phase-dependent interference shift.

14.2 Observable Quantities

The primary observables are:

- work-register rollback fidelity F_W,
- branch interference visibility $\mathcal{V}_B$,
- good-branch probability $p_G(m)$,
- leakage estimate δ,
- effective error ϵ_{eff},
- control action A_H,
- calibration overhead Ω_{cal},
- scaling with macro-cycle number m.

14.3 Predicted Scaling

A stable QFTC experiment should show:

$$1 - F_W(m) = O(m\epsilon_{\text{eff}}), \tag{130}$$

and

$$|\mathcal{V}_B(m) - \mathcal{V}_B^{\text{ideal}}(m)| = O(m\epsilon_{\text{eff}}). \tag{131}$$

For a useful steering process,

$$p_G(m) > p_G(0) \tag{132}$$

over a range

$$m \ll C_{\text{phys}}. \tag{133}$$

If $p_G(m)$ increases only when $m\epsilon_{\text{eff}}$ is already large, the observed effect cannot be attributed to stable coherent QFTC steering.

14.4 Falsifiable Failure Modes

The theory predicts specific failure modes:

(i) If branch interference disappears while work fidelity remains high, the commit phase is not retained in the branch register.

(ii) If work fidelity decays as $O(m)$ but branch gain decays faster, the steering process is more noise-sensitive than the rollback process.

(iii) If increasing predicate depth improves ideal selectivity but destroys rollback fidelity, the advantage is erased by $\mathrm{Cost}(V)$ and ϵ_{eff}.

(iv) If compiled gate protocols outperform geometric rollback under the full cost functional, then no QFTC resource advantage exists for that architecture.

These failure modes make QFTC testable as a physical resource theory rather than merely a speculative complexity claim.

15 Refined Interpretation of QFTC

15.1 What QFTC Is Not

The revised theory explicitly rejects the following interpretations:

- QFTC is not literal time travel.
- QFTC is not postselection.
- QFTC is not nonlinear quantum mechanics.
- QFTC does not automatically exceed BQP.
- QFTC does not beat Grover search under the same oracle accounting.
- QFTC does not eliminate thermodynamic cost.

15.2 What QFTC Is

QFTC is a reversible interference framework in which:

(i) candidate evolutions are explored coherently,

(ii) predicates are evaluated reversibly,

(iii) branch phases are committed without measurement,

(iv) work registers are rolled back by inverse dynamics,

(v) branch-level phase memory is converted into interference,

(vi) physical costs are measured by a resource vector beyond gate depth alone.

In the gate-reducible regime, this is an alternative organization of BQP processes. In the geometric-primitive regime, it is a physical resource model whose possible advantage depends on resource separation.

16 Formal Resource-Separation Framework

16.1 Why the Main Claim Must Be Resource-Theoretic

The previous sections established that gate-reducible QFTC is equivalent to BQP. This result is not merely a defensive correction; it determines the correct logical status of the theory. If QFTC is defined only as a family of polynomial-size unitary circuits with final measurement, then it belongs to ordinary quantum computation.

Therefore, the nontrivial content of QFTC must be formulated as a resource-theoretic claim. The comparison is not simply between language classes. The comparison is between two implementation routes for the same or comparable effective transformation:

(i) a compiled universal-gate implementation,

(ii) a geometric-primitive implementation using native Hamiltonian trial, phase memory, and rollback.

Let F_n be a family of effective branch transformations indexed by input size n. Let

$$C_{\mathrm{gate}}(F_n, \eta) \tag{134}$$

be the minimum known or lower-bounded cost of implementing F_n to error η using standard compiled gates. Let

$$C_{\mathrm{geom}}(F_n, \eta) \tag{135}$$

be the corresponding cost using QFTC geometric primitives under the resource functional defined above.

The strongest conservative QFTC hypothesis is then not a class separation but a cost separation:

$$C_{\mathrm{geom}}(F_n, \eta_n) = o\left(C_{\mathrm{gate}}(F_n, \eta_n)\right) \tag{136}$$

for some physically relevant family F_n and accuracy schedule η_n.

16.2 Resource-Separation Definition

Definition 5 (QFTC resource separation). *A family of effective branch transformations $\{F_n\}$ exhibits QFTC resource separation if there exists an implementation by geometric-primitive QFTC processes such that*

$$C_{\mathrm{geom}}(F_n, \eta_n) = o\left(C_{\mathrm{gate}}(F_n, \eta_n)\right), \tag{137}$$

while satisfying the stability and feasibility constraints

$$m_n \epsilon_{\mathrm{eff},n} \ll 1, \tag{138}$$

$$m_n T_n \ll T_{2,n}, \tag{139}$$

$$\mathrm{Cost}(V_n) \leq \mathrm{poly}(n, \log N_n), \tag{140}$$

$$A_{H,n} \leq A_{\max,n}. \tag{141}$$

This definition does not imply that $\mathsf{QFTC}_{\mathrm{geom}}$ decides languages outside BQP. It states that the physical cost of implementing a transformation may be lower in a geometric rollback architecture than in a universal compiled circuit architecture.

Conjecture 2 (Strong QFTC geometric resource separation). *There exists a physically realizable family $\{F_n\}$ of rollback-induced effective branch transformations for which QFTC resource separation holds.*

Conjecture 3 (Weak QFTC practical resource advantage)**.** *There exists a finite-size experimentally realizable family* $\{F_n\}$ *for which*

$$\mathcal{A}_{\mathrm{QFTC}}(F_n, \eta_n) > 1 \tag{142}$$

under a fully specified architecture-dependent resource functional.

The strong conjecture is asymptotic. The weak conjecture is practical and experimentally testable.

16.3 Equivalence Does Not Remove Architectural Advantage

A key point is that equality at the level of complexity classes does not imply equality at the level of physical architectures. Two computational models may both decide exactly BQP languages while differing greatly in resource efficiency for particular structured families.

Thus, the equality

$$\mathsf{QFTC}_{\mathrm{gate}} = \mathsf{BQP} \tag{143}$$

should be interpreted as follows:

> QFTC does not gain logical complexity power merely by packaging ordinary gates into trial–commit–rollback macro-steps.

It should not be interpreted as:

> QFTC cannot provide any physical or architectural advantage.

The remaining scientific task is to identify whether the physical primitive route can outperform the compiled-gate route under honest resource accounting.

17 Architecture Classes for Geometric QFTC

17.1 Class I: Digital Gate-Reducible Architectures

Class I architectures implement all QFTC components using ordinary quantum gates:

$$U_k, \quad V, \quad S(\phi), \quad R_k \in \text{poly-size circuits.} \tag{144}$$

These architectures belong to $\mathsf{QFTC}_{\mathrm{gate}}$ and therefore to BQP. They are useful for simulation, proof-of-principle demonstrations, and algorithmic organization, but they do not establish a new computational class.

Their advantage can only be constant-factor, architectural, or pedagogical unless a separate physical cost model is introduced.

17.2 Class II: Analog Hamiltonian Rollback Architectures

Class II architectures implement the trial and rollback steps through native Hamiltonian control:

$$U_k(T) = \mathcal{T} \exp\left(-i \int_0^T H_k(t)\, dt\right), \tag{145}$$

$$R_k(T) = \mathcal{T} \exp\left(-i \int_0^T H_k^{\mathrm{rev}}(t)\, dt\right), \tag{146}$$

with

$$H_k^{\mathrm{rev}}(t) \approx -H_k(T - t). \tag{147}$$

Here the primitive is not an abstract inverse circuit but a physically controlled reversal.

Class II is the natural home of $\mathsf{QFTC}_{\mathrm{geom}}$. The central question is whether the physical reversal can be performed with lower cost than compiling the same transformation into universal gates.

17.3 Class III: Hybrid Digital-Geometric Architectures

Class III architectures combine digital predicate circuits with analog trial and rollback dynamics. A typical structure is:

$$\begin{aligned}&\text{digital branch preparation} \to \text{analog trial} \to \text{digital predicate/phase}\\&\to \text{analog rollback} \to \text{digital readout.}\end{aligned} \tag{148}$$

This class is likely the most realistic near-term regime. The predicate V can be implemented digitally, while U_k and R_k may exploit native physical dynamics. The resource comparison must include interface costs between digital and analog layers.

17.4 Class IV: Error-Corrected Rollback Architectures

Class IV architectures embed QFTC macro-steps inside quantum error correction. Let $\mathcal{C}$ be a code space and $P_{\mathcal{C}}$ its projector. A logical rollback condition is

$$P_{\mathcal{C}} R_k U_k P_{\mathcal{C}} = P_{\mathcal{C}} + O(\epsilon_L), \tag{149}$$

where ϵ_L is logical rollback error.

In such architectures, the effective rollback capacity becomes

$$C_{\mathrm{phys}}^{(L)} = \frac{1}{\epsilon_L}. \tag{150}$$

The cost of error correction must be included in Ω_{cal} and C_{geom}.

18 The QFTC Feasibility Theorem

18.1 Statement

A useful QFTC computation requires simultaneous satisfaction of logical, physical, and thermodynamic constraints.

Theorem 5 (QFTC feasibility bound)**.** *Let a QFTC process require m macro-cycles to achieve a target branch success probability $p_G(m) \geq p_\star$. Suppose each macro-cycle has duration T, average Hamiltonian action $\bar{A}$, effective rollback error ϵ_{eff}, and environmental leakage δ. Then a necessary condition for physically meaningful QFTC steering is*

$$m < M_{\mathrm{phys}} = \min\left(\frac{T_2}{T}, \frac{1}{\epsilon_{\mathrm{eff}}}, \frac{1}{\delta}, \frac{A_{\max}}{\bar{A}}\right). \tag{151}$$

Proof. The coherence condition requires $mT < T_2$. Rollback stability requires $m\epsilon_{\mathrm{eff}} \ll 1$, giving $m \ll 1/\epsilon_{\mathrm{eff}}$. Environmental leakage must remain perturbative, giving $m\delta \ll 1$. Bounded control action requires $m\bar{A} < A_{\max}$. Taking the minimum of these upper bounds yields the stated necessary condition. □

Remark 3. *This theorem prevents formal QFTC constructions from being interpreted as physical advantages when they require more cycles than the device can coherently support.*

19 Complexity-Theoretic Position

19.1 Conservative Hierarchy

The conservative complexity-theoretic position is:

$$\mathsf{QFTC}_{\mathrm{gate}} = \mathsf{BQP}. \tag{152}$$

If $\mathsf{QFTC}_{\text{ideal}}$ is defined only through uniformly generated polynomial-size unitary circuits, then

$$\mathsf{QFTC}_{\text{ideal}} = \mathsf{BQP} \tag{153}$$

under the same assumptions.

If $\mathsf{QFTC}_{\text{ideal}}$ includes nonuniform, black-box, or physically primitive transformations not efficiently decomposable into standard circuits, then it is no longer a standard BQP-comparable class without additional resource definitions.

Therefore, the correct hierarchy is not

$$\mathsf{BQP} \subsetneq \mathsf{QFTC}_{\text{ideal}} \tag{154}$$

as an established theorem. The correct conservative statement is:

$$\mathsf{QFTC}_{\text{gate}} = \mathsf{BQP}, \tag{155}$$

and

$$\mathsf{QFTC}_{\text{geom}} \tag{156}$$

is a physical resource model requiring separate cost comparison.

19.2 Relation to Postselection

QFTC commit operations are unitary phase operations. They do not discard branches based on measurement outcomes. Therefore, QFTC should not be equated with postselected quantum computation.

The distinction can be summarized as:

$$\text{Postselection}: \quad \rho \mapsto \frac{P\rho P}{\operatorname{Tr}(P\rho)}, \tag{157}$$

$$\text{QFTC commit}: \quad \rho \mapsto S(\phi)\rho S(\phi)^\dagger. \tag{158}$$

The first is nonlinear after normalization and outcome conditioning. The second is linear and unitary.

Thus, QFTC does not obtain the power of postselection merely by using a good-branch projector inside a phase operator.

19.3 Relation to Amplitude Amplification

QFTC includes amplitude-amplification-like behavior when branch phases are converted into good-branch probability. However, in the gate-reducible regime, this behavior is standard quantum interference.

The novelty of QFTC is not that it discovers interference. The novelty is the macro-structural organization:

$$\text{trial} \to \text{phase commit} \to \text{rollback} \to \text{branch interference}. \tag{159}$$

This organization may be valuable for architectures where trial and rollback are physically natural.

20 Research Roadmap

20.1 Roadmap I: Prove or Disprove Resource Separation

The first theoretical task is to identify a family F_n and prove either:

$$C_{\text{geom}}(F_n, \eta_n) = o(C_{\text{gate}}(F_n, \eta_n)), \tag{160}$$

or that no such separation exists under a chosen physical model.

This requires lower bounds on compiled gate implementation and upper bounds on geometric rollback implementation.

20.2 Roadmap II: Construct Minimal Experimental Demonstrations

The second task is to demonstrate the basic QFTC signature:

$$\text{work restoration} + \text{branch phase retention}. \tag{161}$$

The minimal experiment does not need to solve a useful search problem. It only needs to show that the trial–commit–rollback sequence restores the work register while leaving a measurable branch interference imprint.

20.3 Roadmap III: Identify Native Hamiltonian Families

The third task is to find Hamiltonian families where reversal is physically cheap:

$$H(t) \to -H(T-t) \tag{162}$$

relative to digital compilation. Candidate platforms may include controllable spin systems, superconducting circuits, trapped ions, photonic interferometric networks, and analog quantum simulators.

The relevant question is not which platform is universally best, but where the QFTC resource vector is favorable.

20.4 Roadmap IV: Develop Error-Corrected Rollback

The fourth task is to construct rollback-compatible error correction. Standard error correction protects states against noise, but QFTC also requires preservation of branch-level phase memory through reversible trials. This suggests a specialized objective:

$$\text{protect}\,(\text{branch phase} + \text{work restoration})\,. \tag{163}$$

20.5 Roadmap V: Benchmark Against Compiled Protocols

Finally, any claimed QFTC advantage must be benchmarked against the best compiled standard protocol. The benchmark must include:

- gate depth,
- query count,
- coherence time,
- control action,
- calibration overhead,
- predicate cost,
- error-correction overhead,
- final success probability.

Without this comparison, QFTC remains a conceptual framework rather than a demonstrated resource advantage.

21 Discussion

21.1 Why the Reformulation Strengthens QFTC

The earlier form of QFTC risked being interpreted as a claim that rollback alone creates computational power beyond BQP. That interpretation is not defensible under the standard circuit model. The present reformulation strengthens the theory by making the conservative result explicit:

$$\mathsf{QFTC}_{\text{gate}} = \mathsf{BQP}. \tag{164}$$

Once this is acknowledged, the theory no longer needs to defend an overstrong claim. It can focus on its real contribution: a physically grounded resource model for reversible future-trial interference.

21.2 The Role of Geometric Time

Geometric time remains conceptually important. It provides a natural language for forward trial, rollback, and freeze:

$$d\theta_{\text{FS}} > 0 \quad \text{trial}, \tag{165}$$
$$d\theta_{\text{FS}} < 0 \quad \text{rollback}, \tag{166}$$
$$d\theta_{\text{FS}} \approx 0 \quad \text{freeze}. \tag{167}$$

However, geometric time alone does not create computational complexity advantage. It becomes operationally meaningful only when paired with a cost model involving action, fidelity, and coherence.

21.3 The Central Scientific Question

The final scientific question of QFTC is:

> Can a physical system implement reversible future-trial dynamics and rollback-induced branch phase memory more efficiently than a universal gate model can compile the same transformation?

If the answer is no, QFTC remains a useful reorganization of known quantum techniques. If the answer is yes, QFTC becomes a genuine physical resource theory for quantum computation.

22 Conclusion

This paper reformulated Quantum Future-Trial Computing as a two-regime theory.

First, the gate-reducible regime $\mathsf{QFTC}_{\text{gate}}$ was defined. Under standard uniform polynomial-size circuit assumptions, we proved

$$\mathsf{QFTC}_{\text{gate}} = \mathsf{BQP}. \tag{168}$$

This result directly addresses the main complexity-theoretic criticism of QFTC. If all QFTC macro-step components are efficiently decomposable into ordinary quantum gates, QFTC does not define a strict extension beyond standard quantum computation.

Second, the geometric-primitive regime $\mathsf{QFTC}_{\text{geom}}$ was introduced. In this regime, QFTC is not an immediate language-class extension but a physical resource model. Its resources include Fubini–Study geometric action, Hamiltonian rollback, phase-memory retention, decoherence leakage, predicate construction cost, calibration overhead, and rollback capacity.

The main revised hypothesis is therefore a resource-separation hypothesis:

There may exist physically realizable rollback-induced branch transformations that are cheaper to implement as geometric primitives than as compiled universal-gate circuits.

The theory also established:

- branch-level phase retention after work-register restoration,
- Grover-compatible constraints preventing false unstructured-search speedup claims,
- noisy steering bounds under trace-distance rollback error,
- a feasibility theorem involving coherence time, rollback error, leakage, and control action,
- thermodynamic consistency through the shift from logical erasure to reversible control cost,
- falsifiable experimental signatures.

The refined conclusion is precise:
QFTC does not automatically exceed BQP.
QFTC does not beat Grover search under identical oracle accounting.
QFTC does not use postselection, nonlinear quantum mechanics, or literal time travel.

Instead, QFTC proposes a geometric rollback resource model in which reversible future trials are physically executed, phase information is coherently committed, work registers are restored, and branch-level interference is used as the computational carrier.

In its strongest defensible form, QFTC is not a claim that ordinary quantum complexity theory is immediately surpassed. It is a claim that the physical resource geometry of reversible quantum computation has not yet been fully exhausted by gate-depth accounting alone.

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