

# Supplementary Material

## Forecast-Driven SPC Monitoring Through Multi-Step Recurrent Forecasting in Smart Manufacturing

Table S1 presents detailed sensor-level predictive SPC classification results across all recurrent forecasting architectures, including precision, recall, F1-score, accuracy, cross-trial F1-score coefficient of variation (CV), and Pareto-optimality status.

**Table S1.** Detailed Classification Performance Across Sensors

						CV (F1- score)	Pareto- Optimal
	Model	Precision	Recall	F1-score	Accuracy		
Sensor 1	Encoder–Decoder LSTM	0.3411	0.2125	0.1462	0.1939	0.0607	YES
	GRU	0.3677	0.2089	0.1401	0.1689	0.0555	YES
	Direct Multi-Step LSTM	0.4043	0.1951	0.1370	0.2305	0.0801	NO
	Deep LSTM	0.3925	0.1902	0.1352	0.2175	0.1348	NO
	Vanilla LSTM	0.4116	0.1892	0.1239	0.1429	0.1029	NO
	Bi-LSTM	0.4279	0.1784	0.1160	0.1815	0.1434	NO
	Attention LSTM	0.1832	0.2102	0.0819	0.0518	0.2339	NO
Sensor 2						CV (F1- score)	Pareto- Optimal
	Model	Precision	Recall	F1-score	Accuracy		
	Deep LSTM	0.4387	0.2533	0.2352	0.3695	0.0527	YES
	Encoder–Decoder LSTM	0.4269	0.2611	0.2306	0.3622	0.0615	NO
	GRU	0.4392	0.2400	0.2229	0.3789	0.0747	NO
	Direct Multi-Step LSTM	0.4268	0.2528	0.2222	0.3492	0.0779	NO
	Vanilla LSTM	0.4375	0.2465	0.2211	0.3738	0.0398	YES
	Bi-LSTM	0.4392	0.2390	0.2107	0.3545	0.1322	NO
	Attention LSTM	0.4063	0.2312	0.1980	0.3426	0.0675	NO
Sensor 3						CV (F1- score)	Pareto- Optimal
	Model	Precision	Recall	F1-score	Accuracy		
	Deep LSTM	0.4059	0.3867	0.3150	0.0490	0.0435	YES
	Encoder–Decoder LSTM	0.4179	0.3794	0.3066	0.0501	0.0387	YES
	Direct Multi-Step LSTM	0.4043	0.3809	0.3035	0.0487	0.0247	YES
	GRU	0.4313	0.3594	0.2857	0.0641	0.0578	NO
	Vanilla LSTM	0.4446	0.3560	0.2670	0.0631	0.0492	NO
	Bi-LSTM	0.4419	0.3592	0.2615	0.0429	0.0749	NO
	Attention LSTM	0.3739	0.2981	0.2380	0.0456	0.1554	NO
Sensor 4						CV (F1- score)	Pareto- Optimal
	Model	Precision	Recall	F1-score	Accuracy		
	Encoder–Decoder LSTM	0.3215	0.2533	0.2262	0.0746	0.0319	YES
	Deep LSTM	0.3200	0.2506	0.2247	0.0854	0.0411	NO

	GRU	0.3037	0.2476	0.2194	0.1730	0.0084	YES
	Bi-LSTM	0.3132	0.2436	0.2171	0.1969	0.0337	NO
	Direct Multi-Step LSTM	0.2891	0.2408	0.2124	0.1299	0.0526	NO
	Vanilla LSTM	0.3082	0.2200	0.2012	0.1422	0.0832	NO
	Attention LSTM	0.1995	0.2205	0.1800	0.2657	0.1249	NO
Sensor 5	<b>Model</b>	<b>Precision</b>	<b>Recall</b>	<b>F1-score</b>	<b>Accuracy</b>	<b>CV (F1-score)</b>	<b>Pareto-Optimal</b>
	Bi-LSTM	0.3943	0.2765	0.2472	0.1753	0.0183	YES
	GRU	0.4045	0.2706	0.2472	0.1940	0.0114	YES
	Vanilla LSTM	0.4123	0.2701	0.2373	0.2446	0.0595	NO
	Deep LSTM	0.3888	0.2641	0.2341	0.1185	0.0407	NO
	Direct Multi-Step LSTM	0.3773	0.2635	0.2334	0.0770	0.0173	NO
	Encoder–Decoder LSTM	0.3826	0.2606	0.2326	0.0411	0.0422	NO
	Attention LSTM	0.2530	0.2282	0.1872	0.2455	0.0828	NO
Sensor 6	<b>Model</b>	<b>Precision</b>	<b>Recall</b>	<b>F1-score</b>	<b>Accuracy</b>	<b>CV (F1-score)</b>	<b>Pareto-Optimal</b>
	Direct Multi-Step LSTM	0.3447	0.0339	0.0346	0.0606	0.1265	YES
	Encoder–Decoder LSTM	0.3851	0.0323	0.0325	0.0607	0.1196	YES
	Vanilla LSTM	0.3121	0.0286	0.0307	0.0638	0.0864	YES
	GRU	0.3636	0.0273	0.0301	0.0660	0.0851	YES
	Deep LSTM	0.3587	0.0262	0.0295	0.0659	0.1562	NO
	Bi-LSTM	0.2882	0.0270	0.0293	0.0611	0.0832	YES
	Attention LSTM	0.2508	0.0266	0.0291	0.0643	0.0867	NO
Sensor 7	<b>Model</b>	<b>Precision</b>	<b>Recall</b>	<b>F1-score</b>	<b>Accuracy</b>	<b>CV (F1-score)</b>	<b>Pareto-Optimal</b>
	Bi-LSTM	0.3271	0.2553	0.1391	0.0127	0.0147	YES
	Vanilla LSTM	0.3275	0.2538	0.1376	0.0142	0.0081	YES
	GRU	0.3347	0.2534	0.1371	0.0144	0.0078	YES
	Encoder–Decoder LSTM	0.3329	0.2553	0.1357	0.0076	0.0107	NO
	Direct Multi-Step LSTM	0.3280	0.2560	0.1357	0.0104	0.0078	YES
	Deep LSTM	0.3269	0.2567	0.1354	0.0049	0.0117	NO
	Attention LSTM	0.2415	0.2503	0.1262	0.0281	0.0153	NO
Sensor 8	<b>Model</b>	<b>Precision</b>	<b>Recall</b>	<b>F1-score</b>	<b>Accuracy</b>	<b>CV (F1-score)</b>	<b>Pareto-Optimal</b>
	GRU	0.3746	0.3693	0.3680	0.5443	0.0843	YES
	Direct Multi-Step LSTM	0.3745	0.3669	0.3636	0.6154	0.0759	YES
	Vanilla LSTM	0.4492	0.3643	0.3624	0.5416	0.0201	YES
	Encoder–Decoder LSTM	0.3625	0.3605	0.3566	0.4834	0.0263	NO
	Bi-LSTM	0.3875	0.3602	0.3555	0.4105	0.0081	YES
	Attention LSTM	0.3384	0.3377	0.3365	0.7789	0.0043	YES
	Deep LSTM	0.2361	0.2607	0.2477	0.7872	0.1500	NO
Sensor 9	<b>Model</b>	<b>Precision</b>	<b>Recall</b>	<b>F1-score</b>	<b>Accuracy</b>	<b>CV (F1-score)</b>	<b>Pareto-Optimal</b>

						score)	
	GRU	0.3616	0.2566	0.2583	0.8101	0.0209	YES
	Deep LSTM	0.2336	0.2500	0.2413	0.7820	0.0000	YES
	Attention LSTM	0.2336	0.2487	0.2406	0.8647	0.0011	NO
	Vanilla LSTM	0.2336	0.2485	0.2405	0.8226	0.0013	NO
	Encoder–Decoder LSTM	0.2336	0.2481	0.2403	0.8075	0.0024	NO
	Direct Multi-Step LSTM	0.2336	0.2472	0.2399	0.8145	0.0026	NO
	Bi-LSTM	0.2336	0.2415	0.2368	0.8349	0.0159	NO
Sensor 10						CV (F1- score)	Pareto- Optimal
	<b>Model</b>	<b>Precision</b>	<b>Recall</b>	<b>F1-score</b>	<b>Accuracy</b>		
	Bi-LSTM	0.3625	0.3483	0.3538	0.6819	0.0032	YES
	Vanilla LSTM	0.3628	0.3459	0.3523	0.8861	0.0047	NO
	Attention LSTM	0.3573	0.3393	0.3459	0.8227	0.0089	NO
	GRU	0.3622	0.3285	0.3425	0.7127	0.0240	NO
	Encoder–Decoder LSTM	0.3603	0.3299	0.3402	0.7945	0.0178	NO
	Direct Multi-Step LSTM	0.3485	0.3351	0.3399	0.8243	0.1429	NO
	Deep LSTM	0.2271	0.2301	0.2235	0.6825	0.1294	NO
Sensor 11						CV (F1- score)	Pareto- Optimal
	<b>Model</b>	<b>Precision</b>	<b>Recall</b>	<b>F1-score</b>	<b>Accuracy</b>		
	Attention LSTM	0.3379	0.3593	0.3473	0.7676	0.0004	YES
	Vanilla LSTM	0.3451	0.3443	0.3427	0.7140	0.0237	NO
	Bi-LSTM	0.3348	0.3533	0.3408	0.6262	0.0451	NO
	GRU	0.3182	0.3326	0.3119	0.6853	0.0500	NO
	Encoder–Decoder LSTM	0.3030	0.3352	0.3023	0.4541	0.0054	NO
	Direct Multi-Step LSTM	0.3026	0.3344	0.3007	0.5415	0.0089	NO
	Deep LSTM	0.2420	0.2624	0.2467	0.7213	0.1197	NO

Table S2 summarizes the adjacent-rank Wilcoxon Signed-Rank statistical comparisons performed within the Transitional-Dynamic regime to evaluate pairwise performance separation among recurrent forecasting architectures.

**Table S2.** Adjacent-Rank Wilcoxon Signed-Rank Results for the Transitional-Dynamic Regime

Comparison	p-value	Significant
Encoder–Decoder vs Deep LSTM	1.000	No
Deep LSTM vs Direct Multi–Step LSTM	0.8125	No
Direct Multi–Step LSTM vs GRU	0.8125	No
GRU vs Vanilla LSTM	0.3125	No
Vanilla LSTM vs Bi–LSTM	0.1875	No
Bi–LSTM vs Attention LSTM	0.0625	No

Table S3 summarizes the final sensor-to-regime assignments obtained from the K-Means clustering analysis based on 34 SPC-aligned descriptors. Three distinct thermal signal regimes were identified: Transitional-Dynamic, Structured-Persistent, and Spike-Dominated. The resulting assignments were subsequently used for regime-specific SPC performance analysis and statistical validation throughout the study.

**Table S3.** Sensor-to-Regime Assignment

<b>Sensor</b>	<b>Cluster</b>	<b>Regime</b>
Sensor 1	0	Transitional-Dynamic
Sensor 2	0	Transitional-Dynamic
Sensor 3	0	Transitional-Dynamic
Sensor 4	1	Structured-Persistent
Sensor 5	1	Structured-Persistent
Sensor 6	0	Transitional-Dynamic
Sensor 7	0	Transitional-Dynamic
Sensor 8	1	Structured-Persistent
Sensor 9	2	Spike-Dominated
Sensor 10	1	Structured-Persistent
Sensor 11	1	Structured-Persistent

Table S4 summarizes the centroid values of the 34 SPC-aligned descriptors for the three identified signal regimes. The centroid profiles reveal distinct differences in variability, persistence, oscillatory behavior, deviation structure, and signal complexity, providing the quantitative foundation for the interpretation of the Transitional-Dynamic, Structured-Persistent, and Spike-Dominated thermal regimes discussed in Section 3.2.1.

**Table S4.** Cluster Centroid Characteristics Based on 34 SPC-Aligned Descriptors

<b>Feature</b>	<b>Transitional-Dynamic</b>	<b>Structured-Persistent</b>	<b>Spike-Dominated</b>
STD	0.1510	0.2307	1.3626
Kurtosis	2.7878	10.2471	3.4414
Skewness	-0.3503	0.1542	-1.9657
Max_Z_Score	6.5370	5.9402	4.8852
Max_Deviation_from_CL	0.9594	1.4834	6.6565
Autocorr_Lag1	0.7165	0.8351	0.9997
Max_Run_Above_CL	73.4	1846.8	11293
Max_Run_Below_CL	68	1703.6	4252
Max_Run_Length_Total	84.8	2212.8	11293
Run_Length_Mean	5.8629	22.7367	754.0244
Run_Length_STD	7.7441	123.1507	2148.2293
Run_Length_Max	84.8	2212.8	11293
Run_Count	6288	1719.4	41
Trend_Slope	0.000000009842	-0.0000054399285	0.0000551029388
Mann_Kendall_Tau	0.0036	-0.1392	0.0116
Mann_Kendall_pvalue	0.2125	0.0029	0.0038
Longest_Monotonic_Sequence	8.0000	4.0000	3.0000
Zero_Crossing_Rate_CL	0.2034	0.0556	0.0013
Spectral_Entropy	6.2756	3.8947	1.6415
Alternation_Count	10449.8	4032.4	3908
Sign_Change_Frequency	0.3380	0.1304	0.1264
Rolling_STD_max	0.5637	0.2047	0.2349
Proportion_Above_2Sigma	0.0592	0.0658	0.0686
High_Deviation_Cluster_Rule5	1371.6	1909.6	2119
Rolling_STD_mean	0.1275	0.0248	0.0248
Rolling_Mean_Deviation_mean	0.0452	0.1168	1.0247
Proportion_Above_1Sigma	0.3732	0.1529	0.1508
Consecutive_Above_1Sigma	54.4	1048.4	2192
Deviation_Cluster_Rule6	5829.6	3603.2	4648
PE	1.2548	0.5640	0.5883
SE	1.0550	0.1832	0.0236
Proportion_Within_1Sigma	0.6268	0.8471	0.8492
Low_Variance_Duration_Within_1Sigma	216.6	4297	12003
Spread_Index	0.6946	0.5032	0.7531

Table S5 summarizes the complete set of 34 SPC-aligned descriptors used for regime clustering. The descriptors were specifically designed to capture complementary aspects of thermal signal behavior associated with Nelson SPC Rules 1–8, including variability structure, directional persistence, trend continuity, oscillatory organization, deviation clustering, signal complexity, and mixture behavior. All 34 descriptors were utilized simultaneously during the clustering process without additional feature selection or dimensionality reduction. The resulting feature representation provided the foundation for identifying the Transitional-Dynamic, Structured-Persistent, and Spike-Dominated thermal signal regimes.

**Table S5.** SPC-Aligned Descriptors Used for Regime Clustering

SPC Rule	Descriptor	Primary Function
SPC-1 Point Beyond Control Limits	STD	Measures overall process variability
	Kurtosis	Quantifies tail heaviness and extreme deviations
	Skewness	Measures distribution asymmetry
	Max_Z_Score	Captures the largest standardized deviation
	Max_Deviation_from_CL	Measures the maximum deviation from the center line
SPC-2 Run on One Side of CL	Autocorr_Lag1	Measures short-term persistence and directional continuity
	Max_Run_Above_CL	Longest consecutive run above the center line
	Max_Run_Below_CL	Longest consecutive run below the center line
	Max_Run_Length_Total	Maximum run length regardless of direction
	Run_Length_Mean	Average run duration
	Run_Length_STD	Variability of run lengths
	Run_Length_Max	Longest observed run
	Run_Count	Total number of directional runs
SPC-3 Trend Behavior	Trend_Slope	Overall trend magnitude and direction
	Mann_Kendall_Tau	Non-parametric trend strength indicator
	Mann_Kendall_pvalue	Statistical significance of monotonic trend
	Longest_Monotonic_Sequence	Longest continuously increasing or decreasing

		sequence
SPC-4 Alternating Pattern	Zero_Crossing_Rate_CL	Frequency of crossings around the center line
	Spectral_Entropy	Complexity of oscillatory frequency content
	Alternation_Count	Number of directional alternations
	Sign_Change_Frequency	Frequency of sign changes between successive observations
SPC-5 Moderate Extreme Clustering	Rolling_STD_max	Maximum local variability observed in rolling windows
	Proportion_Above_2Sigma	Fraction of observations beyond $\pm 2\sigma$
	High_Deviation_Cluster_Rule5	Concentration of moderate extreme deviations
SPC-6 Moderate Shift Accumulation	Rolling_STD_mean	Average local variability
	Rolling_Mean_Deviation_mean	Average local deviation from the center line
	Proportion_Above_1Sigma	Fraction of observations beyond $\pm 1\sigma$
	Consecutive_Above_1Sigma	Persistence of deviations beyond $\pm 1\sigma$
	Deviation_Cluster_Rule6	Accumulation of moderate deviations on one side of the center line
SPC-7 Reduced Variability / Stable Region	PE (Permutation Entropy)	Sequential complexity and pattern diversity
	SE (Sample Entropy)	Signal irregularity and unpredictability
	Proportion_Within_1Sigma	Fraction of observations within $\pm 1\sigma$
	Low_Variance_Duration_Within_1Sigma	Duration of sustained low-variability behavior
SPC-8 (Mixture Pattern)	Spread_Index	Dispersion and spread structure of observations