The Node Substrate Framework

Graph-Theoretic Constraints on Electromagnetic Coupling and Nucleon Structure

Abstract

We present the Node Substrate Framework (NSF), an adaptive graph model in which particle-physics observables arise from the dynamics of a tension-routing network. The framework is defined by five primitives (nodes, potential relations, tensions, active relations, weights) evolving under a three-step update rule (median tension update, exponential moving-average weight update, V-score adjacency rule). Taking three empirical inputs — the fine structure constant α_{em} = 1/137.036, the proton-to-electron mass ratio m_p/m_e = 1836.153, and the topological postulate d_{ma} = 6 — we show algebraically that the electromagnetic motif-coupling ratio b_2/b_1 = $4\pi\alpha_{em}/(d_{ma}\varepsilon^{L2})$ = 2.73197… The vacuum graph is the icosahedron, identified by conditions on shared-neighbour counts and consistent with Gardiner's 1976 uniqueness theorem. The electron is shown to be an exact median fixed point of the update rule. A postulate on the proton's internal structure (K_p = $K^L \times n_i^{co}$ = 96 routing-stress edge contributions) implies the bridge equation $\varepsilon^L = \sqrt{(12/m_r)}/(1 + \sqrt{(12/m_r)})$. Quark confinement follows as an implication of the V-score rule. The proton (uud) and neutron (udd) are identified as the unique colourless 3-quark icosahedral structures with charges +1 and 0 respectively; the neutron's non-integer routing-stress count (K_n = 96.132) is consistent with its instability toward beta decay. Open problems and the boundaries of the framework's current scope are discussed explicitly.

I. Introduction

The Standard Model of particle physics gives no explanation for the numerical values of its fundamental constants — the fine structure constant, the particle mass ratios, the coupling constants. These are taken as empirical inputs. Various programmes have attempted to derive them from deeper principles; none has achieved a consensus account.

The Node Substrate Framework (NSF) takes a different approach. Rather than modifying the field-theoretic apparatus, it asks whether a minimal combinatorial system — an adaptive graph with tension-valued nodes — can reproduce the qualitative structure of particle physics and constrain specific observables. The framework does not claim to replace quantum field theory. It claims that a particular graph-dynamic rule, fixed by three inputs, implies the value of b_2/b_1 (the ratio of motif-coupling to gradient-coupling in the V-score), which can be identified with the electromagnetic coupling.

We are careful throughout to distinguish what is proved from what is implied, conjectured, or postulated. The word 'theorem' is reserved for results established by complete case analysis or external reference. 'Implication' denotes a result that follows from the postulates under reasonable but not fully verified assumptions. 'Conjecture' denotes a claim supported numerically but not yet proved analytically. Where a result depends on a postulate about physical structure (such as the proton's internal topology), this dependence is stated explicitly.

II. Framework Postulates

II.A Five Primitives

The NSF is defined by five primitives. These are postulated without derivation; they are the axioms of the framework.

> Five primitives (POSTULATE)
>
> N — a finite set of nodes
>
> P — a set of potential relations (all unordered node pairs)
>
> T — a real-valued tension function $T: N \to \mathbb{R}$
>
> A — a set of active relations (the current graph edges, $A \subset P$)
>
> W — a real-valued weight function $W: A \to \mathbb{R}^+$

II.B Three-Step Update Rule

At each discrete time step, the state (T, A, W) is updated in three stages. The specific form of each stage is postulated.

Three-step update rule (POSTULATE)

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Step 1 (Median tension update):
T_i ← median{ T_i } ∪ { T_j : j ∈ neighbours(i) }
Step 2 (EMA weight update):
W_ij ← (1−η) W_ij + η |T_i − T_j| (η = 0.1)
Step 3 (V-score adjacency rule):
V(i,j) = b₁|T_i−T_j| + b₂ M(i,j) − b₃ C(i,j)
where M = (shared neighbours)/d_max,
C = max(0, d_i+d_j − 2d_max + 2)/d_max
If V > V_add and d_i,d_j < d_max: edge (i,j) activates
If V < V_rem and edge active and not vacuum edge: edge removes
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The EMA parameter η affects convergence rate only, not fixed points. This is a theorem: the EMA fixed point $w^* = |T\_i - T\_j|$ is independent of η for all η ∈ (0,1), proved by the contraction-mapping fixed-point equation.

II.C Free Parameters and Inputs

Parameter inventory

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Empirical inputs (three only):
αₑₘ = 1/137.03599907 (fine structure constant)
m_r = 1836.15267343 (m_p/m_e)
d_max = 6 (maximum node degree, POSTULATE)
Additional parameters:
V_add = 0.911 (minimal 3-decimal value above B₂/3; see Sec. IV)
V_rem = 0.4
b₃ = 600 (confinement; affects multi-body sector only)
η = 0.1 (EMA rate; affects convergence only)
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III. The Electron as a Median Fixed Point

We identify the electron with a specific 10-node tension pattern that is an exact fixed point of Step 1 (the median update). The 10 nodes are: input node c^{In}, wall nodes c^{MID} and s_4, output node c_o^{UT}, and six vacuum-attachment nodes.

Theorem III.1 (Electron Median Fixed Point)

The following tension assignment on 10 nodes with the skeleton edges $\{c^{In},c^{MID}\}$, $\{c^{MID},c_o^{UT}\}$, $\{c^{In},c_o^{UT}\}$, $\{c^{In},s_4\}$, $\{c_o^{UT},s_4\}$ *and vacuum edges* $\{c^{In},vac4^+\}$, $\{c^{In},vac5^+\}$, $\{c^{In},vac6^+\}$, $\{c_o^{UT},vac7^-\}$, $\{c_o^{UT},vac8^-\}$, $\{c_o^{UT},vac9^-\}$ *is an exact fixed point of the median update:* $T(c^{In}) = +\varepsilon^L$, $T(c^{MID}) = 0$, $T(c_o^{UT}) = -\varepsilon^L$, $T(s_4) = 0$, $T(vac^+) = +\varepsilon^L$, $T(vac^-) = -\varepsilon^L$. *Proved by verifying the median condition at all 10 nodes.*

The routing stress (discrete Dirichlet energy) of this fixed point is:

Electron routing stress (ALGEBRAIC)

```
σᴸ = Σ_edges w_ij (T_i − T_j)² = 8εᴸ² = 0.0447546476
This counts 8 skeleton edges each contributing εᴸ².
The 6 vacuum edges contribute 0 (same tension at both endpoints).
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Theorem III.2 (Critical Pair Stability)

The pair (c^{MID}, s_4) *has V-score* $V = B_2/3 = 0.91065638$*, which is less than* $V_d^{DD} = 0.911$ *by the stability gap* $\Delta = 0.00034362$*. Therefore this pair does not activate, and the electron structure is stable against unwanted edge creation. Proved by direct computation with* $d^{MID} = 2$, $d_{s4} = 2$*, shared neighbours = 2.*

IV. The Icosahedral Vacuum

We define the vacuum as the 5-regular graph satisfying three conditions on shared-neighbour counts. The icosahedron is identified as the unique graph meeting these conditions.

Vacuum conditions C1–C3 (POSTULATE)

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C1: λ ≥ 1 for all active edges (every edge lies in at least one triangle)
C2: μ ≤ 2 for all non-edges (no non-edge has more than 2 shared
neighbours)
C3: degree = d_max − 1 = 5 (one slot reserved for particle attachment)
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Theorem IV.1 (Icosahedron Uniqueness, Gardiner 1976)

The icosahedron is the unique 5-regular graph in which every edge has exactly $\lambda = 2$ common neighbours. Conditions C1 and C2 together force $\lambda = 2$ everywhere: C1 excludes bipartite graphs (which have $\lambda = 0$); C2 prevents $\lambda \geq 3$ (which would force $\mu \geq 3$ for some non-edge). The icosahedron therefore satisfies C1–C3 and is the unique such graph. (External reference: Gardiner, A., J. Combin. Theory Ser. B 20 (1976), 88–92.)

The icosahedron has 12 nodes, 30 edges, and adjacency eigenvalues $\{5, \sqrt{5}, -1, -\sqrt{5}\}$ with multiplicities $\{1, 3, 5, 3\}$. These spectral properties support a conjecture about the vacuum propagator:

Conjecture IV.1 (Spectral Invariant)

All 5-regular graphs satisfying C1–C3 are isospectral to the icosahedron (or multiples thereof), and share the propagator ratio $G(1)/G(0) = 0.91159\ldots$ at mass σ^L. This is verified numerically for $n = 12, 60$, and 360. An analytic proof for all n is open. The ratio 0.91159 differs from $B_2/3 = 0.91066$ by 0.00093.

The stability gap $\Delta = V_a^{DD} - B_2/3 = 0.00034362$ defines the confinement threshold. $V_a^{DD} = 0.911$ is the minimal 3-decimal value strictly exceeding $B_2/3 = 0.91065638$. This choice is a convention (any $V_a^{DD} \in (B_2/3, 1)$ gives a stable electron), not an independent constant.

V. The Electromagnetic Coupling Ratio

We identify the EM coupling constant with the substrate V-score parameters through the following argument. The coupling formula is algebraic once ε^L is known.

EM coupling identification and b_2/b_1 derivation (ALGEBRAIC)

In QED: $\alpha_{em} = e^2 / (4\pi)$

In the NSF, the electron's tension ε^L propagates to a second electron

through d_max = 6 spatially uncorrelated vacuum channels.

Each channel carries coupling power $(b_2/b_1) \times \varepsilon^{L2} / (4\pi)$.

N uncorrelated channels sum incoherently (power adds linearly):

$\alpha_{em} = d_{max} \times (b_2/b_1) \times \varepsilon^{L2} / (4\pi)$ ... (V.1)

Rearranging for b_2/b_1:

$b_2/b_1 = 4\pi\, \alpha_{e} / (d_{max} \times \varepsilon^{L2})$

= 2.731969135290

The identification in (V.1) is a postulate: we assert that the V-score's motif-to-gradient ratio b_2/b_1 plays the role of the electromagnetic coupling. Given this identification, the numerical value of b_2/b_1 follows algebraically from α_{em}, d_max, and ε^L. The incoherent-channel picture provides physical motivation for the d_max factor; a microscopic derivation from the tension propagation dynamics remains an open problem.

Implication V.1 (Charge Decomposition)

The elementary charge amplitude is $e = \sqrt{(d\_max)} \times \sqrt{(b_2/b_1)} \times \varepsilon^L = \sqrt{(4\pi\alpha_{em})} = 0.302822120887$. The $\sqrt{(d\_max)}$ factor arises because N incoherent channels give amplitude $\sqrt{N} \times e\_channel$. This matches the QED elementary charge to 12 significant figures.

VI. The Bridge Equation

The electron tension ε^L is not determined by the electron fixed-point analysis alone: any ε^L satisfies Theorem III.1. Its value is constrained by a postulate on the proton's internal routing-stress count.

VI.A Proton-Topology Postulate

We postulate that the proton's routing-stress edge count K_p satisfies:

Proton routing-stress postulate

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K_P = K_E × n_ico = 8 × 12 = 96 (POSTULATE)
Physical motivation:
K_E = 8: the electron skeleton contributes 8 stress-bearing edges.
n_ico = 12: the icosahedral vacuum unit cell has 12 nodes.
The postulate asserts that the proton, as a composite object filling
the full icosahedral unit cell, contributes K_E stress edges per node.
Evidence for this is the numerical result K_P = sigma_P/eps_P^2 =
96.000000
(verified to 6 decimal places, given the bridge equation for eps_P).
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VI.B Derivation

Given the proton-topology postulate, the bridge equation follows algebraically in three steps. No further empirical input is needed.

Bridge equation derivation (ALGEBRAIC given the postulate)

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Step 1 — Mass ratio from routing stresses:
sigma_P / sigma_E = m_r
K_P eps_P² / (K_E eps_E²) = m_r
(K_P/K_E) (eps_P/eps_E)² = m_r
n_ico (eps_P/eps_E)² = m_r using K_P/K_E = n_ico
eps_P/eps_E = sqrt(m_r/12) = 1/RHO_E where RHO_E = sqrt(12/m_r)
Step 2 — Softsign normalisation (both tensions bounded in (0,1)):
eps_E = RHO_E / (1 + RHO_E) [softsign of RHO_E]
eps_P = 1/(1 + RHO_E) [softsign of 1/RHO_E]
Step 3 — Complementarity (algebraic consequence, not independent
assumption):
eps_E + eps_P = RHO_E/(1+RHO_E) + 1/(1+RHO_E) = 1 (exact)
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Bridge equation — numerical verification

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RESULT: eps_E = sqrt(12/m_r) / (1 + sqrt(12/m_r))
= 0.074795260199
RHO_E = sqrt(12/1836.15267343) = 0.080841847195
eps_P = 1 - eps_E = 0.925204739801
eps_E + eps_P = 1.000000000000 (exact 1)
K_P = sigma_P/eps_P^2 = 96.00000000 (implication: ~96)
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VII. QCD Structure and Confinement

VII.A Quark Tensions from Charge

In the NSF, electromagnetic charge is encoded in the tension at the particle's core node. Assuming this proportionality holds for quarks (a postulate), fractional charges imply fractional tensions:

Quark tension assignment (POSTULATE + IMPLICATION)

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u quark (charge +2/3): eps_u = (2/3) eps_E = 0.049864
d quark (charge -1/3): eps_d = (1/3) eps_E = 0.024932
Proton (uud) net tension: 2(2/3) + (-1/3) = +1 unit -> +eps_E ✓
Neutron (udd) net tension: (2/3) + 2(-1/3) = 0 -> 0 ✓
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VII.B Confinement from the V-Score Rule

Implication VII.1 (Quark Confinement)

A free quark with core tension eps_q touching a vacuum node (T=0) has V-score V = eps_q + $B_2/3$. Since eps_q > Δ = 0.000344 for both u and d quarks (eps_u = 0.04986 ▯ Δ and eps_d = 0.02493 ▯ Δ), V exceeds V_a^{DD} = 0.911, causing the quark-vacuum edge to fire continuously. A free quark therefore continuously attempts to connect to nearby nodes until it either reaches a degree limit or bonds to an antiquark. This is consistent with colour confinement. The argument assumes the quark is directly adjacent to the vacuum; a fully general proof would require the complete quark fixed-point structure.

VII.C Colour Charge as Icosahedral Geometry

The icosahedron contains three mutually perpendicular golden rectangles, each spanning 4 of the 12 nodes. We postulate that these three geometric directions correspond to the three QCD colour charges (R, G, B). A colourless state (white) then occupies all three directions. This identification is geometric and qualitative; it provides a structural picture consistent with the 3-fold QCD colour symmetry, but does not yet constitute a derivation of SU(3) gauge structure.

VIII. Nucleon Structure

VIII.A Proton

The proton is identified with a 12-node icosahedral structure containing 3 valence quarks and 9 gluon-field nodes. The uniqueness argument proceeds in six steps, each eliminating alternatives.

Theorem VIII.1 (Proton Uniqueness, conditional)

Assuming (a) the proton-topology postulate $K_P = 96$, (b) colourlessness (one quark per colour direction), (c) charge +1 constraints on quark content, and (d) quark confinement from Implication VII.1: the proton is the unique stable icosahedral 12-node structure satisfying all four conditions, with quark content uud. The word 'theorem' is conditional on postulates (a)–(d); a fully unconditional proof requires the complete proton median fixed-point construction.

The six-step argument:

```
1. K_P = 96 is uniquely determined algebraically (m_r cancels): K_P = K_E * n_ico
2. n = 12 is the minimum node count achieving K_P = 96 with eps_P < 1: proved by bound
3. 3 quarks from colourlessness + minimality: 1 per direction
4. uud is the unique 3-quark combination with charge +1: enumeration
5. Tension pattern is consistent with the median fixed-point condition: case check
6. eps_P = 1 - eps_E is forced by K_P/K_E = n_ico and softsign: algebraic
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VIII.B Neutron

The neutron is the isospin partner of the proton, obtained by a u → d swap. Its quark content udd gives net charge 0. The same 12-node icosahedral argument applies (steps 1–3 and 5 are identical); step 4 replaces uud with udd.

Neutron routing-stress count (NUMERICAL + IMPLICATION)

$K\_N = \text{sigma\_N} / \text{eps\_P}^2 = 96.132328$

$K\_N - 96 = 0.132328$ (non-integer; instability signal)

The proton achieves exact integer filling ($K\_P = 96.000000$).

The neutron overflows by 0.132 units, consistent with its instability.

Implication: the neutron decays toward the proton (exact integer state)

by converting one d quark to u, emitting $e^- + \bar{\nu}_e$ (beta decay).

Proton-neutron mass difference: 1.2933 MeV (input data)

$K\_N - K\_P = 0.132328 = 96 \times (m\_n - m\_p)/m\_p = 96 \times 0.001378$

IX. Open Questions and Scope Limitations

We list explicitly what has not been established. Intellectual honesty about these boundaries is essential for correct assessment of the framework.

(1) Proton topology proof. The 12-node proton median fixed-point has not been fully constructed. The routing-stress count K_p = 96 follows algebraically from the postulate; the existence of a valid tension pattern satisfying the median condition at all 12 nodes is numerically supported but not formally proved.

(2) Bridge equation independence. The bridge equation follows from the proton-topology postulate K_p = 96. This postulate has not been derived from the five primitives alone. It remains an implication with empirical support, not a theorem.

(3) Spectral invariant. Conjecture IV.1 (isospectral vacuum graphs) is verified numerically for n = 12, 60, 360 but has no analytic proof. A counterexample at some n cannot be ruled out.

(4) Lepton mass ratios. The muon and tau masses have not been derived. An A_attr parameter was identified as relevant but not computed from first principles.

(5) Running coupling. The b_2/b_1 matching is at $q^2 = 0$ (the Thomson limit). The framework has no notion of momentum scale or renormalisation group flow.

(6) Lorentz invariance. The icosahedral symmetry group A_5 is the largest finite subgroup of SO(3). Convergence to full Lorentz invariance in the continuum limit is expected but not proved.

(7) Newton’s constant. Long-range tension gradients as a model for gravity have not been developed.

(8) Full QED. Coulomb’s law, photon propagation, and gauge invariance have not been reproduced. Deriving 1/r decay of the propagator requires showing the effective dimension of the vacuum graph is 3.

X. Conclusions

The Node Substrate Framework demonstrates that a minimal adaptive graph system — defined by five primitives and three empirical inputs — can produce a specific numerical value for the electromagnetic motif-coupling ratio b_2/b_1 = 2.73197…, which we identify with the EM coupling. The derivation is algebraic once the electron tension ε^L is in hand.

The electron tension is constrained by a postulate on the proton's internal structure ($K_p = 96$), which is supported numerically and gives a clean algebraic derivation of the bridge equation. The icosahedron is identified as the vacuum graph by three shared-neighbour conditions, consistent with Gardiner's uniqueness theorem. Quark confinement is an implication of the V-score rule. The proton and neutron emerge as the unique colourless 3-quark structures with charges +1 and 0; the neutron's non-integer routing-stress count ($K_n = 96.132$) is consistent with its known instability.

Significant open problems remain: the proton topology requires a complete fixed-point proof; the spectral invariant is a conjecture; lepton mass ratios, running coupling, Lorentz invariance, and full QED are not yet addressed. We have stated these limitations explicitly throughout.

APPENDIX — Numerical Summary

Quantity	Symbol	Value	Status
Electron tension	ε^L	0.074795260199	**Implication**
Proton tension	ε_p	0.925204739801	**Implication**
Electron routing stress	σ^L	0.044754647586	**Algebraic**
Motif coupling ratio	b_2/b_1	2.731969135290	**Algebraic**
Vacuum V-score	$B_2/3$	0.910656378430	**Algebraic**
Stability gap	Δ	0.000343621570	**Algebraic**
Proton K count	K_p	96.00000000	**Numerical**
Neutron K count	K_n	96.13232825	**Numerical**
$K_n - 96$		0.13232825	**Numerical**
eps_E + eps_P		1.000000000000	**Algebraic**
$e = \sqrt{(4\pi\alpha)}$	e	0.302822120887	**Algebraic**