Electron Mass Emerging from the Vacuum Acceleration and the Hubble Parameter's Relation to the Fine Structure Constant in Six Dimensional Geometry

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Abstract

In this work, it is shown that the vacuum acceleration is related to the electron mass, and the Hubble parameter exhibits a reciprocal scaling law with respect to the square of the fine-structure constant, such that $H_0 \propto \alpha^{-2}$. The present model extends Kaluza-Klein theory to include an orthogonal compact timelike dimension, where the compact spacelike and timelike dimensions comprise a phase-plane. The division of energy and the projection onto 3+1D spacetime from this 4+2D geometry gives $a_{\text{vac}} = \frac{cH_0}{4\sqrt{2}} \approx 1.2 \times 10^{-10}\text{m/s}^2$. Coincidentally, this matches acceleration a_0 noted by Milgrom, where galactic rotation curves transition from Newtonian dynamics. We use this same geometric model to describe the closed topology of an electron satisfying $m_{\text{e}} = \left(\frac{\alpha^2\hbar^2 a_{\text{vac}}}{16Gc^2}\right)^{1/3}$. This derivation comes from two postulates. First, that relativistic effects are a rotation of the vector $\mathbf{x}\,(t)$ towards the orthogonal, compact phase-plane. Second, that the particle's internal dynamics are in equilibrium with the vacuum acceleration. Measured values of the Hubble parameter from SH0ES and Planck give us values that are a bit too high and a bit too low for the electron mass, respectively. When we input the precisely measured value of the electron mass into the present equation, it yields the local Hubble parameter as $71.3273\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$, consistent with the Chicago-Carnegie Hubble Program.

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I. INTRODUCTION

We derive a cubic equation for $m_{\rm e}$ by setting the electromagnetic self-energy equal to the rest energy. We then equate the electromagnetic radius to Milgrom's MOND radius, replacing the empirically measured a_0 inferred from galactic rotation curves with our derived vacuum acceleration $a_{\rm vac}$. The present model extends Kaluza-Klein theory to include a compact timelike dimension, for a total of 6 dimensions. In this model, the energy of the universe is partitioned between two orthogonal planes, and partitioned again between the electromagnetic an gravitational planes, giving us the appropriate factors for both the vacuum acceleration and the 6 dimensional momentum that is projected onto 4D spacetime as the electron's rest mass.

II. 6D GEOMETRIC FRAMEWORK

A. The 6D Model: Key Postulates

We combine the extra spacelike dimension from Kaluza-Klein theory [1, 2] with the extra timelike dimension t_φ from Forte [3] into a 6D geometry with the signature $(-,+,+,+,-,+)$. The addition of one timelike and one spacelike dimension allows a photon to carry momentum on a complex "phase" plane $(r_w\,\varphi, i\,t_\varphi)$, representing an internal and inaccessible geometric limit of the observed 4D spacetime. The extra spatial dimension $r_w\,\varphi$, projects momentum onto 4D as electromagnetism (as described by Kaluza-Klein theory), while Forte's extra timelike dimension t_φ projects momentum onto 4D as mass. The 6D interval is expressed as:

$$d\mathcal{S}^2 = -c^2\left(dt^2 + dt_\varphi^2\right) + \left(dx^2 + r_{\rm w}^2\, d\varphi^2\right) + dy^2 + dz^2. \tag{1}$$

In polar coordinates, φ represents the angular phase and $r_{\rm w}$ is the winding radius. At rest, $dt_\varphi \to 0$ and $d\varphi \to 0$, recovering the standard 4D Minkowski metric. At light speed, $t \to 0$ and $dx \to 0$, and the null interval becomes $d\mathcal{S}^2 = -c^2 dt_\varphi^2 + r_{\rm w}^2\, d\varphi^2 + dy^2 + dz^2$, which is a 90 degree rotation onto the phase plane according to the relation $\tau^2 = t^2 + t_\varphi^2$ [3, Eq. (4)]. Similarly, $dx_0^2 = dx^2 + r_{\rm w}^2 d\varphi^2$, where relativistic contraction of dx is a rotation onto the phase plane. The total 6D velocity remains c.

Six-dimensional spacetime is partitioned into two orthogonal planes:

$$\text{Plane A (observable):} \quad (t, x)\,,$$
$$\text{Plane B (phase):} \quad (t_\varphi, r_w\,\varphi)\,.$$

with y and z as shared transverse dimensions. It is further split between electromagnetic $(t, r_w\,\varphi)$ and gravitational (t_φ, x) planes.

In a photon's proper time, it winds in a closed loop in the complex phase-plane, orthogonal to the gravitational plane. In the observer's frame, the photon's 6D worldline is a helix, where the circulation combines with a 4D translation, and projects onto 4D as a wave propagating at c. When the photon forms a closed 4D loop, it can be modelled as a loop that is tilted, with t as the axis, so that it is no longer at 90° to the x axis, projecting onto the gravitational plane and oscillating at c, forming a massive particle. The angle of projection onto 4D spacetime is observer dependent, based on Einstein's relativity, giving an effective wavelength and mass.

B. 6D Energy Partition and Dark Energy

From the present time-plane framework Ref. [3], we postulate the 4D-projected energy (fraction of total energy visible to 4D observers) is

$$E_{4D} = \frac{E_{6D}}{\sqrt{2}}, \tag{2}$$

where the dark-energy density (at equilibrium) is

$$\Omega_\Lambda = \frac{1}{\sqrt{2}} \approx 0.7071, \tag{3}$$

in agreement with DESI DR2 results $\Omega_\Lambda = 1 - \Omega_\mathrm{m} = 0.7025 \pm 0.0086$ [4].

III. ELECTRON MASS

We postulate rest mass emerges from a closed circular path of the photon. A geometric division appears in earlier zitterbewegung models [5–7]. However, the present 6D model has two geometric divisions. We model the massless photon carrying momentum with a 6D trajectory that is at 45° between the observable spacetime plane and compact phase plane (Sec. II B). When the photon forms a closed 4D loop, we postulate a second split between gravitational and electromagnetic planes. In 4D spacetime, this circular momentum manifests as the internal clock resonance observed in Gouanère's channelling experiments [8], where a dip observed at 81.1 MeV for electrons channelled through a silicon crystal is close to the theoretically predicted resonance condition at the de Broglie frequency.

A. Deriving the Equilibrium Condition

The classical electromagnetic radius is defined by equating the self-energy to the particle's rest-energy [9],

$$r_{\rm EM} = \frac{e^2}{4\pi\epsilon_0 mc^2} = \alpha\frac{\hbar}{mc}. \tag{4}$$

We postulate that the MOND scale is related to the classical electromagnetic scale by a factor of four,

$$R_{\rm MOND} = \sqrt{\frac{Gm}{a_{\rm vac}}} = \frac{1}{4}r_{\rm EM}. \tag{5}$$

Here, we use the vacuum acceleration $a_{\rm vac}$ in place of the MOND parameter a_0 empirically inferred from galactic rotation curves [10–12].

The "4/3 problem" identifies a discrepancy between the field energy and the inertial mass of a spherical charge distribution, necessitating the introduction of non-electromagnetic "Poincaré stresses" to maintain stability [13]. We model the electron with a closed one-dimensional trajectory (topology S^1), eliminating the factor of 3 from the assumed spherical three-dimensional structure - the symmetric charge is an equal projection on 3 space dimensions from 6D. The factor of 4 in Eq. (5) contains a factor of 2 for the division of energy between the spacetime and phase planes, since proper time $\tau^2 = t^2 + t_\varphi^2$ Ref. [3], and another factor of 2 since half of the 6D action is coupled to the gravitational plane since proper space $dx_0^2 = dx^2 + r_{\rm w}^2 d\varphi^2$. Squaring $R_{\rm MOND}$ and rearranging:

$$\sqrt{\frac{Gm}{a_{\rm vac}}} = \frac{\alpha\hbar}{4mc}, \tag{6}$$

$$\frac{Gm}{a_{\rm vac}} = \frac{\alpha^2\hbar^2}{16m^2c^2}, \tag{7}$$

which gives

$$\boxed{m_{\rm e}^3 = \frac{\alpha^2\hbar^2 a_{\rm vac}}{16Gc^2}}. \tag{8}$$

1. *Calculating the Electron Mass*

We substitute $\Omega_\Lambda = 1/\sqrt{2}$ from Sec. II B and introduce measured values for constants that are not part of the present geometric model:

$$a_{\mathrm{vac}} = \frac{cH_0\Omega_\Lambda}{4} = \frac{cH_0}{4\sqrt{2}}. \tag{9}$$

Substituting a_{vac}:

$$m_{\mathrm{e}}^3 = \frac{\alpha^2\hbar^2 cH_0}{64\sqrt{2}Gc^2}, \tag{10}$$

$$m_{\mathrm{e}} = \left(\frac{\alpha^2\hbar^2 H_0}{64\sqrt{2}Gc}\right)^{1/3}. \tag{11}$$

Using standard constants and observational values (see Appendix A), we compute the vacuum acceleration (Appendix B) for measured values of the Hubble parameter, and substitute it into Eq. (11) to produce:

$$m_{\mathrm{e}}^{\mathrm{SH0ES}} = 9.18171764 \times 10^{-31}\,\mathrm{kg},$$
$$m_{\mathrm{e}}^{\mathrm{Freedman}} = 9.06930486 \times 10^{-31}\,\mathrm{kg}, \textit{and}$$
$$m_{\mathrm{e}}^{\mathrm{Planck}} = 8.93902829 \times 10^{-31}\,\mathrm{kg}.$$

all within 2% of the CODATA 2022 value $9.1093837139 \times 10^{-31}$ kg [14].

IV. H_0 PREDICTION

Using laboratory-measured values of $m_e, \alpha, G, \hbar$, and c, the framework predicts

$$H_0 = \frac{64\sqrt{2}Gcm_{\mathrm{e}}^3}{\alpha^2\hbar^2} = 71.3273\,\mathrm{km\,s^{-1}\,Mpc^{-1}}. \tag{12}$$

This is within 1σ agreement with Freedman CCHP $70.39 \pm 1.22\,(\mathrm{stat}) \pm 1.33\,(\mathrm{sys}) \pm 0.70\,(\sigma\mathrm{SN})\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ [15]. However, using $\Omega_\Lambda = 1/\sqrt{2}$ assumes perfect geometric equilibrium. Independent of this assumption, the framework predicts

$$H_0\Omega_\Lambda = \frac{64\,G\,m_e^3\,c}{\alpha^2\,\hbar^2} = 50.4360\mathrm{km\,s^{-1}\,Mpc^{-1}}. \tag{13}$$

This is in agreement with $H_0^{\mathrm{SH0ES}}\,\Omega_\Lambda^{\mathrm{Planck}} = 50.01 \pm 0.89\mathrm{km\,s^{-1}\,Mpc^{-1}}$ [16, 17]. Laboratory-measured inputs yield a present-epoch prediction, consistent with late-universe H_0 determinations rather than CMB-inferred values.

V. THE FACTOR 16 AND CONSISTENCY WITH 6D GEOMETRY

The factor $16 = 4^2$ in the denominator of Eq. (8) is consistent with a 6D geometric structure. If we did not divide by a factor of 2 for the partition of energy between the spacetime and phase planes, or the second factor of 2 for the partition of energy between electromagnetic and gravitational planes, the cubic equation would produce a value of H_0 inconsistent with observation by a factor of 16. The predicted value of H_0 in Eq. (12) falls within the range of current observational estimates.

VI. VACUUM ACCELERATION AND HUBBLE TENSION

We postulate that acceleration emerges from non-zero energy density and vacuum permittivity, where $H_0 \propto E^3\,\varepsilon_0^2$. This is scaled by $c/4\sqrt{2}$ as one quarter of the energy projects onto the gravitational plane and it is scaled a further $\sqrt{2}$ when projected onto the observable time dimension. The Hubble parameter exhibits a reciprocal scaling law with respect to the square of the fine-structure constant, such that $H_0 \propto 1/\alpha^2$.

VII. RELATION TO HAWKING RADIATION

In the present model, we postulate that a black hole is the relaxation of 6D into 2D, where $dt_0/d\tau \to 0$ and proper time rotates entirely into t_φ. This is the limit where the photon's 6D circular trajectory dilates to the Schwarzschild circumference, or it's 4D wavelength to the Schwarzschild diameter. If the horizon is a spin $-1/2$ closure, the wavelength is $\lambda_{\rm BH} = 4\pi r_s = 8\pi GM/c^2$, giving

$$\begin{aligned} E_{\rm 2D} &= \frac{hc}{\lambda_{\rm BH}}, && (14) \\ &= \frac{hc^3}{8\pi GM} = E_{\rm Hawking}, && \text{(Ref. [18, 19])} \end{aligned}$$

consistent with the equilibrium condition that fixes the radius for the electron. Hawking radiation arises from vacuum fluctuations external to the horizon, where the present model describes the internal configuration of in-falling mass and photons asymptotically driven to wavelength $\lambda_{\rm BH}$.

VIII. DISCUSSION

Equation (8) expresses m_e entirely in terms of independently measured constants, with $a_{vac} = cH_0/4\sqrt{2}$ introducing a cosmological dependence. Since m_e is measured in a local laboratory environment, the appropriate value of H_0 is the local expansion rate rather than the CMB-inferred global average. The Freedman et al.(2025) measurement [15], based on ten nearby galaxies, provides a local H_0 that is more representative of the vacuum energy density experienced in the present-day low-redshift universe, and yields a closer prediction of m_e than either Planck or SH0ES.

The equilibrium condition depends on the vacuum acceleration, or background energy density (dark energy). In areas of lower energy density, the radius of a particle may dilate, slowing the internal clock, consistent with general relativity. We predict that the inverse relationship between dark energy density and baryonic density implies shallower potential wells in the outer galaxy, favouring atomic over molecular hydrogen. This is consistent with the observed transition from H_2-dominated inner disk to HI-dominated outer disk, at $R \simeq 7.5$ to $8.5\,\mathrm{kpc}$ in the Milky Way [20, 21].

IX. CONCLUSIONS

1. Under the postulate that there is equal energy partition between the spacetime and phase planes, the dark-energy fraction is projected onto 4D spacetime as $\Omega_\Lambda = 1/\sqrt{2}$.

2. Under the postulate that there is a second energy partition between the electromagnetic and gravitational planes, the 6D geometry introduces a factor of 4 in the gravitational equilibrium condition $R_{\mathrm{MOND}} = r_{\mathrm{EM}}/4$.

3. Under the postulate that vacuum acceleration emerges from the energy density in relation to vacuum permittivity, $H_0 \propto E^3\,\varepsilon_0^2$, the present model justifies $a_{vac} = cH_0/4\sqrt{2}$.

4. These two geometric inputs (the energy split and projection onto 3+1D spacetime) yield the electron mass, Eq. (8), within 2% of the Hubble parameter from Planck (2018), SH0ES (2022), and Freedman et al. (2025), with no free parameters.

5. Using locally measured inputs, the present model predicts $H_0 = 71.3273\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ as a local geometric equilibrium, and $H_0\Omega_\Lambda = 50.4360\mathrm{km\,s^{-1}\,Mpc^{-1}}$, with precision

limited by the gravitational constant G.

6. Replacing the 6D model with 4D or 5D geometry removes a factor of 2^2 from Eq. 8, ruling out these alternatives.

7. The equilibrium condition links m_{e} directly to H_0, providing a connection between particle physics and cosmology through an underlying six dimensional spacetime.

Appendix A: Input Parameters

We use the following independently measured values. Constants are taken from CODATA 2022 [14], and measurements from Planck 2018 [17], Chicago-Carnegie Hubble Program [15], and SH0ES [16].

$$
\begin{aligned}
\alpha &= 7.2973525643(11) \times 10^{-3} \\
\hbar &= 1.054571817 \times 10^{-34}\,\mathrm{J \cdot s} \\
G &= 6.67430(15) \times 10^{-11}\,\mathrm{m^3 \cdot kg^{-1} \cdot s^{-2}} \\
c &= 2.99792458 \times 10^{8}\,\mathrm{m \cdot s^{-1}} \\
H_{0,\text{Planck 2018}} &= 67.4 \pm 0.54\,\mathrm{km \cdot s^{-1} \cdot Mpc^{-1}} \\
H_{0,\text{Freedman 2025}} &= 70.39 \pm 1.22\,\mathrm{km \cdot s^{-1} \cdot Mpc^{-1}} \\
H_{0,\text{SH0ES 2022}} &= 73.04 \pm 1.04\,\mathrm{km \cdot s^{-1} \cdot Mpc^{-1}}
\end{aligned}
$$

Appendix B: Derived Vacuum Acceleration

The vacuum acceleration scale is converted to SI:

$$
a_{\mathrm{vac}} = \frac{cH_0}{4\sqrt{2}} \frac{1000\,\mathrm{m \cdot km^{-1}}}{3.08567758 \times 10^{22}\,\mathrm{m \cdot Mpc^{-1}}} \tag{B1}
$$

Evaluating for different measurements of H_0:

$$
a_{\text{vac, SH0ES 2022}} = 1.25445745 \times 10^{-10}\,\mathrm{m \cdot s^{-2}} \tag{B2}
$$

$$
a_{\text{vac, Freedman 2025}} = 1.20894386 \times 10^{-10}\,\mathrm{m \cdot s^{-2}} \tag{B3}
$$

$$
a_{\text{vac, Planck 2018}} = 1.15759080 \times 10^{-10}\,\mathrm{m \cdot s^{-2}} \tag{B4}
$$

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DATA AVAILABILITY

No new data were generated or analyzed in this study.

CONFLICTS OF INTEREST

The author declares no conflict of interest.

AUTHOR CONTRIBUTIONS

The author was solely responsible for the conceptualization, analysis, and preparation of this manuscript.

GENERATIVE AI DISCLOSURE

The author takes full responsibility for all content, including the accuracy of the scientific results, derivations, and conclusions. Generative AI tools were used to assist with literature search, equation verification, and editorial review for clarity and consistency (Claude Sonnet

4.5 and 4.6).

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