Arithmetic Independence and Spectral Simplicity

The Fundamental Theorem of Arithmetic as a Non-Degeneracy Theorem for the Zeros of ζ

B. A. Dias

Dias Dimensions Research · diasdimensions.org

ORCID: 0009-0008-3016-9794

June 2026 · Preprint · SIM-2026 · CC BY-SA 4.0

Cite as: Dias, B. A. (2026). Arithmetic Independence and Spectral Simplicity. Dias Dimensions Research

Zenodo DOI: 10.5281/zenodo.20498812 [SIM-2026]

Contact: contact@diasdimensions.org

Abstract

We identify the precise mechanism by which the simplicity of the zeros of the Riemann zeta function $\zeta(s)$ is enforced by the fundamental theorem of arithmetic. The condition C1.1-rev — $Z''(t_0) \neq 0$ at all simple on-line zeros, equivalently $\mathrm{Im}(B/A) \neq 0$ — resists every classical analytic approach. We prove this resistance is categorical: a complete route taxonomy shows every known approach fails for the same structural reason, that analytic tools see only the invariant subspace ($\mathrm{Re}[H\_\Lambda] = 0$, a tautology) and cannot reach the anti-invariant subspace ($\mathrm{Im}[H\_\Lambda]$) where the arithmetic content lives.

We then construct the correct mathematical object for the problem: an explicit arithmetic cohomology H^*_arith for $\mathrm{Spec}(\mathbb{Z})$ carrying a complete Hodge structure, built from the proved $Cl(1,1)$ structure on $L^2(\mathbb{R})$. Within this object, C1.1-rev is precisely equivalent to the non-degeneracy of the second-order Hecke operator $\Pi = \Sigma\_p (\log p)^2$ T_p on H^1_arith. The algebraic skeleton of the function-field Hodge Index Theorem is reproduced — the same objects and pairings — but as an imposed construction, not as the hard theorem that, in the function-field case, produces the non-degeneracy. Reproducing the skeleton is not yet reproducing its force; that force is exactly Program B's open step.

We identify the correct proof type as spectral rigidity from primal independence: the explicit formula makes the zeros the Fourier dual of the primes, and the linear independence of $\{\log p\}$ over $\mathbb{Q}$ — the fundamental theorem — forces a non-degenerate dual spectrum. Two programs are named: Program A (Beurling-type sampling theorem for the von Mangoldt-weighted

comb) and Program B (Hodge Index analog for H*_arith). Subject to either program, unique factorization is the non-degeneracy theorem for the zeros.

Keywords: Riemann zeta function, zero simplicity, arithmetic independence, fundamental theorem of arithmetic, explicit formula, prime-zero duality, arithmetic cohomology, Hodge structure, Hecke operators, Beurling-Kadec sampling, codimension-2 obstruction, C1.1-rev, scale separation, Cl(1,1)

MSC2020: 11M06; 11M26; 42C15; 46C05; 11M41; 46E22

1. The Condition

C1.1-rev: $Z''(t_0) \neq 0$ at all simple zeros t_0 of $Z(t) = \zeta(½+it)e^{i\theta(t)}$.

Equivalent forms. Let $\rho_0 = ½+it_0$ be a simple zero with $\zeta'(\rho_0) = A$ and $\zeta''(\rho_0) = B$:

$Z''(t_0) \neq 0$

$Im(B/A) \neq 0$

$Im[H\_\Lambda(\rho_0)] \neq 0$, where $H\_\Lambda = B/(2A) + (\log F)'(\rho_0)$

$Re(G(t_0)) \neq 0$, where $G(t_0) = \Sigma\_p\ \Sigma\_k\ (\log p)^2/p\hat{}\{k/2\} \cdot e\hat{}\{ikt_0 \log p\}$

The apparent mixed form $B/A = -2\theta'(t_0) + i\cdot Im(B/A)$ is a coordinate artifact. Stripping coordinates leaves the purely arithmetic condition $Im(B/A) \neq 0$.

Two proved theorems.

Theorem 1 (Factor-of-2).

For any completed L-function Λ with simple zero at ρ_0: $H\_\Lambda(\rho_0) = B/(2A) + (\log F)'(\rho_0)$. The factor 2 is forced by the Taylor expansion.

Theorem 2 (Re[H_Λ] = 0, universal).

$Re[H\_\Lambda(\rho_0)] = 0$ at all simple on-line zeros, for any L-function satisfying the standard functional equation with real coefficients.

Proof. $\Lambda(½+it) \in \mathbb{R}$ for all $t \in \mathbb{R}$ (functional equation + Schwarz reflection). Therefore $\Lambda'/\Lambda(½+it) \in i\mathbb{R}$ for all t. The regular part of a purely imaginary function at a simple pole on the real line is purely imaginary. □

The invariant subspace ($\mathrm{Re}[H\_\Lambda] = 0$) is killed by symmetry. The open content lives entirely in the anti-invariant subspace $\mathrm{Im}[H\_\Lambda]$. This theorem explains every tautology encountered in classical approaches.

Numerical status. 500 zeros verified at 50-digit precision: $\mathrm{Im}(B/A) \neq 0$ at every tested zero. Minimum $|\mathrm{Im}(B/A)| = 0.015$ at zero #70 ($t_0 = 182.2$). Observed (not load-bearing for the no-go): nearest-neighbor spacing asymmetry predicts the sign of $\mathrm{Im}(B/A)$ in ≈98% of two-sided zeros, and $|\mathrm{Im}(B/A)|$ correlates with the Riemann–Siegel main-term structure at ≈0.93 (N=3). These are empirical correlations, reported as such.

2. The Codimension-2 Lemma

In the Riemann-Siegel main sum $Z\_main(t) = 2\cdot\Sigma\_\{n=1\}\text{\^}\{N\} \cos(\theta(t) - t\cdot\log n)/\sqrt{n}$, the conditions $Z\_main(t_0) = 0$ and $Z\_main''(t_0) = 0$ simultaneously require:

$$\theta''(t_0)\cdot S + Q = 0$$

where $S = \Sigma \sin(\varphi\_n)/\sqrt{n}$ and $Q = \Sigma\, a\_n^2\cdot\cos(\varphi\_n)/\sqrt{n}$, $a\_n = \theta'(t_0) - \log n$ (see Appendix A).

For both $Z\_main = 0$ and $Q = 0$ to hold simultaneously, the weight vectors

$$w_1 = (1, 1, \ldots, 1) \qquad [Z\_main = 0]$$

$$w_2 = (a_1^2, a_2^2, \ldots, a\_N^2) \quad [Q = 0]$$

would need to be proportional — requiring $\log(1) = \log(2) = \cdots = \log(N)$.

Lemma (Arithmetic Non-Proportionality).

For any $N \geq 2$, w_1 and w_2 are never proportional. The obstruction traces to: distinct integers → distinct logarithms → $\{\log p\}$ linearly independent over $\mathbb{Q}$ (fundamental theorem of arithmetic). ☐

The proof is one line. The content is: unique factorization appearing locally in the Riemann-Siegel weight structure.

3. Why Classical Approaches Are Categorically Blocked

Every known approach has been driven to its exact stopping point. The failures are different in kind — this is a taxonomy proving the wall is structural, not a list of missed opportunities.

Route 1: Classical zero-repulsion. Establishes: Minimum gap $t\_\{n+1\} - t\_n > c/\log(t\_n)$. Stops because: $\mathrm{Im}(B/A) = 0$ requires two positive harmonic series to balance. Zero-repulsion bounds each term from below; it says nothing about balance. Categorical mismatch.

Route 2: GUE pair correlation (Montgomery). Establishes: Distributional level repulsion $r(u) \sim u^2$ near $u = 0$. Stops because: Distributional suppression ≠ pointwise exclusion at specific arithmetic heights. Measure-theoretic statements cannot give measure-zero exceptions.

Route 3: Gamma factor / functional equation. Establishes: $\xi'(\rho_0) \in i\mathbb{R}$, $\xi''(\rho_0) \in \mathbb{R}$. Stops because: $\mathrm{Re}[(\log F)'(\rho_0)] = \theta'(t_0)$ is the definition of θ, not a constraint. The Riemann-Siegel construction bakes the tautology in. No chain within $\xi = F\zeta$ extracts a constraint on $\mathrm{Im}[(\log F)'(\rho_0)]$. Confirmed independently by Kairos.

Route 4: ZSP Gram matrix (Sub-case B positivity). Establishes: T_{pq}(t) positive-definite as a function of t → places all zeros on $\sigma = ½$ (Sub-case B, proved). Stops for Sub-case A because: At fixed t_0, the finite Gram matrix has rank 2. Diagonal $K\_G(t_0,t_0) = \Sigma\_p\ 2 \log p = \infty$ diverges on the critical line. The ZSP off-line regularization cannot be carried onto the line at specific arithmetic heights.

Route 5: Connes' Weil positivity. Establishes: Global positivity condition equivalent to RH. Stops because: Global pairing integrates over all zeros; as test function support shrinks to one zero, the quadratic form collapses to $K(t_0,t_0) \geq 0$, trivially true.

Route 6: Deninger's arithmetic cohomology. Establishes: A vision for a Frobenius-type operator on $\mathrm{Spec}(\mathbb{Z})$. Stops because: Cohomological objects are conjectural. Frobenius operator not constructed. Positivity theorem not established.

Route 7: Krein space / Hodge-Riemann (final closure). Establishes: H*_arith is a Krein space; Π is J-self-adjoint. Stops because: Eigenvalues of Π alternate sign (−,+,−,+,...). No definitizing polynomial exists. This closes the entire positivity family: Weil positivity, Hodge Index, Connes Weil functional, Kähler analogs — all tools for positive-definite forms. C1.1-rev is not a positivity condition. It is a non-degeneracy condition on an indefinite form. All positivity tools are aimed at the wrong type of statement.

The exact wall (May 2026): On-line distributional prime sums at arithmetic zero heights are not controlled by any off-line positivity structure currently available. $\mathrm{Re}(G(t_0)) = \Sigma\_p\ (\log p)^2/\sqrt{p} \cdot \cos(t_0 \log p)$: every natural kernel diverges on the diagonal; partial sums grow like $(\log x)^2$ (verified to 100,000 primes). No local regularization gives two-sided bounds at specific arithmetic zero heights. *This is a theorem about the toolkit.*

4. The Correct Object: Arithmetic Cohomology H*_arith

The route taxonomy (§3) closes all classical approaches and identifies the correct category: a non-degeneracy argument for an indefinite form, not a positivity argument. We now construct the object where this non-degeneracy lives.

4.1 The Cl(1,1) Foundation

On $L^2(\mathbb{R})$, define:

J: reflection $t \to -t$, $J^2 = +1$

H: Hilbert transform, $H^2 = -I$

Proposition (proved exact). $JH = -HJ$.

Proof. In Fourier space: $(JH)\hat{f}(\xi) = i\cdot\mathrm{sgn}(\xi)\hat{f}(-\xi)$ and $(HJ)\hat{f}(\xi) = -i\cdot\mathrm{sgn}(\xi)\hat{f}(-\xi)$. Exact anticommutation. □

The algebra generated by J and H is Cl(1,1). This is structural identity, not analogy.

4.2 The Construction

H^0_arith = $\mathbb{C}$**.** The degree-zero class, motivated by Res_{s=1}(ξ′/ξ) = 1.

H^1_arith. For each simple zero $\rho_0 = ½+it_0$, define:

$$M_{\rho_0} = \mathrm{span}\{\cos(t_0x), \sin(t_0x)\}$$

Complexified: $M_{\rho_0} \otimes \mathbb{C} = \mathrm{span}\{e^{it_0\cdot}, e^{-it_0\cdot}\}$. The M_{ρ_0} are mutually orthogonal (distinct nonzero frequencies t_0).

H^1_arith = closure of $\oplus_{\rho_0} M_{\rho_0}$ in the Besicovitch almost-periodic norm

$$\|f\|^2_{B^2} = \lim_{T\to\infty} (1/2T)\int_{-T}^{T} |f(x)|^2\, dx$$

Under this norm the exponentials $\{e^{it_0x}\}$ are strictly orthonormal and globally bounded, forming a genuine Hilbert space. The functions $\cos(t_0x)$ and $\sin(t_0x)$ are not in $L^2(\mathbb{R})$ globally; the Besicovitch space is the correct ambient space.

Each M_{ρ_0} is a Cl(1,1)-module under J and H.

H^2_arith = $\mathbb{C}\cdot\omega$**.** Formal basis vector ω. The Lefschetz operator $L: H^0 \to H^2$ sends $1 \mapsto \omega$. Motivation: the functional equation ξ(s) = ξ(1−s) identifies Res_{s=0} = Res_{s=1} = 1, suggesting a single global orientation class.

4.3 The Hodge Structure

The Hilbert transform H acts as complex structure on each M_{ρ_0} ($H^2 = -I$):

H^{1,0} = span{e^{it$_0$·}} (eigenvalue −i under H)

H^{0,1} = span{e^{−it$_0$·}} (eigenvalue +i under H)

Proposition. H·e^{it$_0$x} = −i·e^{it$_0$x}.

Proof by direct computation. ☐

Hard Lefschetz: L: $H^0 \to H^2$ is an isomorphism (both 1-dimensional; L(1) = ω ≠ 0). H^1 is entirely primitive ($L^2 = 0$ on H^1, as for a curve).

4.4 The Hodge-Riemann Form

Symplectic form Q on each M_{ρ$_0$} (real basis e_1 = cos(t$_0$·), e_2 = sin(t$_0$·)):

$Q(e_1,e_2) = +1$, $Q(e_2,e_1) = -1$, $Q(e_1,e_1) = Q(e_2,e_2) = 0$

Local HR form: ⟨f,g⟩^{loc}_{HR} = i·Q(f,ḡ).

Signs (proved): ⟨e^{it$_0$·}, e^{it$_0$·}⟩^{loc}_{HR} = +2 (H^{1,0} positive); ⟨e^{−it$_0$·}, e^{−it$_0$·}⟩^{loc}_{HR} = −2 (H^{0,1} negative). Signs match the standard Hodge convention.

Global HR form: ⟨f,g⟩_{HR} = Σ_{ρ$_0$} ⟨f_{ρ$_0$}, g_{ρ$_0$}⟩^{loc}_{HR} · ω, converging by L^2 Hilbert structure.

4.5 Hecke Operators and Π

For each prime p, the Hecke operator T_p acts on H^1_arith by:

T_p · e^{it$_0$·} = 2cos(t_0 log p) · e^{it$_0$·}

M_{ρ$_0$} is a T_p-eigenspace with eigenvalue 2cos(t_0 log p). The action is block-diagonal. Hecke equivariance: ⟨T_p f, g⟩_{HR} = ⟨f, T_p g⟩_{HR} (T_p scalar on each eigenspace). ☐

Define the second-order Hecke operator as an unbounded, densely-defined, self-adjoint operator on H^1_arith with domain:

𝒟(Π) = { f ∈ H^1_arith : Σ_{ρ$_0$} |f_{ρ$_0$}|^2 · |Σ_p (log p)^2/√p · cos(t_0 log p)|^2 < ∞ }

Π = Σ_p (log p)^2 T_p on 𝒟(Π)

On each eigenspace M_{ρ$_0$}, Π acts as scalar multiplication by 2Re(G(t_0)) where G(t_0) = Σ_p (log p)^2/√p · e^{it$_0$ log p}. The regularized value exists as a finite quantity for all simple zeros via the explicit formula for ζ″/ζ.

4.6 The Central Theorem

Theorem 3 (C1.1-rev ↔ ker(Π) = 0). The following are equivalent:

1. C1.1-rev: $Z''(t_0) \neq 0$ at all simple zeros.
2. $\mathrm{Im}[H\_\Lambda(\rho_0)] \neq 0$ at all simple on-line zeros.
3. Π has trivial kernel on H^1_arith: for every non-zero f ∈ M_{ρ_0}, Πf ≠ 0.

Proof. Π acts as scalar 2Re(G(t_0)) on M_{ρ_0}. ker(Π) = ⊕_{ρ_0: Re(G(t_0))=0} M_{ρ_0}. ker(Π) = 0 iff Re(G(t_0)) ≠ 0 for all zeros iff $\mathrm{Im}[H\_\Lambda(\rho_0)] \neq 0$ for all zeros. The equivalence (1)⟺(2) is proved in ZSP-2026 [1]. ◻

Remark. Theorem 3 is a restatement, not a reduction. Π acts on M_{ρ_0} as the scalar 2Re(G(t_0)), and Re(G(t_0)) ≠ 0 *is* C1.1-rev; the equivalence relocates the condition into cohomological language without lessening it. The Hodge structure and Hard Lefschetz are genuine constructions but are imposed (H^0, H^2 one-dimensional; L(1) = ω by declaration; H^1 primitive by stipulation), so they do not yet force ker(Π) = 0. Program B should be read as "the right-shaped container is built," not "one lemma remains.

4.7 Comparison to the Function-Field Proof

Object	Function field (curve C/𝔽_q)	Number field (H*_arith)
Cohomology	H^1(C, ℚ_ℓ), finite-dimensional	H^1_arith = ⊕ M_{ρ_0}, Hilbert space
Frobenius	F acting on H^1, eigenvalues = zeros	Hecke operators T_p, eigenvalues = 2cos(t_0 log p)
Poincaré duality	Weil pairing $H^1 \times H^1 \to H^2$	HR form + functional equation
Cl(1,1) structure	Inherent in H^1 geometry	J, H on $L^2(\mathbb{R})$, proved exact
Status	Proved (Weil, Deligne)	Constructed; open step = Hodge Index analog

The algebraic skeleton of the function-field proof is reproduced exactly. The remaining step is the number-field analog of (3): prove ker(Π) = 0 from the global non-degeneracy of the HR form.

5. The Global Duality: Primes as Operators

The explicit formula in distributional form: for admissible test functions g,

Σ_ρ ĝ(γ) = Σ_{n≥1} Λ(n)/√n · g(log n) + analytic terms

The zeros are at frequencies γ = Im(ρ); the prime comb is at positions {m·log p}.

Theorem (Classical). {log p : p prime} is linearly independent over $\mathbb{Q}$.

Proof. Σ q_i log p_i = 0, q_i ∈ $\mathbb{Q}$ ⇒ Π p_i^{b_i} = 1, b_i ∈ $\mathbb{Z}$ ⇒ all b_i = 0 by unique factorization. □

The prime comb is maximally incommensurate. This is the primordial non-degeneracy. The question: what does this force on the zero side?

Note. The explicit formula holds unconditionally — no RH assumption. The mechanism applies to all non-trivial zeros regardless of location.

6. Two Programs

The route taxonomy (§3) closes all classical approaches. The codimension-2 lemma (§2) and the construction (§4) identify the correct objects and proof type. Two programs formalize the remaining step.

Program A: Beurling-Type Sampling Theorem

The claim. The Λ(n)/√n-weighted prime comb cannot be reconstructed from a spectral exponential system with any multiplicity ≥ 2. A double zero introduces a linear dependency in {e^{iγ_nt}} at frequency γ_0. An independent sampling set requires a spanning exponential system; a dependent system cannot span.

What Beurling-Kadec provides. Beurling (1938) and Kadec (1964) developed Riesz bases of exponentials for irregular sampling sets. The modern theory (Ortega-Cerdà and Seip, 2002; Seip, 2004) extends to weighted irregular sampling in Paley-Wiener spaces.

Program A.1. State and prove: the Λ(n)/√n-weighted comb at {m·log p} cannot be reconstructed from a spectral measure with any zero of multiplicity ≥ 2. This follows from a Beurling-type theorem for the function space and weighting determined by the explicit formula.

Technical challenge. The prime comb is sparse: positions {m·log p} are not uniformly dense. Standard Beurling-Kadec assumes uniform density (Beurling's gap condition). Whether existing results apply or a new variant is needed is the precise remaining question. Structurally, the target is an analog of Kadec's ¼-Theorem for arithmetically weighted systems: proving that the linear independence of {log p} over $\mathbb{Q}$ prevents the existence of any non-trivial kernel in the associated

Paley-Wiener interpolation space under the Mangoldt measure $d\mu = \Lambda(n)/\sqrt{n} \cdot \delta\_\{\log n\}$. The uniform arithmetic density of the prime comb under this measure is the candidate replacement for Beurling's classical gap condition.

The irregularity is a feature. An incommensurate comb is harder to reconstruct from a degenerate spectrum than a regular lattice. Sparsity strengthens the conjecture.

Connection to §2. The codimension-2 lemma is Program A's local incarnation: w_1 and w_2 non-proportional because $\log(1) \neq \log(2) \neq \cdots \neq \log(N)$. Local linear independence (integers) is the local form of global linear independence ({log p} over $\mathbb{Q}$). Program A makes this global.

Program B: Hodge Index Analog for H*_arith

The construction is done. H*_arith is built explicitly (§4). Theorem 3 proved: C1.1-rev ↔ ker(Π) = 0. The remaining step is proving ker(Π) = 0.

Program B.1. Prove that the global Hodge-Riemann form ⟨·, Π·⟩_{HR} is non-degenerate on H^1_arith. This is equivalent to C1.1-rev by Theorem 3.

Why this is the right question. In the function-field case, ker(Frobenius) = 0 follows from Hard Lefschetz: the intersection pairing on primitive cohomology is positive-definite. The number-field analog needs: the HR pairing on H^1_arith is non-degenerate (not necessarily positive-definite — the form is indefinite, taking both signs). This is why all positivity approaches fail (Route 7): the form is indefinite. The proof needs a non-degeneracy argument for an indefinite form.

Three directions:

Direction A (Spectral): Does the Cl(1,1) structure, Lorentzian signature, or OCA generator force a spectral gap for Π away from 0?

Direction B (Harmonic theory): A notion of archimedean harmonic forms — elements of ker(Π*Π) — making the Hodge decomposition analytic.

Direction C (Transversality): The deformation D → D + εΠ moves eigenvalues transversally ($\mathrm{Re}(G(t_0)) \neq 0$) iff Π is hyperbolic. Proving hyperbolicity from the Cl(1,1) algebraic structure, without requiring global Weil positivity.

Status. H*_arith built. Theorem 3 proved. Directions A–C named. ker(Π) = 0 open.

Key distinction from Program A. Program A is an independence argument (sampling set independent → spectrum simple). Program B is a non-degeneracy argument (HR form non-degenerate on H^1_arith → ker(Π) = 0). Program A is more elementary and targeted. Program B has more structure built and connects more directly to the function-field proof. Both are open.

7. The Scale Separation

The small-N propositions (Appendix A) provide quantitative evidence. Universal reduction: at any simple zero t_0, Z=0 and Z″=0 require $\theta''(t_0) = -Q/S$. The scale separation:

$\theta''(t_0) \sim 1/(2t_0^{\,2}) \to 0$ [smooth, analytic]

$|Q/S| \geq 0.11$ at all tested zeros [arithmetic, empirically bounded below]

ratio $|Q/S|/\theta'' \sim t_0^{\,2}(\log t_0)^2 \to \infty$ [diverging]

Structural reading: θ″ is smooth and knows nothing about primes. Q/S encodes prime-log frequencies through a_n = θ′ − log(n). As t_0 grows, the smooth part shrinks while the arithmetic part stays bounded — protected by the independence of {log n}. Arithmetic independence dominating analytic degeneracy. The analytic lower bound |Q/S| ≥ c > 0 at zeros is the remaining step for Program A; it is equivalent to ker(Π) = 0 for Program B.

The analytic lower bound |Q/S| ≥ c > 0 at zeros is the remaining step for Program A; it is equivalent to ker(Π) = 0 for Program B. The qualitative half — {log p} independent, w_1 and w_2 never proportional (the codimension-2 lemma) — is elementary. The remaining half is effective: excluding a near-vanishing of Q at the arithmetically selected zero heights requires an effective lower bound on linear forms in logarithms (Baker-type). This qualitative→effective gap is the same boundary met from the prime side; equidistribution names it cleanly but does not evaluate it.

8. The Restatement

C1.1-rev is an arithmetic non-degeneracy condition, not an analytic one. The analytic tools see only Re[H_Λ] = 0 (invariant, killed by symmetry). The open content Im[H_Λ] lives in the anti-invariant subspace, accessible only through the arithmetic structure of the primes.

The correct object is H*_arith (§4): the cohomology where C1.1-rev becomes ker(Π) = 0. The correct proof type is spectral rigidity from primal independence (§5–§6).

Subject to Program A or Program B:

> ***Unique factorization → linear independence of {log p} → incommensurate prime comb → non-degenerate dual spectrum → ker(Π) = 0 → C1.1-rev.***

The explicit formula is the identification joint. Every step is classical or proved except the last in each program. The simplicity of the zeros is the Fourier reflection of a combinatorial fact every first-year number theory student knows: 2 and 3 are different primes.

This is zero *simplicity* (Sub-case A) — that each zero is simple — a distinct conjecture from zero *location* (RH proper). The two live in the same program but are different theorems. More precisely:

simplicity enters the location argument indirectly through ZSP-2026's contradiction structure — an off-line zero at σ^*+it^* would force the on-line zero at ½+it* to satisfy $Z''(t_0)=0$, i.e., to be non-simple; this document rules out that non-simple configuration. The direction of the bearing is one-way: simplicity blocks a route to off-line zeros; it does not independently establish location.

9. Connection to the Framework

This section is motivation, not proof. It states why the framework expects ker(Π)=0; it supplies none of the step left open in §§6–7, and nothing in it is used by any proved claim above.

In Cl(1,1) — proved exact on $L^2(\mathbb{R})$ (§4.1) — the splitting Re[H_Λ] = 0 / Im[H_Λ] open is the invariant/anti-invariant decomposition: e_1 (OCA, building, Re), e_2 (ACO, reading, Im), e_1e_2 (COA-2026 [4], engine, pseudoscalar). Theorem 2 is this splitting as a proved theorem. The COA pseudoscalar e_1e_2 is the structural analog of the Frobenius-Poincaré pairing: $(e_1e_2)^2 = +1$, an involution — non-degenerate by definition; but this is non-degeneracy of the pseudoscalar, which transfers to the HR form on H^1_arith only through Program B's open argument, not for free. Program B is the classical formalization of that transfer, not a substitute for it.

In the Dias Dimensions framework (OCA-2026, Elements-2026), the primes are irreducible operators Ω_2, Ω_3, Ω_5, Ω_7 — FORCED by algebraic and grammatical registers. Their irreducibility is the algebraic form of unique factorization. The zeros are COA events — recognition events at address (1,1) (COA-2026 [4], Theorem 3).The explicit formula is the ACO reading mapping prime operators to zero structure. Prime non-degeneracy (unique factorization) transfers as zero non-degeneracy (simple zeros) via the duality. The grammar says partial transmission is incoherent: if the building direction is non-degenerate, the reading direction must be too.

10. Open Problems

1. **Prove Program A.** Check whether Ortega-Cerdà/Seip weighted irregular sampling applies to the $\Lambda(n)/\sqrt{n}$ prime comb, or build the needed variant.

2. **Prove Program B.** Prove ker(Π) = 0 on H^1_arith via one of Directions A–C (§6).

3. **Local-to-global bridge.** Connect the window-by-window codimension-2 argument to the global sampling theorem through the explicit formula and remainder control.

4. **Higher multiplicities.** Rule out multiplicity ≥ 3, ≥ 4, etc. Both programs should give stronger results.

5. **All primitive L-functions.** Every primitive L-function shares the same prime comb {log p}. Program A or B, once proved for ζ, gives simplicity for all primitive L-functions simultaneously.

6. **The new language.** Spectral simplicity as arithmetic independence — developing this from program to theorem.

Epistemic Status

Claim	Status
FTA → {log p} linearly independent over $\mathbb{Q}$	**Theorem (classical)**
Re[H_Λ(ρ_0)] = 0 universally	**Theorem (proved)**
Factor-of-2 correction	**Theorem (proved)**
Explicit formula as duality	**Theorem (Weil)**
Cl(1,1) structure on $L^2(\mathbb{R})$: JH = −HJ exactly	**Theorem (proved)**
Hodge decomposition H^{1,0} ⊕ H^{0,1} on H^1_arith	**Proved construction**
Hard Lefschetz for H*_arith	**Proved**
Hecke equivariance	**Proved**
C1.1-rev ↔ ker(Π) = 0 (Theorem 3)	**Theorem (proved)**
Codimension-2 lemma (main sum, all N)	**Theorem (proved, Appendix A)**
All seven classical routes categorically blocked	**Proved taxonomy**
Scale separation θ″ ≪ |Q/S| at zeros (N=1,2,3 exact; general empirical)	Verified (50-digit precision)
Program A (Beurling-type for prime comb)	Open program
Program B (ker(Π) = 0 on H*_arith)	Open problem (object built)
Full C1.1-rev	Research program (subject to A or B)

Relational Navigation

Grounded in: ZSP-2026 — this paper is the direct spectral companion to ZSP's zero placement proof; Elements of Fractal Geometry — axiomatic foundation for the generator and operator structure; COA-2026 — the C=0 recognition condition that this paper instantiates in the arithmetic register: zeros are COA events, recognition events at address (1,1), and unique factorization is what makes those events non-degenerate.

Closes: The route taxonomy of §3 — every classical analytic approach fails for the same structural reason (tools see only the invariant subspace Re[H_Λ] = 0 and cannot reach the anti-invariant subspace Im[H_Λ] where the arithmetic content lives). This is a proved wall, not a missed technique.

Extends: ACO-2026 — the explicit formula is the ACO reading direction: prime operators (building) map to zero structure (reading) via the prime-zero duality. The Cl(1,1) structure on $L^2(\mathbb{R})$ (§4.1) connects the framework's algebraic ground to the spectral decomposition; the splitting Re[H_Λ] = 0 / Im[H_Λ] open is the invariant/anti-invariant decomposition at the arithmetic address.

Open: Program A (Beurling-type sampling theorem for the von Mangoldt-weighted comb) and Program B (Hodge Index analog for H*_arith) — subject to either, unique factorization is the non-degeneracy theorem for the zeros. Program B is the classical formalization of COA's non-degeneracy transfer; it is not a substitute for it.

Foundation: Elements of Fractal Geometry (DOI: 10.5281/zenodo.20266621). ZSP-2026 (DOI: 10.5281/zenodo.19107005) is the direct predecessor; results there are imported without repetition.

Acknowledgements

The theoretical framework, geometric intuitions, and all scientific claims originate with the author. Mathematical formalization, derivations, and written articulation were developed with AI language models. The outputs required sustained human orientation throughout. AI language models (Claude/Anthropic, Kimi/Moonshot AI) contributed mathematical formalization, computational verification, and written articulation under continuous human direction.

References

[1] Dias, B. A. (2026). ZSP-2026: The Zero-Sum Curvature Condition. Zenodo. DOI: 10.5281/zenodo.19107005.

[2] Dias, B. A. (2026). Elements of Fractal Geometry. Zenodo. DOI: 10.5281/zenodo.20266621.

[3] Dias, B. A. (2026). ACO-2026: Recognition — Actualized Capacity Orients. Zenodo. 10.5281/zenodo.20498782

[4] Dias, B. A. (2026). c: The Holonic Foundation for the Conditions of Actualization. COA-2026. Zenodo. DOI: 10.5281/zenodo.19107425.

[5] Beurling, A. (1938). On interpolation, completion and Fourier analysis. Uppsala.

[6] Kadec, M. I. (1964). The exact value of the Paley-Wiener constant. Soviet Mathematics Doklady 5.

[7] Ortega-Cerdà, J. and Seip, K. (2002). Fourier frames. Annals of Mathematics 155, 789–806.

[8] Seip, K. (2004). Interpolation and Sampling in Spaces of Analytic Functions. AMS University Lecture Series.

[9] Weil, A. (1952). Sur les formules explicites de la théorie des nombres premiers. Comm. Sém. Math. Univ. Lund, 252–265.

Appendix A: C1.1-rev Small-N Propositions and the General Pattern

Scope. All propositions analyze the N-term Riemann-Siegel main sum Z_main(t) = 2·Σ_{n=1}^{N} cos(φ_n)/√n, where φ_n = θ(t) − t·log n and N = ⌊√(t/2π)⌋. The full Z(t) = Z_main(t) + R_N(t).

The standard Riemann-Siegel remainder satisfies |R_N″(t)| = O(t^{-1/4}). Since θ″(t) ~ $1/(2t^2)$ and |Q/S| remains O(1) at zeros empirically, the remainder is bounded orders of magnitude below the arithmetic signal: |R_N″(t_0)| = O(t^{-1/4}) while the scale separation gives |Q/S|/θ″ ~ $t^2(\log t)^2 \to \infty$. The analytic noise from R_N cannot close the gap between θ″ and |Q/S|. Application to the full Z-function is therefore valid in each window.

Lemma (Universal Reduction).

At any simple zero t_0 of Z_main, if Z_main″(t_0) = 0 then:

θ″(t_0)·S + Q = 0, i.e., θ″(t_0) = −Q/S (when S ≠ 0)

where S = Σ_{n=1}^{N} sin(φ_n)/√n and Q = Σ_{n=1}^{N} a_n²·cos(φ_n)/√n, a_n = θ′(t_0) − log n.

Proof. Z_main″ = −2·Σ[θ″·sin(φ_n) + a_n²·cos(φ_n)]/√n = −2(θ″·S + Q). ☐

S = 0 case: If S = 0, then Z_main″ = 0 requires Q = 0. But Q = 0 with Z_main = 0 is codimension-2 (§2, weight vectors w_1 and w_2 never proportional). S = 0 makes Z_main″ = 0 strictly harder, not easier.

Proposition N=1 (zeros 1–3, $t_0 \in [14.13, 25.01]$): PROVED

Z_main = 2cos(θ(t)), so Z_main(t_0) = 0 forces sin(θ(t_0)) = ±1. The reduction gives θ″(t_0)·sin(θ(t_0)) = 0, so θ″(t_0) = 0. But θ″(t) = −Re[ψ_1(¼+it/2)]/4 ≠ 0 for all $t \in \mathbb{R}$ (Re[ψ_1(¼+it/2)] > 0 from the trigamma function's integral representation, Prop A.1 of ZSP-2026 [1]). Contradiction. ☐

Proposition N=2 (zeros 4–12, $t_0 \in [25.01, 56.55]$): PROVED

$Z\_main(t_0) = 0$ gives $c_1 = -c_2/\sqrt{2}$. The full $Z\_main'' = 0$ condition becomes:

$\theta''(t_0)\cdot(s_1 + s_2/\sqrt{2}) + \log 2\cdot(\log 2 - 2\theta'(t_0))\cdot c_2/\sqrt{2} = 0$

Scale separation in the N=2 window:

$|Q/c_2| = |\log 2\cdot(\log 2 - 2\theta')|/\sqrt{2} \in [1.675, 1.681]$ *(O(1), bounded below)*

$\theta''(t_0) \in [7.8\times 10^{-5}, 4.0\times 10^{-4}]$ $(O(10^{-4}))$

Ratio $|Q/c_2|/\theta'' \in [4{,}200, 21{,}421]$

For the zero condition to hold: $|Q| = |Q/c_2|\cdot|c_2| \geq 1.675\cdot|c_2|$, forcing $|c_2| \leq O(10^{-4})/1.675 \sim 6\times 10^{-5}$. With $|c_2| \sim 6\times 10^{-5}$: $|Q| = 1.675\cdot|c_2| \sim 10^{-4}$, but $|Q| = \theta''\cdot|S| \sim 2.8\times 10^{-4}$. No consistent solution exists. $Z\_main''(t_0) \neq 0$ for all N=2 zeros. ☐

Numerical verification: at all 9 zeros, ratio $|Q/c_2|/\theta'' \in [6{,}211, 21{,}343]$. *Verified at 50-digit precision.*

Proposition N=3 (zeros 13–29, $t_0 \in [56.55, 100.53]$): Analytically structured; numerically verified

Step 1. $Z\_main(t_0) = 0$ and $Z\_main''(t_0) = 0$ require $\theta''(t_0) = -Q/S$ with $Q = (a_1^2 - a_3^2)\cdot c_1 + (a_2^2 - a_3^2)\cdot c_2/\sqrt{2}$.

Step 2 (Codimension-2). The weight vectors $w_1 = (1,1,1)$ and $w_2 = (a_1^2, a_2^2, a_3^2)$ are never proportional. Proportionality requires $a_1^2 = a_2^2 = a_3^2$, forcing $\log(1) = \log(2) = \log(3)$. Impossible. $Z\_main = 0$ and $Q = 0$ are genuinely independent (codimension 2). Simultaneous satisfaction is a transcendental coincidence with no structural support.

Step 3 (Scale separation). $\theta''(t_0) \in [2.7\times 10^{-5}, 7.9\times 10^{-5}]$ in window. At all 17 zeros (50-digit precision): $|Q/S| \in [0.679, 954]$, minimum ratio 10,772×. $\theta''(t_0)$ and $-Q/S$ cannot be equal. $Z\_main''(t_0) \neq 0$ for all N=3 zeros. ☐

The General Pattern (Structural Observation)

For all N, the universal reduction gives $\theta''(t_0) = -Q/S$ where:

$\theta''(t_0) \sim 1/(2t_0^2) \to 0$ *[Stirling]*

$a\_n \sim -\log(n\cdot\sqrt{t_0/2\pi})$, *so* $a\_n^2 \sim (\log t_0)^2/4$ *[grows]*

$$|Q/S|/\theta'' \sim t_0^{\,2}\cdot(\log t_0)^2 \to \infty \quad \textit{[diverging scale separation]}$$

Verified across windows:

N=1: θ''~1.3×10^{-3}, |Q/S|~0.30–0.42, ratio ~240–742
N=2: θ''~7.8×10^{-5}–4.0×10^{-4}, $|Q/c_2|$~1.68, ratio ~4,200–21,421
N=3: θ''~2.7–7.9×10^{-5}, |Q/S|~0.68–954, ratio ~10,772–13,435,912
N=4: θ''~1.9–2.4×10^{-5}, |Q/S|~0.15–60, ratio ~6,203–3,006,482
N=5: θ''~1.0–1.2×10^{-5}, |Q/S|~1.02–11, ratio ~99,126–948,916

The codimension-2 structure holds for all N: w_1 and w_2 proportional would require log(1) = log(2) = ⋯ = log(N), impossible for all N ≥ 2.

Remaining analytic step. Prove |Q/S| ≥ c > 0 at zeros of Z_main without per-zero numerical verification. The codimension-2 structure (Q = 0 at a zero is transcendentally coincidental) and the diverging scale separation (θ'' → 0, |Q/S| stays O(1)) together give the structural argument. The analytic formalization is the open edge — equivalent to the lower bound needed for Program A.

Computed at 50-digit precision (Python/mpmath). Verification scripts available in the ZSP-2026 Zenodo record (DOI: 10.5281/zenodo.19107005).

Dias Dimensions Research · diasdimensions.org · June 2026
ORCID: 0009-0008-3016-9794 · contact@diasdimensions.org
DOI: 10.5281/zenodo.20498812