

the Indians, and he was suffered to depart in peace. These, Dr E. thinks, may have been descendants of *Madoc's followers*; and he seems inclined to ascribe to them also those very remarkable mounds, fortifications, and enclosures, which are found in such quantity in the valleys of the *Mississippi* and the *Ohio*. He is inclined also to trace to these Welsh adventurers, or at least to some early Europeans, the now almost extinct tribe, the *Mandans*—a people fairer and handsomer than the *Red men*,—that are now found 1800 miles above St Louis, on the Missouri, as described by Lewis and Clarke, and Catlin, the American travellers.

These, and several other circumstances, which might have been adduced, prove that Columbus cannot be regarded as the original discoverer of the New World.

The following Donations to the Library were announced :

Philosophical Transactions of the Royal Society of London, 1849.

Part II. 4to.—*By the Society.*

Kongl. Vetenskaps. Akademiens Handlingar, för 1847 & 1848. 8vo.

Årsberättelser om Botaniske Arbeten och Uptäckter för 1843 & 1844. 8vo.

Årsberättelse om Framstegen i Kemi under År. 1847. 8vo.

Årsberättelse om Technologiens Framsteg. 1842, 1843, 1844, 1846. 8vo.

Öfversigt af Kongl. Vetenskaps. Akademiens Forhandlingar. 1848. 8vo.—*By the Academy.*

*Monday, 18th February 1850.*

The Right Rev. BISHOP TERROT, V.P., in the Chair.

The following Communications were read :—

1. On the Equilibrium of Elastic Solids. By James Clerk Maxwell, Esq. Communicated by the Secretary.

This paper commenced by pointing out the insufficiency of all theories of elastic solids, in which the equations do not contain two

independent constants deduced from experiments. One of these constants is common to liquids and solids, and is called the modulus of *cubical* elasticity. The other is peculiar to solids, and is here called the modulus of *linear* elasticity. The equations of Navier, Poisson, and Lamé and Clapeyron, contain only one coefficient; and Professor G. G. Stokes of Cambridge, seems to have formed the first theory of elastic solids which recognised the independence of cubical and linear elasticity, although M. Cauchy seems to have suggested a modification of the old theories, which made the ratio of linear to cubical elasticity the same for all substances. Professor Stokes has deduced the theory of elastic solids from that of the motion of fluids, and his equations are identical with those of this paper, which are deduced from the two following assumptions.

In an element of an elastic solid, acted on by three pressures at right angles to one another, as long as the compressions do not pass the limits of perfect elasticity—

1st, The sum of the pressures, in three rectangular axes, is proportional to the sum of the compressions in those axes.

2d, The difference of the pressures in two axes at right angles to one another, is proportional to the difference of the compressions in those axes.

Or, in symbols :—

$$1. \quad (P_1 + P_2 + P_3) = 3 \mu \left( \frac{\delta x}{x} + \frac{\delta y}{y} + \frac{\delta z}{z} \right)$$

$$2. \quad \begin{cases} (P_1 - P_2) = m \left( \frac{\delta x}{x} - \frac{\delta y}{y} \right) \\ (P_2 - P_3) = m \left( \frac{\delta y}{y} - \frac{\delta z}{z} \right) \\ (P_3 - P_1) = m \left( \frac{\delta z}{z} - \frac{\delta x}{x} \right) \end{cases}$$

$\mu$  being the modulus of *cubical*, and  $m$  that of *linear* elasticity.

These equations are found to be very convenient for the solution of problems, some of which were given in the latter part of the paper.

These particular cases were—

That of an elastic hollow cylinder, the exterior surface of which was fixed, while the interior was turned through a small angle. The action of a transparent solid thus twisted on polarized light, was calculated, and the calculation confirmed by experiment.

The second case related to the torsion of cylindric rods, and a method was given by which  $m$  may be found. The quantity  $E = \frac{9 m n}{m + 6 n}$  was found by elongating, or by bending the rod used to determine  $m$ , and  $\mu$  is found by the equation,

$$\mu = \frac{E m}{9 m - 6 E}.$$

The effect of pressure on the surfaces of a hollow sphere or cylinder was calculated, and the result applied to the determination of the cubical compressibility of liquids and solids.

An expression was found for the curvature of an elastic plate exposed to pressure on one side ; and the state of cylinders acted on by centrifugal force and by heat was determined.

The principle of the superposition of compressions and pressures was applied to the case of a bent beam, and a formula was given to determine  $E$  from the deflection of a beam supported at both ends and loaded at the middle.

The paper concluded with a conjecture, that as the quantity  $\omega$ , (which expresses the relation of the inequality of pressure in a solid to the doubly-refracting force produced) is probably a function of  $m$  ; the determination of these quantities for different substances might lead to a more complete theory of double refraction, and extend our knowledge of the laws of optics.

## 2. Two Letters from W. E. Logan, Esq., to Earl Cathcart.

These letters were dated in August 1846 and September 1847. Earl Cathcart intended himself to have read them to the Society, but, having been prevented by his official duties from coming to Edinburgh, had sent them, to be communicated in his name.

In the first letter, the author, who had been sent to examine the geology of Canada, describes a visit which he made, on his way to Fort-William, Lake Superior, to the silver and copper mines on the south side of the lake, in the territory of the United States.

He considers the formation in which the mines occur as being older than the new red. They consist of parallel ranges of trap and conglomerate, apparently interstratified. They are well displayed at and near Copper Harbour. They are sometimes so thick as to