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are variable, if one drives the cone and the other the screw, for $\frac{\omega}{\omega'} = \frac{r'}{r}$, in which r is constant.

We desire in conclusion to express the obligation we are under to Mr. Hilger for the admirable skill and patient intelligence with which he has successfully carried out our idea.

VI. *The Gas-Engine Indicator-Diagram.* *By* Professors W. E. Ayrton, *F.R.S.*, *and* John Perry, *M.E.**

[Plate III.]

THE members of this Society are probably aware that gas-engines are now largely in use, and that their use is still extending rapidly. There can be no doubt that gas-engines would be largely used, even if they were wasteful of fuel, on account of their being always ready to start or stop, and their requiring so little attention; but it is gradually becoming clearer that even small specimens of this kind of engine, whose history is merely beginning, are in actual fact less wasteful of fuel than the largest and most carefully constructed steam-engines. It is, for example, a demonstrated fact that, with a Dowson's generator not larger than the boiler used in the corresponding steam-engine, an Otto engine uses only 1·1 lb. of coal per indicated horse-power per hour. It is therefore not unreasonable to suppose that gas-engines will soon be employed even in the propulsion of ships.

The rapid growth of this new application of science renders it necessary that help should be given to practical men to enable them to use such observations as they are constantly making. This paper is intended to teach such men a *method* of obtaining information from the indicator-diagram of a gas-engine.

2. *The Action in the Otto Gas-Engine.*—This large model which we exhibit has been constructed in the workshop of the Finsbury College, to enable students to follow the motions of different parts of the gas-engine. It will be seen that when the piston is at the end of its stroke, only what we call the clearance-space behind it is filled with fluid. This fluid is a mixture of carbonic acid, water-vapour, and nitrogen, whose temperature is about 410° C.†, if there was an immediately

* Communicated by the Physical Society. Read April 26, 1884.

† In our own observations at Finsbury we have not used any specially contrived apparatus, as our investigations were really for the purpose of

previous explosion. The temperature of the clearance-space fluid may be anything between 410° C. and the temperature of the atmosphere, depending on how recently an explosion has taken place. As the piston moves forward it draws into the space a mixture of gas and air. At the end of the forward stroke the pressure of the mixed fluid is nearly that of the atmosphere; in the back stroke the fluid is compressed. At the beginning of the next forward stroke the fluid is ignited, and rapid development of heat results, causing great increase in pressure, the pressure gradually diminishing until, just before the end of the forward stroke, the fluid is allowed to escape. In the next back stroke the piston drives the fluid out of the cylinder with the exception of what remains in the clearance, and thus completes a cycle of operations. In fig. 1 (Pl. III.) indicator-diagrams show the nature of the alterations in pressure and volume going on during the compression and working parts of the cycle; distances measured from O L representing pressure in pounds per square inch from vacuum, and distances measured from O P representing volume of the fluid, the unit of volume being the volume described by the piston moving through one foot of the length of the cylinder.

Four different diagrams are given whose compression parts practically coincide, the differences in their ignition parts being due to differences in the amounts of gas supplied. We have not thought it necessary to give a complete diagram in which the dismission and suction parts of the cycle should be represented.

The shape of the diagram is materially modified by the recentness of the last explosion, as this affects the temperature of the fluid before compression, and so modifies the actual amount of the mixture of gas and air entering the cylinder. To a less degree the shape of the diagram is affected in the discharge part of the cycle by the recentness of an explosion, as a recent explosion will have given the exhaust passages a higher temperature.

3. *The Nature of the Working Fluid.*—For the purpose of showing the nature of the working fluid we have constructed Table I. It will be seen from this that a mixture of 6·760

enabling students to illustrate for themselves a course of lectures delivered by one of us on the Gas-engine. Hence we have taken the temperature of 410° C. as the exhaust temperature, instead of 300° C. given by our own measurements with the Siemens pyrometer. A correction of this temperature would perhaps lessen the number 1·57 W and increase 0·37 W, given in § 9.

cubic feet of air and coal-gas becomes, after complete combustion, 6·4977 cubic feet of carbonic acid, water-vapour, and nitrogen, reduced to the same pressure and temperature without condensation of the water-vapour. Now as there is always an excess of air, and as the mixture before combustion has added to it nearly six cubic feet of the products of previous combustions, we have something like 13·3 cubic feet before combustion becoming 13·0377 cubic feet after combustion, at equivalent pressures and temperatures. The contraction is only about 2 per cent. We therefore conclude that we may regard the fluid in a gas-engine as a fluid which, however it may receive heat, obeys approximately the characteristic law,

$$\frac{pv}{\mathrm{T}} \text{ constant}$$

(where p is pressure, v volume, and T the absolute temperature), in so far as mechanical actions are concerned. That is, we may speak of it as a perfect gas, which receives heat from without, neglecting the fact that it is its own molecular energy which appears as heat. It is also approximately true that the ratio of the specific heats of the fluid is unchanged by combustion taking place.

In Table II. we give a similar comparison for Dowson gas :—

2·1325 cubic feet of combustible mixture become 1·9143 cubic feet ;

or, taking into account the clearance-space and its products of past combustions, we have, say,

4 cubic feet becoming 3·7818 cubic feet ;

or contracting by nearly $5\frac{1}{2}$ per cent. of its volume.

In Tables III. and IV. we calculate the specific heat of a mixture of 1 cubic foot of coal-gas, 5·76 cubic feet of air, and 4·5 cubic feet of products of a previous combustion, before and after combustion takes place. The Centigrade scale of temperature is employed.

In Tables V. and VI. similar calculations are made for the usual mixture of Dowson gas with air and products of previous combustion.

TABLE I.—Coal-Gas.

	Hydrogen, H_2.	Carbonic oxide, CO.	Marsh-gas, CH_4.	Olefiant gas, C_2H_4.	Tetrylene, C_4H_8.	Nitrogen, N.	Carbonic acid, CO_2.	Water-vapour. H_2O.	Total.
Cubic feet in one cubic foot of gas	0·4600	0·0750	0·3950	0·0253	0·0127	0·0050	0	0·0200	
Heat evolved per cubic foot of each	191·12	190·02	592·6	932	1702	0	0	0	
Heat evolved by given amount of each	87·92	14·25	234·1	23·58	21·61	0	0	0	381·46 or 530,230 foot-pounds.
Oxygen required by one cubic foot of each	0·5	0·5	2	3	6	0	0	0	
Product of combustion per cubic foot of each	1·0	1·0	3	4	8	0	0	0	
Oxygen required by given amount of each	0·2300	0·0375	0·790	0·0759	0·0762	0	0	0	1·2096 cub. ft. of oxygen per cub. ft. of gas. or 5·760 ,, ,, air ,, ,, ,,
Volume of product of combustion	0·460	0·075	1·1850	0·1012	0·1016	0·005	0	0·020	1·9478+4·5499 of nitrogen, or 6·4977.

TABLE II.—Dowson Gas.

	Hydrogen, H_2.	Carbonic oxide, CO.	Marsh-gas, CH_4.	Olefiant gas, C_2H_4.	Tetrylene, C_4H_8.	Noncombustible.	Total.
Cubic feet in one cubic foot of gas	0·1873	0·2507	0·0031	0·0020	0·0011	0·5555	
Heat evolved per cubic foot of each	191·12	190·02	592·6	932·05	1702	0	
Heat evolved by given amount of each	35·79	47·63	1·837	1·864	1·872	0	88·99, or 123696·1 foot-pounds.
Oxygen required by given amount of each	0·0936	0·1254	0·0062	0·0060	0·0066	0	0·2378 cub. ft. of oxygen per cub. ft. of gas, or 1·1325 ,, ,, air ,, ,, ,,
Volume of product of combustion	0·1873	0·2507	0·0093	0·0080	0·0088	0	·4641+·4898 of N in gas +·0657 of CO_2 in gas +·8947 of N in air introduced =1·9143.

TABLE III.—Coal-Gas before Combustion.

Constituent.	Amount in fluid.	Specific heat at constant pressure per unit volume.	Specific heat at constant volume per unit volume.		
	Cubic feet. q.	C_p.	C_v.	qC_p.	qC_v.
Hydrogen	0·46	·2359	·99 × ·168	·1085	·4554 × ·168
Carbon monoxide	0·075	·237	1 ,,	·0178	·0750 ,,
Marsh-gas	0·3950	·3277	1·54 ,,	·1294	·6082 ,,
Olefiant gas	0·0380	·4106	2·03 ,,	·0156	·0771 ,,
Nitrogen	0·0050	·237	1 ,,	·0012	·0050 ,,
Water-vapour ...	0·0200	·2984	1·36 ,,	·0060	·0272 ,,
Air	5·76	·2374	1 ,,	1·3680	5·760 ,,
Products	4·5	·2581	1·124 ,,	1·1614	5·058 ,,
Total	11·253	......	......	2·8079	12·066 × ·168 or 2·027
Or C_p=0·2496, C_v=0·1802. Ratio .... 1·385.					

TABLE IV.—Coal-Gas after Combustion.

	Cubic feet. q.	C_p.	C_v.	qC_p.	qC_v.
Water-vapour ...	1·3714	·2984	1·36 × ·168	·4092	1·865 × ·168
Carbon dioxide ...	0·5714	·3307	1·55 ,,	·1889	·8855 ,,
Nitrogen	4·5554	·2370	1 ,,	1·0790	4·5554 ,,
Total	......	......	......	1 6771	7·3059 × ·168 or 1·227
Or C_p=0·2581, C_v=0·1889. Ratio ...... 1·367.					

TABLE V.—Dowson Gas before Combustion.

	Cubic feet. q.	C_p.	C_v.	qC_p.	qC_v.
Hydrogen	·1873	·2359	·99 × ·168	·0442	·1854 × ·168
Carbon monoxide.	·2507	·237	1 ,,	·0594	·2507 ,,
Marsh-gas	·0031	·3277	1·54 ,,	·0010	·0048 ,,
Olefiant gas	·0031	·4106	2·03 ,,	·0013	·0063 ,,
Nitrogen	·4898	·237	1 ,,	·1161	·4898 ,,
Carbon dioxide ...	·0657	·3307	1·55 ,,	·0217	·1018 ,
Air	1·1325	·2374	1 ,,	·2689	1·1325 ,,
Products	2	·2594	1·1323 ,,	·5188	2·2646 ,,
Total	4·1322	......	......	1·0314	4·4359 × ·168
				·2496	1·0735 × ·168 or ·1803
Ratio .... 1·385.					

TABLE VI.—Dowson Gas after Combustion.

	q.	C_p.	C_v.	qC_p.	qC_v.
Water-vapour ...	0·2019	·2934	1·36×·168	·0602	·2746×·168
Carbon dioxide ...	0·3279	·3307	1·55 „	·1084	·5083 „
Nitrogen	1·3845	·2370	1 „	·3281	1·3845 „
				·4967	2·1674 „
Total	1·9143	......	......	·2594	1·1323×·168 or ·1902
Ratio .... 1·3637.					

The specific heats here given are those of the constituents in the cold state. Seeing, however, that there is no great change due to combustion, we may for many practical purposes assume that the specific heats remain the same at all temperatures. We have less right when using Dowson gas than when using coal-gas to make our assumption, which is that the fluid in the gas-engine, from the beginning of its compression until it is allowed to escape, behaves like a perfect gas, receiving heat from an outside source.

4. *Study of the Shapes of the Compression and Expansion parts of the Diagram.*—The first problem which comes before us is the determination of the shape of the indicator diagram. For the sake of illustration we mean to investigate the series of diagrams shown in fig. 1, diagrams taken from a 6-horse power Otto engine at the Finsbury College, using coal-gas. It will be observed that, although the ignition parts of the various diagrams differ greatly, on account of differences *in the amounts of gas supplied*, the curves agree in their compression and expansion parts. It is known to us that the clearance-space is 0·4 of the whole space occupied by fluid when the piston is fully at the end of its stroke. If λ is the distance in feet passed through by the piston from the end of its stroke, the stroke being $1\frac{1}{3}$ foot, the clearance is 0·889 foot, and $\lambda+0·889$, or l feet expresses the volume of the fluid at any instant. Measurements of l and p were made on the expansion-curves of the indicator-diagrams, and tabulated with $\log l$ and $\log p$.

We are making arrangements for measuring the clearance-space with accuracy. The above assumption is made from statements of the manufacturers, and may possibly be slightly in error. Such an error is of but small importance to us at present, as our object is to teach a method of study rather than to give results of the use of the method.

If we assume the law of the expansion-curve to be

$$pl^m \text{ constant},$$

we have $\log p + m \log l = k$; so that when we plot $\log p$ and $\log l$ on squared paper as coordinates of points, these points ought to lie in a straight line if our assumption is correct. Fig. 2 (Pl. III.) shows how the points determined by the measured numbers lie; and it is obvious that they lie very nearly in a straight line. Taking the straight line which lies most evenly among them, we find that it is defined by

$$\log l = 0\cdot313, \quad \text{when } \log p = 1\cdot7,$$

$$\log l = 0\cdot0425, \text{ when } \log p = 2\cdot1.$$

Hence

$$1\cdot7 + 0\cdot313\, m = k,$$

$$2\cdot1 + 0\cdot0425\, m = k.$$

Hence

$$m = 1\cdot479 \text{ and } k = 2\cdot1629.$$

Now

$$\log 145\cdot5 = 2\cdot1629.$$

Hence the law of expansion is

$$pl^{1\cdot479} = 145\cdot5. \qquad \ldots \qquad (1)$$

In the same way we find that the law of the compression-curve is

$$pl^{1\cdot304} = 39\cdot36. \qquad \ldots \qquad (2)$$

It is obvious that the expansion-curve (1) is steeper than the adiabatic, if we assume that the ratio of specific heats of the fluid is 1·37, as the equation of the adiabatic curve is

$$pl^{1\cdot37} = \text{constant}.$$

Again, the compression-curve (2) has less slope than the adiabatic.

5. *Influence of Vibrations of the Indicator-spring.*—We wish to point out that it is exceedingly necessary, in obtaining the law of expansion, to take many points in the curve, and, either by using the algebraic method or by the use of squared paper, to determine the most probable values of the constants. Engineers are constantly in the habit of determining these constants from measurements of the coordinates of two points only in the curve, forgetting that the position of either point may be much influenced by the vibrations of the spring of the indicator. The sinuous shape of the expansion-curve is

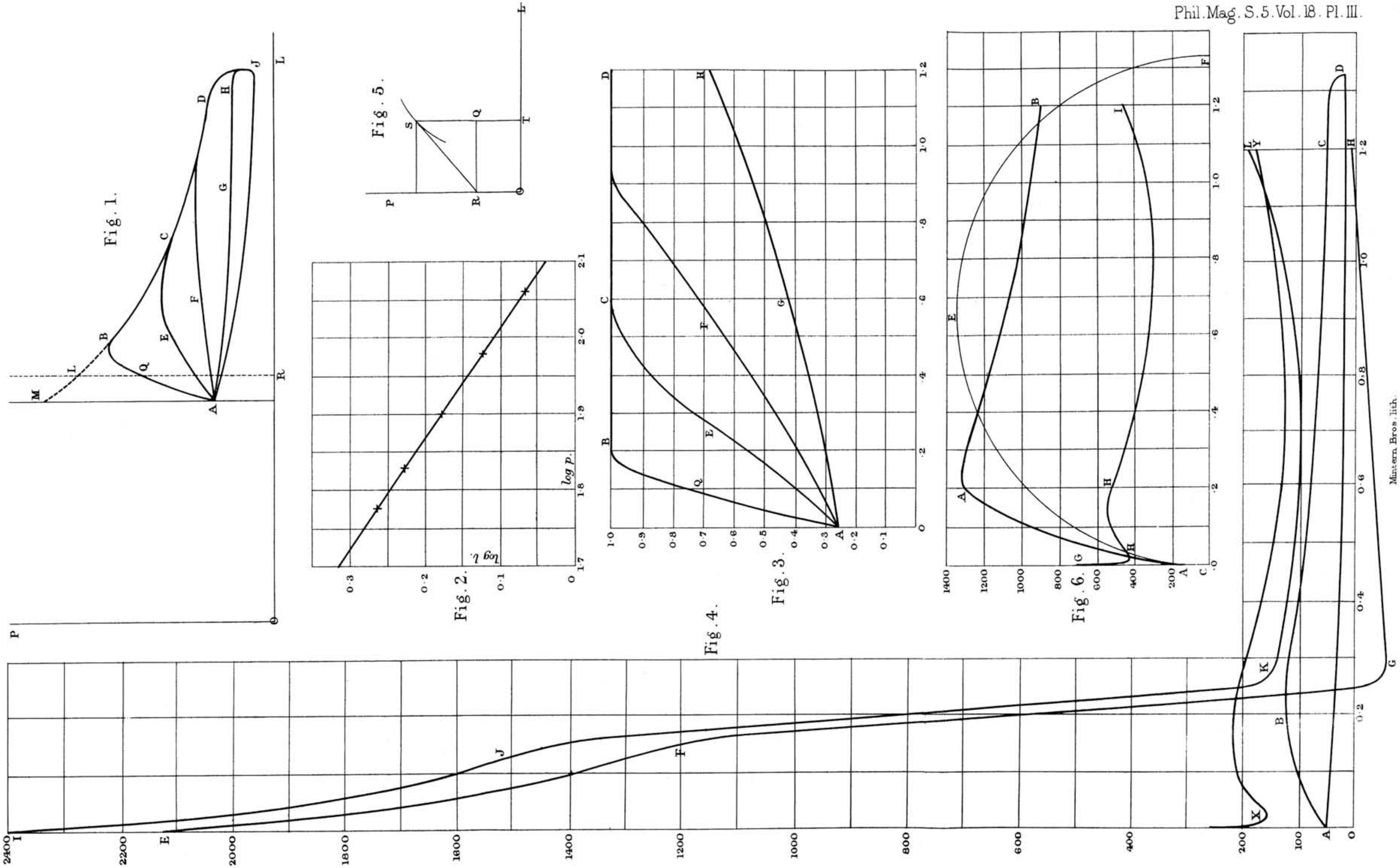
Fig. 1.
Fig. 2.
log p.
log l.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.
Mintern Bros. lith.

specially noticeable in the early part of the expansion-curves of gas-engine diagrams.

It is further to be remarked that the vibration of the indicator-spring is very visible in the expansion part, because we have every reason to believe that the expansion part ought to have no sinuosities; but it is our belief that this vibration has its effect on the explosion part of the curve as well, and that it is of the utmost importance to find some means of eliminating the effects produced by these vibrations of the indicator-spring.

In our communication to the Journal of the Society of Telegraph Engineers, p. 391, vol. v. (1876), we showed how to eliminate such vibrations in any case where the effect to be measured followed a regular law of increase or diminution, as in the case of the expansion part of this indicator-diagram. We have not yet sufficient information to enable us to employ this method on the explosion part.

The rule which we arrived at is as follows:—Draw two curves, A and B, through the highest and lowest points of the wavy curve which represents the actual observations: draw ordinates: the points of bisection of the parts of the ordinates intercepted between A and B lie on the correct curve.

6. *Empirical Formula for the whole Diagram.*—Now, inasmuch as the compression and expansion parts of all the curves follow the same laws, it would seem to be important to obtain one general formula for all the diagrams with one or more variable parameters. We have found that when we produce the expansion-curve (using formula 1), as shown at B M (fig. 1), and when we divide the pressure Q R at any part of the stroke by the corresponding ordinate L R of the expansion-curve, doing this for many parts of the stroke, we get the ordinate of an interesting curve. We have done this for the four diagrams of fig. 1, and obtain the four curves A Q B D, A E C D, A F D, A G H of fig. 3. From a study of these curves, which are nearly formed of straight lines, it will be found that the ignition and expansion parts of any diagram satisfy approximately the law

$$p=145{\cdot}5\, l^{-1{\cdot}479}\left\{\kappa'+n\lambda-\sqrt{(\kappa-n\lambda)^2+s}\right\}. \quad . \quad . \quad (3)$$

The smaller the value of s the more nearly do the curves of fig. 3 approach straight lines. In our present case $\lambda=l-0{\cdot}889$, $\kappa'=0{\cdot}6343$, $\kappa=0{\cdot}3637$; $s=0{\cdot}0087$, and n has different values for the four diagrams. These constants are evidently easily calculated from any diagrams. For the curve shown as A B C D in fig. 1 (Pl. III.) the value of n is $2{\cdot}2876$. Using this value and calculating p for the following values of l, we have

TABLE VII.

λ.	l.	Observed. p.	Calculated. p.	Expansion-curve calculated. p.
0	·889	45	45	173
·061	·950	86	83	157
·111	1·000	108	108	146
·170	1·059	123	124	134
·211	1·100	126	125	126
·311	1·200	111	111	111
·511	1·400	88	88	89
·711	1·600	72	73	73
·911	1·800	60	61	61
1·111	2·000	52	52	52

The differences between the observed and calculated values of p are within the limits of errors of observation therefore. It may be that there is a discontinuity in the ignition-curve at $\lambda = \cdot 061$ for a mixture of gas and air; but no discontinuity shows itself in the indicator-diagram in a sufficiently marked manner to be distinguished from vibrations due to the spring of the indicator.

As we have already stated, the recentness of the last explosion affects the temperature of the mixture of gas and air, and therefore the mass of the mixture. Hence we often find that the expansion parts of successive indicator-diagrams do not coincide. The law

$$p = \mathrm{M} l^{-1\cdot 479}\left\{\kappa' + n\lambda - \sqrt{(\kappa - n\lambda)^2 + s}\right\},$$

where κ', κ, and n are constants, satisfies all the diagrams.

n is a constant whose value depends on the point in the stroke at which the maximum pressure occurs; and this, for a given speed of engine, depends principally on the proportion of gas to air in the mixture;

M is a constant which depends on the recentness of the last explosion.

7. *Simple Formulæ for the Ignition and Expansion parts of the Diagram.*—The complete formula (3) assumes no want of continuity in proceeding from the ignition to the expansion part of the diagram. For the sake of ease in practical calculations we may, however, assume that the ignition- and expansion-curves are quite discontinuous, employing some method of indicating continuity when results are plotted on squared paper. In this case we assume that the lines of fig. 3 are

quite straight, so that

the ignition part of curve is $(a+b\lambda)\kappa l^{-m}$, . . (4)

the expansion part of curve is κl^{-m}. . . . . (5)

In our diagrams,

$$\kappa=145{\cdot}5, \quad m=1{\cdot}479\,;$$

Also $a=0{\cdot}257$.

In curve A B C D, $b=4{\cdot}372$;

" A E D, $b=1{\cdot}457$;

" A F D, $b=0{\cdot}782$;

" A G H, $b=0{\cdot}313$.

To use formula (4) in any given case. Find by the method already given in § 4 the constants of equation (5) to the expansion part. Assume that the ignition is complete when $\lambda=\lambda_1$. Let the pressure at the beginning of the stroke be p_0; calculate the value of κl^{-m} when $\lambda=0$, that is, when l=clearance or l_0, say; then

$$p_0 \div \kappa l_0^{-m} \text{ is } a,$$

and

$$a+b\lambda_1=1;$$

so that

$$b=\frac{1-a}{\lambda_1},$$

or

$$a=\frac{p_0}{\kappa}l_0^m \text{ and } b=\frac{1-\frac{p_0}{\kappa}l_0^m}{\lambda_1}.$$

8. *The Rate at which the Fluid receives Heat as calculated from its Volume and Pressure.*—We shall now proceed to calculate the heat received by the fluid. This we shall do as if, instead of there being combustion going on among the particles of the fluid, we had the fluid a perfect gas receiving heat from a great number of little furnaces, or pieces of hot wire immersed in the fluid. Besides the heat here considered we have the heat radiated to the cold cylinder.

If A is the area of the piston in square inches, p the pressure of the fluid in pounds per square inch, and l the distance moved through by piston in feet, the work done by the fluid on the piston in an element of length dl is

$$\mathrm{A}p\,dl \text{ foot-pounds.}$$

It is evident that if we represent the heat which is received by the fluid in the length dl by

$$\mathrm{A}q\,dl \text{ foot-pounds,}$$

we may regard q as the rate (per foot-travel of the piston) at which heat is received by the fluid, just in the same sense as p the pressure is the rate at which work is done by the fluid; and a comparison of q and p shows at once the comparison between the rates at which heat is being received and work is being done.

Now, since we consider the fluid to behave like a perfect gas, we know from thermodynamics that

$$q=\frac{1}{\gamma-1}\left(\gamma p+l\frac{dp}{dl}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where $\gamma=1{\cdot}37$ (see §§ 3 and 4); and it is obvious that the relation of p to q is not altered in any way by altering the scale to which l or p is represented in the indicator-diagram.

We have taken three methods of drawing the curve whose ordinate is q. In the first method a list of the values of p was made out from careful measurements of the curve ABCD (fig. 1), for values of l, 0·889, 0·9, 0·911, 0·922, 0·933, 0·944, &c. The observed increment of p divided by the increment of l was taken to represent the value of $\frac{dp}{dl}$ for the mean value of l. In this laborious way q was obtained for many values of l, and the curve E F G H (fig. 4) represents our result. It is obvious that the rate at which the fluid receives heat is greatest at the very beginning of the stroke, falling off during the ignition period, much more rapidly at the end of the ignition period; and in G H we see that the fluid is losing heat during what we call the expansion part of the stroke. In the same figure the actual indicator-diagram is shown in A B C D, the pressure being shown to the same scale as the ordinate of the heat-diagram.

The area between E F G H, the line Oλ, and any two ordinates, shows in foot-pounds the heat given to the fluid between the two positions, to the same scale as that to which the area of the indicator-diagram represents work done.

Another method which we have taken is this. To find q corresponding to a point S (fig. 5) on the indicator-curve. Draw a tangent S R to the indicator-curve, meeting the line O P in R. O P and O L are the lines O P and O L of fig. 1. Draw from R a line R Q parallel to O L, meeting the ordinate from S in the point Q. Then the distance S Q, or rather TS$-$TQ, represents

$$l\frac{dp}{dl}.$$

Measuring S Q, therefore, paying attention to the fact that it may be negative or positive, adding to it γ times S T and dividing by $\gamma-1$, we obtain the ordinate q of the heat-curve.

Another method is to calculate q as given in (6), by actual differentiation of p as given in (3). We have employed this method, and believe that our result, in which the early part of the curve E F (fig. 4) is less steep than there shown, is probably more nearly true than what is given in fig. 4. We do not put our result forward at present because there may be a discontinuity in the explosion part of the curve, as Table VII. shows that at $\lambda=\cdot061$ the empirical formula does not give the observed pressure; and until we know how to eliminate vibrational effects of the indicator-spring, we have preferred only to publish the curve E F G H which has been obtained from the actual diagram, the expansion part only having been corrected for vibrations. As we know that the expansion part follows a law

$$p=\kappa l^{-m},$$

it is obvious that

$$q=\frac{\gamma-m}{\gamma-1}p,$$

being proportional to the pressure. Now, comparing this result with the part G H (fig. 4), we see that G H is not quite correct, although determined from most careful measurements of the indicator-diagram. We see here an illustration of the great importance of obtaining a formula such as that given in (3) for the shape of the indicator-diagram.

9. *Total Heat and Work of One Cycle.*—The integral of $q\,.\,dl$ multiplied by the area A of the piston in square inches gives the total heat received by the fluid during any part of the stroke, and is evidently

$$A\int_{l_1}^{l_2} q\,.\,dl=\frac{A}{\gamma-1}(p_2l_2-p_1l_1)+A\int_{l_1}^{l_2} p\,.\,dl.$$

Taking from the diagram

$$p_1=44\cdot5,\quad l_1=\cdot889,$$

$$p_2=49\cdot0,\quad l_2=2\cdot089,$$

so that $l_2-l_1=1\cdot2$ foot (l_2 corresponds to the part of the stroke at which the exhaust-valve opens), it is evident that

$$\int_{l_1}^{l_2} q\,.\,dl=169\cdot5+\int_{l_1}^{l_2} p\,.\,dl. \quad . \quad . \quad . \quad . \quad (7)$$

Or if q_m is the mean value of q during this portion of the stroke, and if p_m is the mean value of p, then

$$1{\cdot}2\, q_m = 169{\cdot}5 + 1{\cdot}2\, p_m,$$

or

$$q_m = 141{\cdot}25 + p_m.$$

We find from the diagram that $p_m = 94{\cdot}5$, so that $q_m = 235{\cdot}75$.

Now $1{\cdot}2\, \mathrm{A} q_m$ is H, the total heat given to the fluid to alter its volume and pressure until the exhaust-valve opens;

$1{\cdot}333\, \mathrm{A} \times 61{\cdot}52$ is the indicated work W of the cycle, as calculated from the total indicator-diagram, including the dismission and suction parts not shown in fig. 1.

Hence $$\mathrm{H} = \frac{1{\cdot}2 \times 235{\cdot}75}{1{\cdot}333 \times 61{\cdot}52} \mathrm{W}, \text{ or } 3{\cdot}45\, \mathrm{W}.$$

As we know that combustion, about the period of opening of the exhaust-valve, is just sufficient to supply the loss by radiation to the cylinder without having much effect on the volume and pressure of the fluid, we can assume that any combustion after that time produces heat which is radiated to the cylinder. We are told that there is no combustion in the exhaust. For the small amount of combustion after the exhaust-valve opens we do not see our way at present to the basis of any but the very roughest assumption, and we think that attention ought to be paid to this matter in future investigations. What complicates the question is the fact that the mass of the fluid which radiates heat to the cylinder rapidly gets less after the exhaust-valve opens. To obtain a first approximation, we may assume that the heat of combustion after the exhaust-valve opens is equal to the work done in the forward stroke after that time—that is, 0·14 W.

The heat retained by the fluid is $141{\cdot}25 \times \mathrm{A} \times 1{\cdot}2$, or 1·94 W.

The gases in the exhaust-pipe close to the cylinder are known to have a temperature not much greater than 400° C. Hitherto it has been customary to calculate the amount of heat carried off by the gases through the exhaust-pipe from the heat-capacity multiplied by the difference of temperature from that of the atmosphere. It must be remembered, however, that the total heat of combustion of coal-gas contains the latent heat of the steam produced, and that the exhaust gases carry off this heat. Hence the amount of about 0·95 W, deducible from the experiments of Messrs. Brooks and Steward, must be increased by two thirds of itself, giving 1·57 W as the energy carried off by the gases in the exhaust-pipe.

Hence (1·94−1·57) W, or 0·37 W, is the amount of energy which is lost by the fluid from the opening of the exhaust-valve until the fluid is passing along the exhaust-pipe outside the cylinder. This is largely expended in heating the cylinder itself, in friction at bends in the exhaust-valve, &c.; so that it disappears as radiated heat, and as heat given to the water-jacket during the remaining parts of the cycle. Now we may safely take it that the expenditure of gas is about 22 cubic feet per hour per indicated horse-power; so that, using the heat of combustion calculated in Table I., the total energy of combustion of the gas used is 5·91 W; and we are now in a position to make the following table. Of the 5·91 W, the total energy of combustion of a charge, we have:—

1·38 W. Work of forward working-stroke till exhaust-valve opens.

0·14 W. Work of forward working-stroke after exhaust-valve opens.

2·31 W. Heat given to the cylinder during forward working stroke by radiation before exhaust-valve opens*.

0·14 W. Heat of combustion after exhaust-valve opens and which is radiated to the cylinder.

1·57 W. Heat carried off by gases in exhaust-pipe.

0·37 W. Given to the cylinder as heat after the exhaust-valve opens, partly by friction at the exhaust-valve, partly during the succeeding three fourths or non-working parts of the cycle.

It is found by experiment that the water from the water-jacket carries off somewhat more than 50 per cent. of the total heat of combustion, or 2·96 W; but it is almost impossible to make this measurement accurately for one cycle. It is sometimes as much as 62 per cent. and sometimes as little as 35 per cent. We have not employed such a measurement here, partly for this reason, and partly on account of the rate of loss during three fourths of the cycle being indeterminate. Again, the cylinder loses heat by radiation as well as by the water-jacket; so that, even if we could assume that such a number as

* It may be well to state here that we do not know with certainty the amount of gas used per indicated horse-power when the particular diagram which we are discussing was being taken, nor are we quite sure that Mr. Clerk is right in saying that the exhaust-gases show complete combustion when the engine is working to its full power. Our arrangements for determining this latter point are now nearly complete. We consider that there was no possibility of the expenditure having been less than 20 cubic feet per hour per indicated horse-power when our diagram was taken; and if we take 24 as a higher limit, and assume with Mr. Clerk that combustion is complete in the exhaust-pipe, we find the amount of heat given to the cylinder during the forward-working stroke by radiation before the exhaust-valve opens could not have been less than 1·77 W, the higher limit giving 2·85 W.

50 per cent. is correct, we are still not in a position to state the total loss of heat from the cylinder.

It is to be remembered that W is the indicated work. The useful work of a gas-engine, given out by the crank-shaft, is about 0·8 W, there being an expenditure of 0·2 W in overcoming the mechanical friction of the engine.

10. It is unnecessary to put before the Society the curves obtained by us by employing (6) on the discontinuous expressions of § 7 for the indicator-diagrams.

11. *Rate of Loss of Heat by the Fluid during Compression.*—For the compression part of the diagrams,

$$\gamma = 1{\cdot}385,$$

and

$$p = 39{\cdot}36\, l^{1{\cdot}304}.$$

Rate at which heat is received by fluid is $-q$, if

$$q = \frac{1}{\gamma - 1}\left(\gamma p + l\frac{dp}{dl}\right);$$

and by § 8,

$$-q = \frac{36{\cdot}36\, l^{1{\cdot}304}(m-\gamma)}{\gamma - 1}$$

$$-q = l^{1{\cdot}304}\, 39{\cdot}36 \times \frac{m-\gamma}{\gamma-1},$$

$$m - \gamma = -{\cdot}081,$$

$$\gamma - 1 = {\cdot}385.$$

Therefore the rate at which heat is received by fluid is

$$-0{\cdot}2104 p,$$

being proportional to the pressure, and is negative—that is, the fluid is radiating heat to the cylinder.

12. *Rate at which Fluid radiates Heat to cold Cylinder.*—It was found by the pyrometric measurements of Messrs. Brooks and Steward that the temperature of the products of combustion in the clearance-space, if there has been a recent explosion, is about 410° C.; and for the purpose of determining the temperature of the fluid before compression, they take 1·4 volume of coal-gas and 9·25 of air at 22° C. with 7·94 volumes of products of past combustions at 410° C., from which, assuming that the specific heats of the constituents do not alter with temperature, they find that the temperature of the mixture before compression is 120° C. This is sufficiently correct for our present purpose, and if we take it as the temperature when $l = 2{\cdot}222$, $p_0 = 14{\cdot}7$, we can find the temperature corresponding with any point of any of the indicator-diagrams.

We have made the calculations for various points on curve A B C D (fig. 1), knowing that $pl \div T$ is constant (see § 3).

TABLE VIII.

l.	p.	Absolute temperature. T.	Temperature of fluid. $t°$ C.	$t°$ −60° C.	Ordinate of semicircle.	Ratio.
·889	44·5	476	203	143	0	∞
·949	86	982	709	649	28	23·18
·989	105	1244	974	914	35·5	25·74
1·089	126	1644	1371	1311	48	27·33
1·389	90	1496	1223	1163	65	17·89
1·689	66	1344	1071	1011	66	15·32
2·089	49	1231	958	898	40	22·45

From some diagrams examined by us we find that the fluid may have as high a temperature as 1900° C.

The temperature of the water leaving the water-jacket was about 60° C.; and we may for our purposes assume that the rate at which the fluid loses heat to the cylinder is proportional to the excess of its temperature above 60° C., so that $t-60°$ represents, to some scale, the fluid's loss of heat to the cylinder per second. This rate of loss is shown on the curve A B (fig. 6).

Now if H is the quantity of heat which has been given by the fluid to the cylinder-jacket at the time τ, then $\frac{dH}{d\tau}$ is represented by the ordinate of the curve A B. But

$$\frac{dH}{d\lambda}=\frac{dH}{d\tau}\frac{d\tau}{d\lambda} \text{ or } \frac{dH}{d\tau}\div\frac{d\lambda}{d\tau};$$

and as the piston-motion is very nearly

$$\lambda=r(1-\cos a\tau),$$

$$\frac{d\lambda}{d\tau}=ra\sin a\tau;$$

that is, $\frac{d\lambda}{d\tau}$ is proportional to the ordinate of a semicircle C E F. Describing the semicircle C E F with a diameter equal to the length of the stroke, and dividing the ordinates of A B by those of C E F, we obtain the curve G H I, whose ordinate at any point represents the rate of the fluid's loss of heat to the cylinder per foot of piston-motion; so that the whole area

C G H I F represents the total loss of heat to the cold cylinder during the stroke. We are in a position to speak of the loss of heat from $\lambda=0$ to $\lambda=1{\cdot}2$ (see § 9).

This loss must be represented on the diagram to the same scale as the indicator-diagram represents work done, and it is expended in the part of the working stroke from $\lambda=0$ to $\lambda=1{\cdot}2$. Hence if k is the mean ordinate of such a diagram,

$$k=2{\cdot}31\times 61{\cdot}52\times 1{\cdot}330\div 1{\cdot}2=158.$$

Now the mean ordinate of the curve G H I (fig. 6) from $\lambda=0$ to $\lambda=1{\cdot}2$ being taken and found different from 158, all the ordinates of G H I (fig. 6) have been diminished in the proportion of 158 to the mean ordinate of G H I (fig. 6) to get the diagram X L Y of fig. 4. This diagram X L Y represents the rate of loss of heat by conduction and convection from the fluid to the cylinder during the working stroke until the exhaust-valve opens.

13. *Rate at which Combustion goes on during the Stroke.*—The curve E F G H represents the rate at which heat is actually gained by the fluid, and X L Y shows the rate at which heat is wasted to the cylinder; so that the curve I J K Y shows at every point the rate at which heat is being generated in the fluid by combustion. It is obvious, then, that combustion is not complete at the end of the explosion part of the curve, although, as Mr. Clerk's experiments prove, the mixture of air and gas is in the proper proportions for explosion immediately behind the piston at all periods of the compression-stroke. The diagram I J K Y is specially valuable, as showing the effect of dissociation of the products of combustion at such temperatures as obtain in the gas-engine, and are shown in Table VIII.

Postscript, *added June 10th.*—We have assumed in the paper that the rate of loss of heat by radiation and convection is proportional to the difference between the mean temperature of the fluid and the temperature of the cylinder. When we have more information concerning the distribution of heat in the fluid, and the way in which a heated fluid loses its heat to a cold enclosing vessel, a more accurate assumption may be made; and it is easy to see what alteration this will introduce in our method of obtaining the curve X L Y (fig. 4).

It is known from the experiments of MM. Dulong and Petit that the rate of cooling by radiation and convection of a solid body increases more rapidly than the difference of temperature, and that it is greater at greater pressures of the gaseous medium between the hot body and the surrounding cold vessel.

We are now engaged in an investigation involving much higher differences of temperature than those of MM. Dulong and Petit; and in so far as we have obtained results for temperatures extending from about 800° C. to 1300° C., we have confirmed the conclusions of these gentlemen. Thus the loss of heat per second at 776° C. being 97·2, the loss at 1292° C. is 253·2.

Our method of experimenting is, we believe, new. The heated body is a spiral of platinum, whose change of electric resistance when there is change of temperature is known. It is surrounded by a vessel, blackened inside and maintained at constant temperature. An electric current is made to pass through the platinum spiral, maintaining it at any required temperature. An ammeter and voltmeter enable the current A and the difference of potentials V between the ends of the platinum spiral to be measured. Then VA is the rate at which heat is being radiated from the platinum, and V / A is the electric resistance of the platinum, from which its temperature is known. We intend to investigate the influence of high pressures of air and other gases.

We are somewhat doubtful as to the weight which we ought to give to the results obtainable from these experiments on the loss of heat by solid bodies, since in our gas-engine investigations, we deal with a mixture of hot gases; and in adopting the law of simple proportionality to difference of temperature, we have been influenced by the fact that rate of loss of heat by the fluid in the compression-stroke increases much more slowly than is indicated in Dulong and Petit's law, although the pressure of the fluid is increasing as well as its temperature. The result given in § 11 is to the effect that during compression the rate of loss of heat by the fluid is nearly proportional to the $\frac{13}{23}$ power of the absolute temperature of the fluid, or to

$$(\theta+333)^{\frac{13}{23}},$$

if θ is the difference of temperature from 60° C., the temperature taken as that of the cold cylinder. If q is this rate of loss, it is obvious that $\frac{dq}{d\theta}$ diminishes as θ increases. It will be observed that the probability of the piston's having a high temperature causes this result to be even more curious than it might otherwise appear to be.