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J. R. M'CLEAN, Vice-President, in the Chair.

No. 1,045.—"On Measuring Distances by the Telescope."

By WILLIAM BAYLEY BRAY, M. Inst. C.E.

THE Author's attention was attracted to this subject by a Paper by Mr. Bowman, which was read before the British Association in 1841.[1] In the method of measuring distances put forward by that gentleman, the number of divisions of a staff, which were subtended by the diaphragm, or field of view of a telescope were observed, and from that measurement, the distance of the staff from the centre of the telescope was ascertained by calculation, or by the use of auxiliary tables. One objection to the practical application of the method, as then put forward, arose from the diaphragm, or field of view of a telescope, not being sufficiently defined, for the purpose of measuring the divisions on the staff. To obtain accurate readings, it would be essential that they should be observed by means of hairs as fine as those which are ordinarily used in taking levels. The subsequent calculation, or tabular inspection, might also be dispensed with, if the distance hairs were so placed on the diaphragm as to subtend 1 foot on a staff, at a distance of 1 chain, and 10 feet on a staff, at a distance of 10 chains. In practice, the Author found that it was more convenient to have two distance hairs on the diaphragm of a levelling instrument, one about $\frac{3}{20}$ of an inch above the level hair, and the other as much below, so that either of the distance hairs could be used as a fellow to the level hair, for the purpose of measuring the distance. He also found, that under certain circumstances, these distance hairs were of great use in levelling. It frequently occurs, that the level of some point is required which is so low, that although the levelling staff is visible in the field of view, yet it cannot be intersected by the central horizontal, or level hair of the instrument. In such cases it is customary for the staff-holder to raise the bottom of the staff on to his chest, or shoulder, and the approximate height of the bottom of the staff, above the ground, is then added to its reading. This mode of overcoming

[1] *Vide* "Report of the British Association for the Advancement of Science, 1841—Transactions of Sections," p. 42. Also "The Civil Engineer and Architect's Journal for 1841," p. 425, where the Paper is given *in extenso*.

the difficulty occasions loss of time, and is by no means trustworthy; but if the level is provided with distance hairs, there is a great probability that the staff will be intersected by the lower distance hair. If the division at which it intersects the staff is noted, and then by means of the level screws the telescope is depressed, until the level hair intersects the staff, it will only be necessary to add the measurement, subtended between the level hair and the lower distance hair, to the original reading of the lower distance hair, and the sum will be the correct height of the level of the instrument above the bottom of the staff. In like manner, in case of need, the height of a bank, or any other point a few feet above the level of the instrument, can be taken by means of the upper distance hair.

The use of these distance hairs for eighteen years has proved their practical value. The widths of rivers and of deep ravines have been rapidly and easily taken by them, and distances of 20 chains have been read in favourable weather. If the compass is sufficiently delicate, any operation of contouring, or of running trial levels, can be performed by their aid with rapidity and exactness, and when they are accurately fixed on the diaphragm, they may be used even for fractions of a link, in taking widths incapable of direct measurement.

As regards the theoretical limit of the accuracy of the method now proposed by the Author, it is evident that if the distance hairs remained at an invariable distance from the object glass, the portions of the staff intercepted by them would be strictly proportional to the distance of the staff from the object glass. Since, however, in focussing the instrument to any object, it is necessary to bring the cross hairs into a new focus, which is proportionally further from the object glass as the object is nearer, the angle which the hairs subtend from the centre of the object glass must be variable, diminishing as the distance diminishes, and in consequence the portion of the staff intersected at short distances is too little. If the distance hairs subtend 10 feet at a distance of 10 chains, it will be found that they subtend 0·985 feet at a distance of 1 chain, instead of subtending 1 foot, which they would do by inverse proportion. Hence a correction is necessary, and this the theory of refraction by lenses furnishes. It shows that the error is constant at all distances, amounting, in every case, to the focal length of the object glass for parallel rays. In fact, the portions of the staff, intercepted by the distance hairs, are strictly proportional to the distance from the anterior focus of the object glass.

If, therefore, the hairs be so placed on the diaphragm, that at the distance of 10 chains from the anterior focus, they subtend 10 feet, at the distance of 1 chain from the anterior focus, they will subtend 1 foot, and at a distance of 20 links from the anterior

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focus they will subtend 0·20 feet. As these distances are all measured from the anterior focus, the constant focal distance for parallel rays of the instrument, must in each case be added to the distance observed by means of the staff, as well as the distance from the object glass to the centre of the instrument, which is also a constant quantity. In the instances of a five-inch theodolite, and of a ten-inch level, the sum of these two constants is about 16 inches, or 2 links. This amount should be added to the distance, when reading the number of divisions subtended on the staff. The easiest mode of doing this, is by bringing the lower cross hair near any even division of feet, but exactly 0·02 feet above it, and then reading the intersection of the upper distance hair as shown on the staff. The difference between the even division at the lower hair, and the upper reading, will then be the number of divisions which represent the distance.

When distance hairs are applied to a theodolite, they can be used for measuring distances on sloping ground. But in that case, since the line of sight is no longer perpendicular to the staff, a larger portion of the staff is subtended by the hairs, than is due to the correct distance. If the correction for sloping ground, graduated at the back of the vertical arc of the theodolite, is deducted from the quantity subtended by the distance hairs, the remainder will be the sloping distance, and if twice that correction is deducted from the quantity subtended, the remainder will then be the horizontal distance. An additional power is obtained both in the theodolite, and in the level, by having a second pair of distance hairs so placed on the diaphragm as to read the distances in feet, instead of in chains, and so that either of these feet distance hairs may be used, in combination with the centre level hair of the instrument. In the theodolite they are of great use in taking cross sections of sidelong ground, or determining by one observation of the staff any rise or fall even of 100 feet, or more. But for these purposes an auxiliary table is requisite, similar to the one on the next page.

In measuring the height or depth by the aid of these Tables: when the horizontal distance to the staff has been ascertained, the telescope of the theodolite is to be elevated to the tabular angle, corresponding to the fractional rise, nearest to the slope of the ground; then that fraction of the horizontal distance, less the reading on the staff, will be the correct rise. Thus, if the horizontal distance is found to be 380 feet, and the fractional rise is 1 in 5, the telescope of the theodolite must be set at an elevation of $11^\circ\ 18\frac{1}{2}'$; and supposing the centre horizontal hair cuts the staff at 3·00 feet, the rise will be $\frac{380}{5}$ feet $-$ 3 feet $=$ 73 feet. A sufficient number of distances and heights to form a cross section, may thus be rapidly taken with sufficient accuracy for ordinary purposes.

TABLE of SLOPES and of their CORRESPONDING ANGLES, together with the amount per cent. to be deducted from the sloping distance, to find the horizontal distance.

Slope.	Corresponding Angle.			Amount to be deducted.	Slope.	Corresponding Angle.			Amount to be deducted.
1 in	°	′	″	Per Cent.	1 in	°	′	″	Per Cent.
1	45	0	0	29·37	16	3	34	35	0·19
2	26	33	50	10·55	17	3	21	59	0·17
3	18	26	5	5·12	18	3	10	47	0·15
4	14	2	10	3·00	19	3	0	46	0·13
5	11	18	35	1·97	20	2	51	45	0·12
6	9	27	44	1·38	21	2	43	35	0·11
7	8	7	49	1·01	22	2	36	9	0·10
8	7	7	30	0·78	23	2	29	22	0·09
9	6	20	25	0·62	24	2	23	9	
10	5	42	38	0·50	25	2	17	26	0·08
11	5	11	40	0·41	26	2	12	9	
12	4	45	49	0·35	27	2	7	16	0·07
13	4	23	55	0·30	28	2	2	43	
14	4	5	8	0·25	29	1	58	30	0·06
15	3	48	51	0·22	30	1	54	32	

By fitting the telescope of a level, or a theodolite, with a second pair of distance hairs to measure the distances in feet, a further advantage is incidentally obtained. It sometimes happens, that the greater part of a staff is hid from view by hedges, or other obstructions; as, for instance, a case may occur when at 10 chains' distance only 3 feet, or 4 feet, of the upper part of the staff are visible. Now at a distance of 10 chains the portion of the staff which would be subtended between the upper foot distance hair, and the upper link distance hair, would be 3·40 feet. Assuming this quantity, or any other, to be observed, the approximate distance can be obtained, by multiplying the observed interval by three, thus: 3·40 feet × 3 = 10·20, and the correct distance can subsequently be obtained by deducting two links from each chain, thus: 10·20 − 0·20 = 10 chains. In clear weather, with a distinct reading staff, a distance of 40 chains has thus been determined, by the interval of the staff which was read between the foot and link hairs.

[Mr. GRAVATT