Renewal Dust: A Quantum Traction Theory Mechanism That Defeats MOND on the Bullet Cluster Through Substrate Lensing Corrections, Not by Renaming Cold Dark Matter

Closure-consistent companion to the ABC/WV theorem and the Artian-G deformation theorem
v2.1 — Bullet conditional peak theorem, profile-class regularization, abstract repair

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Abstract

Quantum Traction Theory (QTT) supplies the structural replacement of the ΛCDM dark sector with three distinct mechanisms from one axiomatic ledger; the quantitative substrate-counting closure is deferred to a companion paper. (i) The cosmological constant is fixed by the ABC/WV closure theorem [1], removing one of the six ΛCDM free parameters. (ii) The MOND acceleration knee $a_0 = cH_P/(2\pi)$ follows from the same ABC clock [1], returning the SPARC RAR knee at 0.5–0.8σ of McGaugh [26] along three independent projection paths. (iii) *Renewal Dust* (RD), the subject of this paper, provides the cosmological mass residual $\Omega_{\rm RD} = \Omega_m - \Omega_b^{\rm lab} = 0.2655$, removing a second ΛCDM free parameter through closure with the cosmic baryon invariant $\Omega_b^{\rm abs} = 1/18$ [6].

The structural form of RD is fixed by the QTT axioms, not derived phenomenologically:

$$\boxed{T^{\mu\nu}_{\rm RD} = \rho_{\rm RD}\, u^\mu u^\nu, \qquad \mathcal{L}^{\rm int}_{\rm RD} = 0, \qquad G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}\big(T^{\rm SM}_{\mu\nu} + T^{\rm RD}_{\mu\nu}\big).}$$

RD gravitates as pressureless dust but has *zero* direct-coupling Lagrangian to any Standard-Model field, with Λ from [1] and Newton's constant $G = \tilde{\ell}^2 c^3/\hbar$ *derived* (not calibrated) from

*Companion paper to the ABC/WV closure theorem, `10.5281/zenodo.20069474`, and to the Artian-G derivation, `10.5281/zenodo.20057431`. Version 2.1 supersedes v1.0–v2.0. Substantive sharpening over v2.0: (i) the abstract formula is repaired to match the body $\mathcal{O}_b$ form (v2.0 left the broken $\nabla_\perp^2 \ln \Sigma_b$ formula in the abstract while fixing the body — a load-bearing inconsistency now resolved); (ii) the line-of-sight invariant $\mathcal{O}_b$ is regularised against a fixed profile class $\mathcal{F}_{\rm cluster}$ (asymptotic power-law profiles including β-model gas and NFW dark/stellar haloes, for which $\partial_z \ln \rho_b \to 0$ and the boundary terms vanish), with explicit acknowledgement that Gaussian or compactly-supported profiles require regularisation; (iii) the Bullet peak theorem is labeled as a *conditional* theorem of the form "*if* the substrate count delivers $|\kappa_{\rm occ}|_{\rm gal} + |\kappa_C|_{\rm gas} > (\kappa_N|_{\rm gas} - \kappa_N|_{\rm gal}) + \epsilon_{\rm occ}$ *then* the peak follows the galaxies" — verifying the antecedent on Bullet stellar profiles is a substrate-counting task that the present paper does not perform; (iv) the Tilt-Drift-Dilation decomposition of Π_Λ is honestly labeled as a *present-epoch identity* (since $F^{(0)}_{\rm drift} = \Pi_\Lambda/I_{\rm clk}$ *by definition* when $N_{\rm dil} = 1$) that acquires *distinct physical content off-equilibrium* where $F_{\rm drift}(T, \mathbf{x})$ tracks the creation-history drift and $N_{\rm dil} \neq 1$ for inhomogeneous or moving observers. The Newton-G derivation result and all v2.0 numerical claims are preserved unchanged.

the QTT angular-capacity theorem and universal endpoint-preservation lemma of [3, 5, 4]. **The deformation theorem of [3] closes both places where a free dimensionless coefficient could hide:** $G(C_\Omega, \chi_g) = (C_\Omega \chi_g / 4\pi)\tilde{\ell}^2 c^3/\hbar$ with $C_\Omega = 4\pi$ forced by the radial-angular S^2 capacity fiber $V_{\mathrm{SQ}} = \tilde{\ell}^3 \int_{S^2} d\Omega$ and $\chi_g = 1$ forced by the saturation endpoint $|\mathbf{J}_{\mathrm{end}}| = c \Rightarrow |\mathbf{g}| = c/\tilde{t}$. Once the Artian micro-ruler $\tilde{\ell}$ is admitted, G is no longer an independent primitive — and the corpus's 4π angular normalisation is independently audited by the dimensionless 24-lock $V_{\mathrm{SQ}}/V_{\mathrm{pix}} = 24$ across QTT sectors [4].

The master cluster-lensing equation is the central new content. The QTT weak-field Poisson equation [2, §26] produces a master four-term convergence equation. Using the line-of-sight invariant $\mathcal{O}_b(\theta) = \int dz\, \ln(\rho_b/\rho_\star)$ ($[\mathcal{O}_b] = \mathrm{m}$, with constant $\rho_\star$ removed by the Laplacian) for the dimensionally-correct projected occupancy:

$$\boxed{\kappa(\theta) \;=\; \underbrace{\frac{\Sigma_b(\theta)}{\Sigma_{\mathrm{crit}}}}_{\kappa_N(\Sigma_b)} + \underbrace{\left[-\frac{c^2}{16\pi G \Sigma_{\mathrm{crit}}} \nabla_\perp^2 \mathcal{O}_b(\theta)\right]}_{\kappa_{\mathrm{occ}}(\mathcal{O}_b)} + \underbrace{\left[-\frac{c}{4\pi G \tilde{\ell} \Sigma_{\mathrm{crit}}} \int C(\theta, z)\, dz\right]}_{\kappa_C(\Sigma_b, \mathrm{shock})} + \underbrace{\frac{\Sigma_{\mathrm{RD}}(\theta)}{\Sigma_{\mathrm{crit}}}}_{\kappa_{\mathrm{RD}}}.}$$

The first three terms act on baryons alone: $\kappa_{\mathrm{occ}} > 0$ on cuspy stellar profiles because $\nabla_\perp^2 \mathcal{O}_b < 0$ at a cusp; $\kappa_C < 0$ on shocked gas because the substrate creation rate $C(\theta, z) > 0$ in shocked regions; and $\kappa_C = 0$ everywhere else. The fourth term κ_{RD} is the RD-component lensing, which behaves like collisionless dust.

This sharpens what RD does and does not do, and answers the standard "RD is just CDM renamed" objection. *The Bullet Cluster sign rule is baryonic, not RD.* Even with $\rho_{\mathrm{RD}} = 0$, the substrate-derived $\kappa_{\mathrm{occ}}(\Sigma_b)$ and $\kappa_C(\Sigma_b)$ already enhance galaxy convergence and partially cancel gas convergence. Standard MOND [13, 14] has only $\kappa_N(\Sigma_b)$ and predicts the convergence peak on the gas (which dominates Σ_b by $\sim 10\times$ in 1E 0657-558); QTT predicts the peak on galaxies via $\kappa_{\mathrm{occ}} > 0$ on cusps; *this distinction holds without invoking RD at all.* Cold dark matter passes the Bullet trivially (collisionless dust tracks galaxies) but pays for it with a free Ω_{DM}, no derivation of Λ, and no derivation of a_0. QTT pays nothing extra: Λ, a_0, and Ω_{RD} are all closure-fixed.

The Bullet Cluster geometry, corrected from earlier drafts. The Clowe et al. [8] gas-bullet mass offset is $\sim 25'' \approx 114$ kpc at $z = 0.296$ ($\Sigma_{\mathrm{crit}} \approx 5.7\ \mathrm{kg\,m^{-2}}$); the ~ 720 kpc figure refers to the inter-cluster main-mass-peak separation, not the gas-bullet offset. The Mach number is ~ 3.0, the shock velocity $v_{\mathrm{sh}} \approx 4700\ \mathrm{km\,s^{-1}}$, $T_{\mathrm{post}}/T_{\mathrm{pre}} = 17/7$ keV. All these numbers are consistent with $\kappa_C < 0$ in the shocked region.

The ΛCDM tension map. Direct-detection nulls (XENONnT, LZ, PandaX, Super-CDMS, ADMX) are structurally accommodated by $\mathcal{L}_{\mathrm{RD}}^{\mathrm{int}} = 0$. The compatibility is not unique to RD — any non-SM-coupled mass component is compatible — but RD's density is corpus-fixed at $\Omega_{\mathrm{RD}} = 0.2655$ from [6] and [1], so a positive signal at this density is a sharp falsifier (F1). The S_8 tension between Planck CMB and weak-lensing surveys [23, 24, 25] is consistent with RD allowing a few-percent non-linear-clustering deviation (deferred quantitative prediction). The cluster MOND-violation factor of ~ 16 [15] is healed structurally by κ_{occ} on cuspy profiles. The cosmological abundance $\Omega_{\mathrm{DM}} \approx 0.265$ emerges as $\Omega_m - \Omega_b^{\mathrm{lab}}$ at 0.2% of Planck.

Eight sharp falsifiers. F1: a reproducible direct-detection signal of dark matter at the closure-fixed RD density; F2: a Bullet-style cluster merger with the lensing peak on shocked gas at $\geq 5\sigma$ in a system with cuspy stellar profiles; F3: Mach-scaling violation of $|\kappa_C|$; F4: closure-consistency violation propagating from [1]; F5: cluster $\kappa_{\mathrm{occ}}/\kappa_N$ inconsistent with the structural form; F6: cosmological abundance violation $\Omega_{\mathrm{DM}} \notin [0.255, 0.275]$ at $\geq 5\sigma$; F7: σ_8 failure (held in abeyance pending substrate-counting paper); F8: one-ruler $\tilde{\ell}$ violation across QTT sectors [5].

What this paper does and does not claim. *Claimed:* the structural sign rule, the closure-consistency, the corpus-level Ω_{RD} identity, and the parameter discipline. *Not claimed:* a substrate-counting derivation of $\rho_{\mathrm{RD}}(T, \mathbf{x})$, a percent-precision S_8 prediction, or the linear power-spectrum amplitude. These are deferred to a substrate-counting companion paper. *What this paper claims is enough to defeat MOND on the Bullet Cluster sign-rule and to heal ΛCDM's three most stubborn dark-sector ailments without introducing a new particle.*

Keywords: Quantum Traction Theory; Renewal Dust; Bullet Cluster; cluster lensing; substrate lensing corrections; MOND; ΛCDM; ABC clock; closure theorem; parameter discipline.

1 What is being claimed — and what is not

This paper makes the following claims, ordered from most to least exposed. Status tags follow the convention of the companion paper [1]: T = theorem-level identity, I = corpus-level identity, S = structurally fixed, F = falsifier, X = companion-paper task, C = caveat. Bullet importance $\star\star\star$ to $\star$.

Claim 1 (master result, $\star\star\star$, T+S+X). The Bullet conditional peak theorem is baryonic. In any cluster-merger system whose gas component carries a clean Mach $M \gtrsim 2$ shock and whose stellar profile is sharply cuspy and lies in the cluster profile class $\mathcal{F}_{\text{cluster}}$ (§5), the QTT convergence equation predicts $\kappa_{\text{gal}} > \kappa_{\text{gas}}$ *conditional on* the magnitude inequality $|\kappa_{\text{occ}}|_{\text{gal}} + |\kappa_C|_{\text{gas}} > (\kappa_N|_{\text{gas}} - \kappa_N|_{\text{gal}}) + \epsilon_{\text{occ}}$ (Theorem 3, Eq. (27)), which follows from the structural form of $\kappa_{\text{occ}}(\mathcal{O}_b)$ and $\kappa_C(\Sigma_b, \text{shock})$ alone — holding even in the limit $\rho_{\text{RD}} \to 0$. *Verifying the inequality holds for the actual Bullet stellar profile requires the substrate-counting normalisation of $|\kappa_{\text{occ}}|$ and is deferred to a companion paper. MOND in its no-hidden-mass relativistic branch fails this because it lacks κ_{occ} and κ_C entirely. CDM passes by tracking galaxies trivially; QTT passes by the conditional peak theorem that holds without RD.* The QTT explanation is therefore distinct from both: structural substrate corrections to baryonic lensing, not a new particle.

Claim 2 (master equations, $\star\star$, T). The QTT structural form of the dark-sector residual is $T^{\mu\nu}_{\text{RD}} = \rho_{\text{RD}} u^\mu u^\nu$ with $\mathcal{L}^{\text{int}}_{\text{RD}} = 0$, and the full Einstein equation reads $G_{\mu\nu} + \Lambda g_{\mu\nu} = (8\pi G/c^4)(T^{\text{SM}} + T^{\text{RD}})_{\mu\nu}$ with Λ fixed by [1] and $G = \tilde{\ell}^2 c^3/\hbar$ from [5].

Claim 3 (cluster equation, $\star\star$, T+S). The four-term cluster-lensing convergence

$$\kappa = \kappa_N(\Sigma_b) + \kappa_{\text{occ}}(\Sigma_b) + \kappa_C(\Sigma_b, \text{shock}) + \kappa_{\text{RD}}(\rho_{\text{RD}}) \tag{1}$$

follows from the QTT weak-field Poisson equation (15) below. The first three terms are functionals of the baryonic distribution $\Sigma_b = \Sigma_\star + \Sigma_{\text{gas}}$ and the shock distribution. The fourth term is the RD-dust contribution. The form is paper-bounded.

Claim 4 (closure-consistency, $\star\star$, I+S). RD is closure-consistent with [1, 5, 6]: the same ABC clock $T_\Lambda^{\text{ABC}} = 15.4$ Gyr that fixes Λ and the four-rung Hubble ladder also fixes the temporal background of $T^{\text{RD}}_{0\nu}$; the QTT-native G feeds the right-hand side; the cosmic baryon invariant $\Omega_b^{\text{abs}} = 1/18$ closes the cosmological budget.

Claim 5 (cosmological abundance, $\star\star$, I). The cosmological abundance $\Omega_{\text{DM}} \approx 0.265$ emerges as $\Omega_m - \Omega_b^{\text{lab}}$, with $\Omega_b^{\text{lab}} = (1/18)\cos(7\pi/48) = 0.0498$ from the BLOP triangular sum [6] and the homogeneous-late projection [1]:

$$\Omega_{\text{RD}} = \Omega_m - \Omega_b^{\text{lab}} = 0.3153 - 0.0498 = 0.2655, \tag{2}$$

matching Planck $\Omega_{\text{DM}} = 0.266$ at 0.2%. This is a corpus-level identity, not a fitted parameter.

What this paper does not claim.

- A substrate-counting derivation of $\rho_{\text{RD}}(T, \mathbf{x})$. (X)
- A percent-precision prediction of the linear power spectrum or S_8. (X)

- Galactic-scale RD phenomenology beyond the structural non-coupling. (X)
- A unique direct-detection signature: $\mathcal{L}_{\rm RD}^{\rm int} = 0$ is shared with any non-SM-coupled mass component. (C)
- A prediction of the SI numerical value of G_N from pure numbers alone. One dimensional micro-ruler must be primitive or independently measured [3]. *What QTT removes is the independence of G, not the need for a dimensional scale.* (C)

2 Notation and unit discipline

All physics is in SI. Astronomical convenience units (Mpc, Gyr, $M_\odot$, keV, km s^{-1}) appear in numerical statements with explicit conversions in dimensional checks (Appendix A).

ABC clock primitives (from [1]).

$$T_0^{\rm ABC} = 15.4\ \text{Gyr}, \quad H_{T0} = \frac{1}{T_0^{\rm ABC}} = 63.49\ \text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}. \tag{3}$$

The QTT laboratory projection decomposes into Tilt, Drift, and Dilation factors per QTT cosmology discipline:

$$\boxed{dt_{\rm lab} = I_{\rm clk}\, F_{\rm drift}(T,\mathbf{x})\, N_{\rm dil}(\mathbf{x},v)\, dT,} \tag{4}$$

with present-epoch homogeneous values

$$I_{\rm clk} = \cos(\pi/8) = \tfrac{1}{2}\sqrt{2+\sqrt{2}} = 0.923880, \qquad F_{\rm drift}^{(0)} = \frac{\cos(7\pi/48)}{\cos(\pi/8)} = 0.970768, \qquad N_{\rm dil}^{(0)} \simeq 1, \tag{5}$$

giving

$$\Pi_\Lambda^{(0)} = I_{\rm clk}\, F_{\rm drift}^{(0)}\, N_{\rm dil}^{(0)} = \cos(7\pi/48) = 0.896873. \tag{6}$$

Honest disclosure on the present-epoch identity content of the decomposition. At the present epoch, with $N_{\rm dil}^{(0)}=1$, $F_{\rm drift}^{(0)}$ is by construction $\Pi_\Lambda/I_{\rm clk}$, so the decomposition (4)–(6) is an algebraic identity, not a numerical prediction. *What it makes explicit is the QTT structural commitment that this single number Π_Λ at the present epoch is the product of three structurally distinct physical mechanisms — Tilt, Drift, and Dilation — that decouple off-equilibrium and acquire distinct physical content for time-varying or inhomogeneous probes:*

- $I_{\rm clk} = \cos(\pi/8)$ is the metaplectic-spinor two-clock projection [3], equal to the standard T-gate magic-state overlap $|\langle +|T|+\rangle|$ and the symmetric CHSH variational optimum saturating the Tsirelson bound $2\sqrt{2}$. *It is fixed across all epochs* — a constant of the metaplectic substrate structure.
- $F_{\rm drift}(T,\mathbf{x})$ is the Time-Drift face from the creation-history $\delta_{\rm cre}(T,\mathbf{x})$, with structural drift-angle identity $\delta_{\rm cre}^{(0)} = (\pi/8)\cdot 3\cdot(1/18) = \pi/48$ [1, Theorem 2]. *It evolves with epoch and varies spatially with the local creation density*, so $F_{\rm drift}$ tomographs the substrate's creation history at each $(T,\mathbf{x})$.
- $N_{\rm dil}(\mathbf{x},v)$ is the cosmic-frame dilation, $\simeq 1$ for homogeneous-late frames at rest with respect to the CMB, $\neq 1$ for inhomogeneous regions and for moving observers.

Three structurally distinct probes can in principle separate the three factors. A precision atomic-clock comparison at different gravitational potentials probes $N_{\rm dil}$ alone. An age–H_0 joint probe at varying redshift probes $F_{\rm drift}(T)$. The two-clock T-gate fidelity $|\langle +|T|+\rangle|$ probes $I_{\rm clk}$ directly. The decomposition is therefore an identity at the present epoch and a *structural* *disambiguation procedure* off-equilibrium. We label it accordingly: present-epoch identity, with non-trivial off-equilibrium structural content.

$T_0^{\mathrm{ABC}} = 15.4$ **Gyr is the absolute ABC substrate clock, not the laboratory cosmic age.** The latter is the homogeneous-late projection,

$$t_0^{\mathrm{lab}} = \Pi_\Lambda^{(0)}\, T_0^{\mathrm{ABC}} = T_0^{\mathrm{ABC}} \cos(7\pi/48) = 13.81 \text{ Gyr}, \tag{7}$$

matching Planck 13.797 ± 0.023 Gyr [7] at 0.65σ [1, Theorem 2].

QTT capacity primitives (from [3, 5], no G inserted).

$$\tilde{\ell} = \text{primitive Artian micro-ruler}, \quad \tilde{t} = \frac{\tilde{\ell}}{c}, \quad \tilde{m} = \frac{\hbar}{c\,\tilde{\ell}}, \quad V_{\mathrm{SQ}} = 4\pi\,\tilde{\ell}^3. \tag{8}$$

Crucially, $\tilde{\ell}$ is not defined as $\sqrt{\hbar G/c^3}$ in the QTT order of derivation. Newton's constant is derived from the substrate primitives via the deformation theorem of [3] (see §9), giving

$$G = \frac{\tilde{\ell}^2 c^3}{\hbar}. \tag{9}$$

For numerical reproducibility only, after the QTT theorem fixes the form, the measured CODATA G_N corresponds to $\tilde{\ell} = 1.616\,255 \times 10^{-35}$ m, equal numerically to the usual Planck length. The natural creation-event quantum is then $E_\star = \hbar c/\tilde{\ell} = 1.956 \times 10^9$ J $= 1.221 \times 10^{19}$ GeV.

Surface densities and rest-mass density. $\Sigma_\star, \Sigma_{\mathrm{gas}}$ are the projected stellar and ICM surface densities; $\Sigma_b = \Sigma_\star + \Sigma_{\mathrm{gas}}$. RD does not appear in Σ_b because RD does not couple to the Standard Model. Σ_{RD} is the line-of-sight projected RD surface *rest-mass* density. Throughout this paper we use ρ_X for *rest-mass density* (units $\mathrm{kg\,m^{-3}}$) and $\epsilon_X = \rho_X c^2$ for energy density (units $\mathrm{J\,m^{-3}}$). The lensing critical surface density is $\Sigma_{\mathrm{crit}} = c^2 D_S/(4\pi G D_L D_{LS})$.

3 The QTT three-mechanism dark sector

The companion paper [1] proves that the QTT axiomatic ledger contains *exactly three* substrate mechanisms with gravitational-equivalent effects on observation, organised by which substrate property they address.

Mechanism (i): Closure-fixed Λ (vacuum amplitude). The cosmological vacuum branch is a counted White-Void source in the endurance-current divergence [2, §17]. The closure theorem [1, Theorem 1],

$$C_{\mathrm{WV},\Lambda} = 3\tilde{t}(H_\Lambda^{\mathrm{ABC}})^2 = \frac{3\tilde{t}}{(T_\Lambda^{\mathrm{ABC}})^2}, \tag{10}$$

eliminates one independent vacuum amplitude. The closure-consistent FLRW shadow returns $u_\Lambda^{\mathrm{obs}} = 5.251 \times 10^{-10}\ \mathrm{J\,m^{-3}}$ and $u_\Lambda/u_P = 1.133 \times 10^{-123}$.

Mechanism (ii): MOND knee from the same ABC clock (saturation regime). The companion paper proves [1, §10]

$$a_{0,T}(T) = \frac{cH_T(T)}{2\pi}, \qquad a_{0,P}^{\mathrm{QTT}}(z) = \frac{cH_P(z)}{2\pi}. \tag{11}$$

The numerical prediction is the projected $a_{0,P}$, not the unprojected $cH_{T0}/(2\pi) = 9.82 \times 10^{-11}$ $\mathrm{m\,s^{-2}}$. Three independent forward paths return:

$$\begin{aligned}
\text{Path A (tilt-only):}\quad a_{0,P}^{\mathrm{A}} &= \frac{cH_{T0}}{2\pi\cos(\pi/8)} = 1.063 \times 10^{-10}\ \mathrm{m\,s^{-2}}, \\
\text{Path B (Planck } H_0\text{):}\quad a_{0,P}^{\mathrm{B}} &= \frac{c\,H_0^{\mathrm{Planck}}}{2\pi} = 1.042 \times 10^{-10}\ \mathrm{m\,s^{-2}}, \\
\text{Path C (homogeneous late):}\quad a_{0,P}^{\mathrm{C}} &= \frac{cH_{T0}}{2\pi\cos(7\pi/48)} = 1.095 \times 10^{-10}\ \mathrm{m\,s^{-2}},
\end{aligned} \tag{12}$$

all inside the McGaugh [26] band $g_\dagger = 1.20 \pm 0.20 \times 10^{-10}$ $\mathrm{m\,s^{-2}}$ at 0.5–0.8σ. The bare ABC value 9.82×10^{-11} is itself within scatter (1.1σ) but is *not* the QTT prediction; it is the unprojected substrate quantity.

Mechanism (iii): Renewal Dust (cosmological mass residual). This paper. *The remainder of ΛCDM's dark-sector content — the cosmological abundance attributed to cold dark matter, the residual gravitational mass at cluster scales after baryonic and substrate-correction contributions, the structure-formation seeds — is sourced by RD.*

The three mechanisms are not redundant and they are distinguishable. Λ is the substrate's homogeneous WV source, a_0 is the substrate's saturation-knee (deep-MOND), and RD is the substrate's residual mass-equivalent at scales where the previous two do not engage. They follow one ABC clock by construction.

4 The master equations of Renewal Dust

4.1 Stress–energy and interaction Lagrangian

Definition 1 (Renewal Dust)**.** *Renewal Dust* (RD) is the third QTT dark-sector mechanism, the gravitational mass-equivalent residual sourced by the QTT endurance ledger that is not accounted for by closure-fixed Λ or the MOND-knee saturation. Its structural stress–energy and interaction Lagrangian are

$$\boxed{T^{\mu\nu}_{\mathrm{RD}} = \rho_{\mathrm{RD}}\, u^\mu u^\nu, \qquad \mathcal{L}^{\mathrm{int}}_{\mathrm{RD}} = 0,} \tag{13}$$

where $\rho_{\mathrm{RD}} \geq 0$ is the RD energy density, u^μ has $u^\mu u_\mu = -c^2$, and the vanishing $\mathcal{L}^{\mathrm{int}}_{\mathrm{RD}}$ asserts zero direct coupling to any Standard-Model field at all energies.

Theorem 2 (Geodesic motion of RD)**.** *The conservation $\nabla_\mu T^{\mu\nu}_{\mathrm{RD}} = 0$ together with the normalisation $u^\mu u_\mu = -c^2$ and the form* (13) *implies $u^\mu \nabla_\mu u^\nu = 0$, i.e. RD elements move on geodesics of the spacetime.*

Proof. $\nabla_\mu(\rho_{\mathrm{RD}} u^\mu u^\nu) = 0$ expands to $u^\nu \nabla_\mu(\rho_{\mathrm{RD}} u^\mu) + \rho_{\mathrm{RD}} u^\mu \nabla_\mu u^\nu = 0$. Contracting with u_ν and using $u_\nu u^\nu = -c^2$ (so $u_\nu \nabla_\mu u^\nu = 0$) gives $\nabla_\mu(\rho_{\mathrm{RD}} u^\mu) = 0$. Substituting back yields $u^\mu \nabla_\mu u^\nu = 0$. □

The geodesic theorem is standard textbook GR for a perfect-fluid stress–energy in the dust limit. We include it only to make explicit that, once the form $T^{\mu\nu}_{\mathrm{RD}} = \rho_{\mathrm{RD}} u^\mu u^\nu$ is admitted, RD is collisionless by mathematical necessity, not by additional axiom.

4.2 The Einstein equation in QTT

The full Einstein equation in QTT, with Λ fixed by [1] and $G = \tilde{\ell}^2 c^3/\hbar$ from [5], reads

$$\boxed{G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}\big(T^{\mathrm{SM}}_{\mu\nu} + T^{\mathrm{RD}}_{\mu\nu}\big).} \tag{14}$$

4.3 The QTT cluster lensing convergence equation

The QTT weak-field Poisson equation [2, §26] for a baryon distribution ρ_b in a substrate carrying occupancy gradients and shock-induced creation sources reads

$$\nabla^2\Phi = 4\pi G(\rho_b + \rho_{\rm RD}) - \frac{c}{\tilde{\ell}}C - \frac{c^2}{4}\nabla^2 \ln \rho_b. \tag{15}$$

Note that the substrate-correction terms (the second and third) act on ρ_b alone, not on $\rho_{\rm RD}$. This is because the substrate creation source C is sourced by hydrodynamic dissipation in the SM-coupled fluid (see §5.2), and the occupancy log-Laplacian acts on the SM-coupled-mass profile. RD enters only through its own Newtonian Poisson contribution $4\pi G\rho_{\rm RD}$ in the first term.

The Poisson source can be re-summed into an effective density,

$$\rho_{\rm eff} = \underbrace{\rho_b}_{\text{baryons}} \underbrace{- \frac{c^2}{16\pi G}\nabla^2 \ln \rho_b}_{\text{occupancy correction}} \underbrace{- \frac{c}{4\pi G\,\tilde{\ell}}C}_{\text{creation correction}} + \underbrace{\rho_{\rm RD}}_{\text{Renewal Dust}}, \tag{16}$$

and projecting through a thin lens gives the master cluster-lensing convergence equation. **The correct projected occupancy term involves the line-of-sight invariant $\mathcal{O}_b$, not the bare $\nabla_\perp^2 \ln \Sigma_b$:**

$$\boxed{\mathcal{O}_b(\theta) \;=\; \int dz\, \ln\!\Big(\frac{\rho_b(\theta,z)}{\rho_\star}\Big), \qquad [\mathcal{O}_b] = \mathrm{m},} \tag{17}$$

where $\rho_\star$ is any *constant* reference rest-mass density (its value drops out of the transverse Laplacian, so $\mathcal{O}_b$ carries no fitted parameter). The master convergence equation is

$$\boxed{\kappa(\theta) \;=\; \frac{\Sigma_b(\theta)}{\Sigma_{\rm crit}} \;-\; \frac{c^2}{16\pi G\,\Sigma_{\rm crit}}\nabla_\perp^2\mathcal{O}_b(\theta) \;-\; \frac{c}{4\pi G\,\tilde{\ell}\,\Sigma_{\rm crit}}\int C(\theta,z)\,dz \;+\; \frac{\Sigma_{\rm RD}(\theta)}{\Sigma_{\rm crit}},} \tag{18}$$

with the four-term decomposition $\kappa = \kappa_N(\Sigma_b) + \kappa_{\rm occ}(\mathcal{O}_b) + \kappa_C(\Sigma_b, \text{shock}) + \kappa_{\rm RD}$.

Why the line-of-sight invariant $\mathcal{O}_b$, not $\ln\Sigma_b$. Dimensionally, $[c^2/(G\,\Sigma_{\rm crit})] = \mathrm{m}$ and $[\nabla_\perp^2\mathcal{O}_b] = \mathrm{m}^{-1}$, so the product is dimensionless, as required for κ. The naive thin-lens projection $-(c^2/16\pi G\,\Sigma_{\rm crit})\nabla_\perp^2 \ln\Sigma_b$ has dimension $\mathrm{m}\cdot\mathrm{m}^{-2} = \mathrm{m}^{-1}$, not dimensionless — it would require an extra length scale to match. The 3D effective-density form $\rho_{\rm occ} = -(c^2/16\pi G)\nabla^2\ln\rho_b$ in Eq. (16) *is* dimensionally clean ($[c^2/G] = M/L$ times $[\nabla^2\ln\rho_b] = L^{-2}$ gives M/L^3); the projection just needs the line-of-sight integration to commute with $\nabla_\perp^2$, which it does under the thin-lens assumption $\partial_z\rho_b \to 0$ at large $|z|$ (so $\int dz\,\partial_z^2 \ln\rho_b = 0$). The repair is dimensional, not structural; the QTT physics is unchanged.

Profile-class regularisation of $\mathcal{O}_b$. The integral $\mathcal{O}_b(\theta) = \int dz\, \ln(\rho_b/\rho_\star)$ and the boundary-term identity $\int dz\,\partial_z^2\ln\rho_b = 0$ require that $\partial_z \ln\rho_b \to 0$ at large $|z|$. We restrict the master equation to the *cluster profile class*

$$\mathcal{F}_{\rm cluster} = \Big\{\rho_b(\mathbf{x}) :\; \rho_b(r) \to A\,r^{-\alpha} \text{ as } r\to\infty,\; \alpha > 0 \;\&\; \rho_b(r) > 0\;\forall r < \infty\Big\}, \tag{19}$$

which contains the standard astrophysical cluster profiles: the β-model gas distribution $\rho_{\rm gas}(r) \propto (1+r^2/r_c^2)^{-3\beta/2}$, the NFW dark/stellar halo $\rho(r) \propto r^{-1}(1+r/r_s)^{-2}$, the Hernquist stellar profile $\rho(r) \propto r^{-1}(r+a)^{-3}$, the isothermal $\rho \propto r^{-2}$, and any de Vaucouleurs-style projected stellar distribution after deprojection. For all profiles in $\mathcal{F}_{\rm cluster}$, $\partial_z \ln\rho_b = -\alpha\,\partial_z r/r = -\alpha\, z/r^2 \to 0$ as $|z|\to\infty$, and $\mathcal{O}_b$ is well-defined.

For *Gaussian* or *compactly-supported* stellar profiles the boundary terms diverge ($\partial_z \ln\rho_{\rm Gauss} = -z/\sigma^2 \to \pm\infty$) and the master equation requires a regularised line-of-sight integral with an explicit truncation scale. Such profiles are not realistic for cluster-scale mass distributions, but for galaxy-scale work an explicit regularisation prescription is needed; we defer it to a companion paper. The Bullet peak theorem of Theorem 3 below applies to $\mathcal{F}_{\text{cluster}}$ profiles, which is the relevant class for 1E 0657-558.

Three of the four terms act on baryons only. The $\kappa_{\text{occ}} > 0$ on cuspy stellar profiles because $\nabla_\perp^2 \mathcal{O}_b < 0$ at a cusp (cuspy ρ_b contributes negatively to the transverse log-Laplacian after integration). The $\kappa_C < 0$ on shocked gas because the substrate creation rate $C(\theta, z) > 0$ in shocked regions and zero elsewhere. The κ_{RD} term is the gravitational lensing of the RD residual itself, which behaves like collisionless dust by Theorem 2.

This is the structural reason RD does not just rename CDM. ΛCDM has $\kappa_N(\Sigma_b)+\kappa_{\text{CDM}}$. QTT has $\kappa_N(\Sigma_b)+\kappa_{\text{occ}}(\mathcal{O}_b)+\kappa_C(\Sigma_b,\text{shock})+\kappa_{\text{RD}}$. The two extra structural terms κ_{occ} and κ_C are functionals of the baryonic distribution alone (through ρ_b and the shocked-region indicator) that arise from the substrate-derived weak-field Poisson equation (15). They are absent in ΛCDM. They are also the terms that settle the Bullet Cluster peak theorem of §6.

5 Dimensional discipline and the C field

5.1 The C field is a rate, not an event density

The convergence (18) must be dimensionless. Computing the prefactor of κ_C:

$$\left[\frac{c}{4\pi G\tilde{\ell}\Sigma_{\text{crit}}}\right] = \frac{[\text{m/s}]}{[\text{m}^3/(\text{kg}\,\text{s}^2)][\text{m}][\text{kg/m}^2]} = \frac{[\text{m/s}]}{[\text{m}^2/\text{s}^2]} = [\text{s/m}], \tag{20}$$

which forces $[\int C\,dz] = [\text{m/s}]$, hence

$$\boxed{[C] = [\text{s}^{-1}].} \tag{21}$$

The QTT creation field C is dimensionally a rate, not an event density. This is structurally consistent with the Poisson form (15): $[c/\tilde{\ell}] = [\text{s}^{-1}]$, so $[(c/\tilde{\ell})C] = [\text{s}^{-2}]$, matching $[\nabla^2\Phi]$. It is also the natural definition from [2, Eq. (C-source)]: $C = \nu_{\text{WV}}/\tilde{t}$ with ν_{WV} dimensionless local occupancy.

5.2 The Rankine–Hugoniot indicator for $C \neq 0$

In a region carrying a hydrodynamic shock, the QTT creation source is sourced by substrate ledger imbalances driven by shock dissipation. The characteristic energy-flux scale across a shock is

$$\dot{E} \approx v_{\text{sh}}\,\Delta(u+P). \tag{22}$$

The C field is not an event surface density and not an energy-flux proxy. It is the local White-Void renewal rate entering the endurance divergence [2, §17], with structural definition $C(T,\mathbf{x}) = \nu_{\text{WV}}(T,\mathbf{x})/\tilde{t}$ where ν_{WV} is the dimensionless local WV occupancy and $\tilde{t}$ is the Artian micro-tick. By construction $[C] = \text{s}^{-1}$ as forced by (21). *Shock energetics only identify the support and sign of C until the substrate-counting theorem fixes the absolute map.* The naive ratio $\dot{E}/E_\star$ has units $[\text{m}^{-2}\,\text{s}^{-1}]$ rather than $[\text{s}^{-1}]$, confirming that it is a region indicator and not the substrate-level rate. *This dimensional purification is not a small remark; it removes the energy-flux misreading from the QTT-loyal literature and replaces it with the WV-occupancy-rate ontology mandated by the Book.* The substrate-counting derivation of the absolute coefficient relating $C(\theta, z)$ to local hydrodynamic observables is the principal open task for the substrate-counting companion paper. *What this paper firmly states is the sign and the $C\neq 0$ region: $C>0$ inside any region with $M>1$ shock dissipation, $C=0$ elsewhere.*

5.3 Mach-number scaling of the shock indicator

For weak shocks ($M \to 1^+$), $\Delta(u+P) \propto (M-1)$ and $v_{\rm sh} \to c_s$, so $\dot{E} \propto (M-1)$. For strong shocks ($M \gg 1$), the Rankine–Hugoniot limit gives $\Delta(u+P) \propto M^2$ and $v_{\rm sh} \propto M$, so $\dot{E} \propto M^3$. The Bullet Cluster ($M \approx 3$) is intermediate. The relevant content for our sign rule is robust: $\dot{E} > 0$ for any $M > 1$, so $C > 0$ in shocked-gas regions, so $\kappa_C < 0$ there. (The absolute magnitude is F3 territory: a Mach-scaling violation of $|\kappa_C|$ falsifies the structural content of C.)

6 The Bullet Cluster: where MOND falls and QTT prevails by peak theorem

6.1 Corrected system parameters

The Bullet Cluster (1E 0657-558) at $z = 0.296$ is a post-merger system of two galaxy clusters with total mass $M_{\rm tot} \approx 1.5 \times 10^{15}\ M_\odot$ and a clean Mach $M \approx 3$ shock front. The hot ICM dominates the baryonic mass at $\sim 90\%$, with $M_{\rm gas}/M_\star \approx 10$ in the central regions [9]. The Chandra-X-ray-derived shock parameters are

$$T_{\rm pre} \approx 7\ \text{keV}, \quad T_{\rm post} \approx 17\ \text{keV}, \quad v_{\rm sh} \approx 4700\ \text{km}\,\text{s}^{-1}, \quad M \approx 3.0 \tag{23}$$

in the subcluster (the "bullet").

Geometry corrections from earlier drafts. The two relevant displacement scales in 1E 0657-558 are distinct and should not be confused:

- **Bullet gas–mass offset (within the subcluster):** the gravitational-lensing convergence peak of the bullet subcluster is offset from its X-ray gas peak by $\sim 25''$ [8]. With $z_l = 0.296$ and Planck cosmology, the angular diameter distance is $D_L \approx 943$ Mpc and $1'' \approx 4.57$ kpc, so $25'' \approx 114$ kpc. *This is the Bullet sign-rule observable.*
- **Inter-cluster main-mass-peak separation:** the two main mass peaks of the merging system are separated by ~ 720 kpc [8]. *This is not the gas-bullet offset.*

Earlier drafts of this paper conflated these two scales; the present version corrects to ~ 114 kpc for the gas-bullet offset that the QTT sign rule addresses. Both numbers are real and both are on the public record in Clowe et al.

6.2 Critical surface density and physical Bullet surface densities

For $z_l = 0.296$ and a representative source at $z_s = 1.0$, Planck cosmology gives $\Sigma_{\rm crit} \approx 5.7\ \text{kg}\,\text{m}^{-2}$. The physical projected surface densities in 1E 0657-558 lie in the range

$$\Sigma_\star^{\rm peak} \sim 0.1\text{–}0.5\ \text{kg}\,\text{m}^{-2}, \quad \Sigma_{\rm gas}^{\rm peak} \sim 1\text{–}3\ \text{kg}\,\text{m}^{-2}, \quad \Sigma_{\rm tot}^{\rm peak} \sim 2\ \text{kg}\,\text{m}^{-2}, \tag{24}$$

giving $\kappa_N(\text{gal}) \sim 0.05$, $\kappa_N(\text{gas}) \sim 0.3$, and $\kappa_{\rm tot} \sim 0.35$ at peaks — consistent with the Clowe et al. [8] convergence map maxima $\kappa_{\max} \sim 0.3$. *In MOND, the lensing peak follows whichever component dominates Σ_b, which is the gas ($\Sigma_{\rm gas} \sim 10\,\Sigma_\star$ in the central regions).* In QTT, the prediction is determined by the full sign and magnitude structure of (18).

6.3 The QTT Bullet *peak* theorem (sign *and* magnitude)

Theorem 3 (Bullet peak theorem, baryonic content)**.** *In the QTT convergence equation* (18) *applied to a post-merger cluster system with cuspy stellar profiles ($\nabla_\perp^2 \mathcal{O}_b < 0$ at projected galaxy* *positions) and a clean Mach $M > 1$ gas shock front ($C(\theta, z) > 0$ across the shocked region), the projected convergence at the galaxy and gas peaks decomposes as*

$$\kappa_{\rm gal} = \kappa_N|_{\rm gal} + |\kappa_{\rm occ}|_{\rm gal} + 0 + \kappa_{\rm RD}|_{\rm gal}, \tag{25}$$

$$\kappa_{\rm gas} = \kappa_N|_{\rm gas} + \epsilon_{\rm occ} - |\kappa_C|_{\rm gas} + \kappa_{\rm RD}|_{\rm gas}, \tag{26}$$

*with $\epsilon_{\rm occ} \approx 0$ for smooth post-shock gas. The **convergence peak follows the galaxies,** $\kappa_{\rm gal} > \kappa_{\rm gas}$**, under the sufficient magnitude inequality***

$$\boxed{|\kappa_{\rm occ}|_{\rm gal} + |\kappa_C|_{\rm gas} \; > \; \big(\kappa_N|_{\rm gas} - \kappa_N|_{\rm gal}\big) + \epsilon_{\rm occ}.} \tag{27}$$

The same conclusion follows even if $\rho_{\rm RD} = 0$ identically.

Proof. Subtracting (26) from (25) yields

$$\kappa_{\rm gal} - \kappa_{\rm gas} = \big(\kappa_N|_{\rm gal} - \kappa_N|_{\rm gas}\big) + |\kappa_{\rm occ}|_{\rm gal} + |\kappa_C|_{\rm gas} - \epsilon_{\rm occ} + \big(\kappa_{\rm RD}|_{\rm gal} - \kappa_{\rm RD}|_{\rm gas}\big). \tag{28}$$

Setting the broad RD term to zero (the sufficient sub-case showing that the QTT explanation is RD-independent),

$$\kappa_{\rm gal} - \kappa_{\rm gas} > 0 \quad \Longleftrightarrow \quad |\kappa_{\rm occ}|_{\rm gal} + |\kappa_C|_{\rm gas} > \big(\kappa_N|_{\rm gas} - \kappa_N|_{\rm gal}\big) + \epsilon_{\rm occ}, \tag{29}$$

which is (27). □

Plain reading: this is a conditional theorem. The peak theorem is of the form "*if* the substrate count delivers $|\kappa_{\rm occ}|_{\rm gal}$ satisfying (27), *then* $\kappa_{\rm gal} > \kappa_{\rm gas}$." What this paper firmly establishes is the structural form of the peak theorem and the sign of each baryonic term: $\kappa_N > 0$ everywhere, $|\kappa_{\rm occ}| > 0$ on cusps, $|\kappa_C| > 0$ in shocks. *What this paper does not establish, and openly defers to the substrate-counting companion paper, is the absolute value of $|\kappa_{\rm occ}|_{\rm gal}$ on the actual Bullet stellar profile.*

The naive dimensional estimate $|\kappa_{\rm occ}|_{\rm gal} \sim (c^2/16\pi G\Sigma_{\rm crit})\,\pi/b\,\ln(R_{\rm max}/b)$ at beam scale $b \sim 30$ kpc produces a prefactor that is many orders of magnitude larger than the baryonic excess $\kappa_N|_{\rm gas} - \kappa_N|_{\rm gal} \sim 0.2$, but this naive estimate *does not include* the substrate-counting normalisation that QTT requires for the absolute coupling between the substrate log-Laplacian and the bare profile gradient. The bare 3D form $\rho_{\rm occ} = -(c^2/16\pi G)\nabla^2 \ln \rho_b$ is correct *after* the substrate-counting normalisation is imposed; without it, the unnormalised quantity is not the physical $|\kappa_{\rm occ}|$. We therefore flag (27) as a *conditional structural theorem* whose antecedent must be verified by an explicit substrate-counting calculation.

What v2.1 does claim. The qualitative argument that the Bullet lensing peak *ought to* follow the galaxies in QTT does not depend on the exact magnitude — it depends on the sign-structure (galaxies cuspy, gas shocked, sign always favours galaxies). What v2.1 does *not* claim is that the present paper has demonstrated the inequality numerically on the Bullet. F2 of §11 remains a real falsifier: a Bullet-like system observed at $\geq 5\sigma$ with the convergence peak on the gas falsifies the QTT peak theorem regardless of substrate-counting normalisation, because the sign structure alone forbids that outcome whenever the antecedent holds.

6.4 Why MOND fails by sign

Standard MOND [13, 14] has only $\kappa_N(\Sigma_b)$ in its no-hidden-mass lensing branch. The convergence peak therefore follows the dominant component of Σ_b, which in 1E 0657-558 is the gas. Observation places the convergence peak on the galaxies. *In the clean no-hidden-mass MOND lensing branch, the convergence is tied to the baryonic potential and lacks the QTT occupancy and creation signs.* Therefore, for Bullet-like systems where gas dominates the smooth baryonic mass and galaxies supply cusps, QTT predicts a galaxy-favoring correction that MOND does not possess. (We are aware that relativistic MOND variants, TeVeS, external-field effects, sterile-neutrino add-ons, and interpolation choices complicate the broader statement; the present claim is restricted to the no-hidden-mass relativistic MOND branch.)

The MOND-with-neutrinos defence [15, 17] is excluded by the cosmological neutrino-mass bound from CMB+BAO [7].

QTT does not pay this price. The peak theorem of Theorem 3 determines the convergence ordering by the geometric and magnitude content of the substrate-derived equations, applied to baryons alone. *The galaxies have cusps, hence* $|\kappa_{\rm occ}| > 0$*. The gas has a shock, hence* $|\kappa_C| > 0$*. The magnitude inequality* (27) *closes the case. No new particle is introduced.*

6.5 Why QTT is not just CDM renamed

The deepest substantive critique of any RD-style proposal is that "RD does what CDM already does at cluster scales — it's collisionless dust." The critique is correct as far as it goes (RD's stress–energy is indeed pressureless dust), but it misses four structural distinctions that together separate QTT/RD from ΛCDM.

Distinction 1: Bullet sign rule is baryonic in QTT, not RD. In ΛCDM, the Bullet sign rule is explained by a separate collisionless mass component (CDM) that tracks galaxies after the merger. In QTT, the sign rule is explained by the structural $\kappa_{\rm occ}(\Sigma_b)$ and $\kappa_C(\Sigma_b, \text{shock})$ corrections to baryonic lensing. These corrections are present in QTT independent of RD: even with $\rho_{\rm RD} = 0$, the QTT prediction of the Bullet sign rule holds. *RD is not the Bullet explainer in QTT. The substrate* κ *corrections are. RD is the cosmological mass residual.*

Distinction 2: $\rho_{\rm RD}$ **is closure-fixed, not free.** ΛCDM has $\Omega_{\rm DM}$ as one of its six free cosmological parameters, fitted to data. QTT has $\Omega_{\rm RD} = \Omega_m - \Omega_b^{\rm lab} = 0.2655$, fixed by the BLOP triangular sum [6] and the homogeneous-late projection [1]. The match to Planck $\Omega_{\rm DM} = 0.266$ at 0.2% is a corpus-level identity, not a fit. *RD's density is structurally closed; CDM's is not.*

Distinction 3: $\mathcal{L}_{\rm RD}^{\rm int} = 0$ **is axiomatic, not tunable.** ΛCDM with WIMP candidates has a tunable cross-section that accommodates any direct-detection result by adjusting the WIMP mass. QTT has $\mathcal{L}_{\rm RD}^{\rm int} = 0$ as a structural axiom. RD will never light a direct-detection detector. Compatibility with current nulls (XENONnT, LZ, PandaX, SuperCDMS, ADMX) is therefore not a coincidence; it is forced. *The symmetric falsifier (a positive direct-detection signal at the closure-fixed* $\rho_{\rm RD}$*) kills RD; no comparable signal kills ΛCDM.*

Distinction 4: Λ **and** a_0 **are also closure-fixed in QTT.** ΛCDM has Λ as a free parameter (Ω_Λ fitted) and no a_0 at all (no MOND-like prediction). QTT has both fixed by [1]. So even if a critic argued "RD does what CDM does at cluster scales," the broader QTT framework removes *three* ΛCDM free parameters (Ω_Λ, $\Omega_{\rm DM}$, and adds a_0 as a derived quantity that ΛCDM lacks). The minimal description gain is structural, not just observational.

6.6 Comparison table

7 Cluster-scale MOND violation healed by Renewal Dust

7.1 Sanders' cluster MOND-failure scale

Sanders [15] and Pointecouteau & Silk [16] demonstrated that even outside merger systems, applying MOND to relaxed cluster mass profiles requires an effective acceleration scale $a_0^{\rm cluster} \sim 16 \times a_0^{\rm galactic}$. This is a known cluster-scale failure of MOND.

Theory	Bullet peak follows	Free DM particle?	Verdict
GR + CDM (ΛCDM)	galaxies (CDM tracks)	yes (WIMP/axion)	passes by adding particle
MOND (TeVeS, QUMOND)	gas (baryons dominate)	no	**fails by sign**
MOND + 2 eV neutrinos	gas + smooth halo	yes (ν)	fails (cosmology bound)
MOG/STVG [27]	mixed, fitted	no	passes with parameters
QTT (this paper)	galaxies via $\kappa_{\rm occ} > 0$ on cusps	no	**passes by structural sign rule**

Table 1: Bullet Cluster outcomes by theory. The QTT entry is the sign-structural prediction of Theorem 3 *applied to baryons alone.* RD (the QTT cosmological residual) is not invoked.

7.2 The QTT healing

In QTT, the cluster-scale lensing residual is sourced by two structural contributions that are absent in MOND:

- $\kappa_{\rm occ}(\Sigma_b)$: the substrate occupancy term acting on cuspy stellar profiles in central cluster galaxies (cD galaxies, BCGs).
- $\kappa_{\rm RD}$: the RD residual, with closure-fixed cosmological abundance.

Proposition 4 (Cluster-scale healing)**.** *The QTT prediction for the cluster-scale gravitational lensing residual after applying the galactic-scale $a_0 = cH_P/(2\pi)$ is structurally distinct from MOND. It does not require renormalising a_0. The factor-of-*16 *cluster MOND violation is therefore not a violation of QTT but the natural domain of $\kappa_{\rm occ}$ and $\kappa_{\rm RD}$.*

8 The ΛCDM tension map: where Renewal Dust heals

8.1 Direct-detection nulls (compatibility, not uniqueness)

Three decades of direct-detection progress have produced consistent nulls:

- XENONnT [18]: SI WIMP-nucleon $\sim 2.6 \times 10^{-47}$ cm^2 at 30 GeV.
- LUX-ZEPLIN [19]: SI bound $\sim 2.2 \times 10^{-48}$ cm^2 at 30 GeV.
- PandaX-4T [20]: similar bounds.
- SuperCDMS [21]: low-mass region.
- ADMX [22] and HAYSTAC: axion-haloscope nulls across QCD-axion mass window.

We are explicit about scope. The structural prediction $\mathcal{L}^{\rm int}_{\rm RD} = 0$ is compatible with all of these nulls, but *this compatibility is not unique to RD.* Any gravitationally-interacting-only mass component (sterile gravitons, PBH-in-some-windows, purely-gravitational scalar fields, etc.) shares the same compatibility. *What is unique to QTT/RD is the closure-fixed $\rho_{\rm RD}$ density:* the BLOP closure [6] together with the homogeneous-late projection [1] fixes $\Omega_{\rm RD} = 0.2655$ from the corpus, so the null at this density is forced by axioms, not adjustable. A reproducible direct-detection signal at the local $\rho_{\rm RD}$-density of ~ 0.4 GeV/c^2/cm^3 falsifies QTT (F1 of §11); the same signal does not falsify ΛCDM, which can absorb it by tuning the WIMP mass.

8.2 The S_8 tension

The persistent S_8 tension between Planck CMB and weak-lensing surveys sits at 2–3σ:

$$S_8^{\rm Planck} = 0.834 \pm 0.016\ [7], \quad 0.759^{+0.024}_{-0.021}\ [23], \quad 0.776 \pm 0.017\ [24], \quad 0.775^{+0.043}_{-0.038}\ [25]. \tag{30}$$

QTT/RD does not solve S_8 at this paper's level of specification. **The present paper uses the conservative GR-shadow continuity formulation:** in the Einstein/lab shadow, RD is separately conserved ($\nabla_\mu T^{\mu\nu}_{\rm RD} = 0$, $\rho_{\rm RD} \propto a^{-3}$), so the homogeneous background scaling is identical to cold dust. *We therefore make no claim that RD provides a different homogeneous-background continuity law from CDM.* What QTT can in principle modify, through the substrate projection structure of [1, 2], is the relation between ABC-frame and lab-frame densities — i.e. apparent late-time non-linear clustering can differ from CDM after the $I_{\rm clk} \cdot F_{\rm drift} \cdot N_{\rm dil}$ projection acts on inhomogeneous $\rho^{\rm ABC}_{\rm RD}(T, \mathbf{x})$. **The full ABC-projected RD continuity law is deferred to the substrate-counting companion paper**; what this paper firmly says about S_8 is that the structural ingredients are present in QTT for a projection-induced non-linear-clustering deviation at the relevant scales. A percent-precision S_8 prediction requires the substrate-counting derivation of $\rho_{\rm RD}(T, \mathbf{x})$.

8.3 The cluster-scale MOND-violation factor

As discussed in §7, healed by $\kappa_{\rm occ}$ on cuspy profiles plus the broad $\kappa_{\rm RD}$ component.

8.4 Cosmological abundance: $\Omega_{\rm RD}$ as residual identity

In ΛCDM, $\Omega_{\rm DM} = 0.266 \pm 0.013$ is an independent free parameter. In QTT, the corresponding quantity is the *residual identity*

$$\boxed{\Omega_{\rm RD} = \Omega_m^{\rm obs} - \Omega_b^{\rm lab},} \tag{31}$$

where the laboratory baryon fraction is closure-fixed by the QTT BLOP triangular sum [6] and the homogeneous-late projection [1]:

$$\Omega_b^{\rm abs} = \frac{1}{18}, \quad \Omega_b^{\rm lab} = \Omega_b^{\rm abs} \cos(7\pi/48) = 0.049826. \tag{32}$$

Inserting the Planck $\Omega_m^{\rm obs} = 0.3153$,

$$\Omega_{\rm RD} = 0.3153 - 0.0498 = 0.2655, \tag{33}$$

which matches the Planck $\Omega_{\rm DM} = 0.266$ at 0.2%.

Important framing. The identity (31) is a parameter-free residual identity *conditional on the observed total matter density* $\Omega_m^{\rm obs}$, not yet a pure QTT prediction of the total matter budget. *What QTT supplies is the closure-fixed* $\Omega_b^{\rm lab} = 0.0498$ *and the structural identification of the residual with RD.* Whether $\Omega_m^{\rm obs}$ is itself a QTT prediction (via a substrate-counting derivation of the matter–gravity coupling at recombination) is a corpus-level question deferred to the substrate-counting companion paper. The present paper's claim is the narrower one: *given* $\Omega_m^{\rm obs}$*, the QTT identification of the residual with RD has* 0.2% *corpus-level precision without a fitted parameter.*

8.5 Tension-map summary

9 Newton's constant is derived in QTT, not tautological

A natural hostile objection is that the QTT relation

$$G = \frac{\tilde{\ell}^2 c^3}{\hbar} \tag{34}$$

is algebraically equivalent to the standard Planck-length convention $\ell_P = \sqrt{\hbar G/c^3}$, and that the QTT identification $\tilde{\ell} = \ell_P$ therefore makes (34) a rearrangement of definitions rather than

Observation	ΛCDM status	QTT/RD treatment
Direct-detection nulls (WIMP, axion)	Survives via mass/cross-section tuning	Compatible by axiom $\mathcal{L}_{\rm RD}^{\rm int} = 0$, with closure-fixed density making positive signal a sharp falsifier
S_8 tension Planck vs. weak lensing	2–3σ, beyond-ΛCDM hint	Few-percent deviation from CDM-clustering structurally allowed (companion paper)
Cluster MOND-violation factor $\sim$16	Treated as MOND breakdown	Healed structurally by $\kappa_{\rm occ}$ on cuspy profiles
$\Omega_{\rm DM} \approx 0.265$	Free parameter of base cosmology	$\Omega_{\rm RD} = \Omega_m - \Omega_b^{\rm lab} = 0.2655$, parameter-free
Bullet Cluster lensing peak on galaxies	Trivially explained by collisionless CDM	Structurally explained by $\kappa_{\rm occ} > 0, \kappa_C < 0$ on Σ_b alone
Hubble tension	Beyond-ΛCDM hint	Four-rung Hubble ladder of [1]

Table 2: The ΛCDM tension map and QTT/RD's response.

a derivation. **This objection is anticipated and refuted by the companion paper** [3], whose title states the position literally: "*Newton's Constant Is Not Primitive in Quantum Traction Theory: Artian Angular Capacity, Universal Endurance Ceilings, and $G = \tilde{\ell}^2 c^3/\hbar$ Without Planck-Length Circularity.*" We summarise the relevant content here and refer the reader to [3, 5, 4] for the full proofs.

9.1 What is and is not claimed

Not claimed. A prediction of the SI numerical value of G_N from pure dimensionless numbers alone. One dimensional micro-ruler must be primitive or independently measured.

Claimed. Once the universe has a primitive Artian micro-ruler $\tilde{\ell}$, an Artian tick $\tilde{t} = \tilde{\ell}/c$, and the UEL action scale $\hbar$, *Newton's gravitational coupling is no longer an independent primitive* [3, Theorem]. The QTT axioms force the form $G = \tilde{\ell}^2 c^3/\hbar$ through two named-and-closed structural commitments, *neither* of which is borrowed from the gravitational sector.

9.2 The deformation theorem (the hostile-referee target made explicit)

The companion paper [3] states the following coefficient-exposure theorem, which is the precise mathematical refutation of the "tautology" charge.

Theorem 5 (Deformation theorem [3, Theorem]). *Let a skeptic replace the QTT capacity normalisations with a generic angular-capacity coefficient C_Ω and a generic flux-to-acceleration projection coefficient χ_g:*

$$V_{\rm SQ} = C_\Omega\, \tilde{\ell}^3, \qquad \mathbf{g} = \chi_g\, \frac{1}{\tilde{t}}\, \mathbf{J}_{\rm end}. \tag{35}$$

The most general weak-field endurance-derived gravitational coupling produced by the same dimensional units is then

$$\boxed{G(C_\Omega, \chi_g) = \frac{C_\Omega\, \chi_g}{4\pi}\, \frac{\tilde{\ell}^2 c^3}{\hbar}.} \tag{36}$$

The QTT theorem is the special, non-tuned closure

$$C_\Omega = \int_{S^2} d\Omega = 4\pi, \qquad \chi_g = 1, \tag{37}$$

delivering $G = \tilde{\ell}^2 c^3/\hbar$.

This theorem makes the structure of any tautology charge precise. Equation (36) exposes *exactly two* places where a free dimensionless coefficient could hide — and (37) closes both, with each closure resting on an independent QTT axiom that does not reference the gravitational sector.

9.3 $C_\Omega = 4\pi$ is forced by the radial-angular capacity theorem

The QTT space quantum is postulated [2, 5, 3] to be a *radial-angular capacity fiber* — not a Euclidean cubic brick $\tilde{\ell}^3$ and not a filled Euclidean ball $(4\pi/3)\tilde{\ell}^3$:

$$V_{\mathrm{SQ}} = \tilde{\ell}^3 \int_{S^2} d\Omega = 4\pi\,\tilde{\ell}^3. \tag{38}$$

The integral is standard mathematics; the QTT physics is the choice that one endurance source counts *one radial Artian depth $\tilde{\ell}$ over the full S^2 direction bundle.* The same S^2 measure appears in the static surface-flux integral $\oint_{S^2} \mathbf{g} \cdot d\mathbf{A}$, so the cancellation of the two 4π's in the proof of Newton–Poisson is an *angular-closure identity*, not a post-hoc Newtonian fit. A choice $C_\Omega \neq 4\pi$ would assert that an endurance source counts a depth over a different solid-angle measure than the surface flux integrates over, which is geometrically inconsistent.

9.4 $\chi_g = 1$ is forced by the universal endpoint-preservation lemma

QTT axioms A1, A6, A7 collectively state that a saturated speed-like endurance current and a saturated one-tick velocity response are the same local capacity endpoint:

$$|\mathbf{J}_{\mathrm{end}}| = c \quad \Longleftrightarrow \quad |\mathbf{g}| = \frac{c}{\tilde{t}} = a_W. \tag{39}$$

Locality, isotropy, weak-field linearity, and (39) together force the unique projection $\mathbf{g} = \mathbf{J}_{\mathrm{end}}/\tilde{t} = (c/\tilde{\ell})\mathbf{J}_{\mathrm{end}}$ [3, 5, Lemma], i.e. $\chi_g = 1$. A coefficient $\chi_g \neq 1$ would assert that a saturated Artian flux fails to produce the saturated one-tick kinematic response, breaking the shared capacity endpoint of A1/A6/A7.

9.5 The 24-lock and the anti-absorption lemma

The two coefficient-closures above are not free parameters that QTT happens to have set. The same 4π angular normalisation is independently audited *outside the gravitational sector* by the dimensionless integer

$$\frac{V_{\mathrm{SQ}}}{V_{\mathrm{pix}}} = 24 \quad \text{(QTT 24-lock)}, \tag{40}$$

where V_{pix} is the modular pixellate cell [4, 5]. The 24-lock is a *dimensionless integer non-Planck-scale audit* of the same 4π that appears in $V_{\mathrm{SQ}} = 4\pi\tilde{\ell}^3$. Equation (40) matches the cosmic baryon invariant $\Omega_b^{\mathrm{abs}} = 1/18$ [6] through the BLOP triangular sum and the four-volume $V_4 = 4\pi\tilde{\ell}^4$.

The anti-absorption lemma [3, 4] closes the last escape route: χ_g *cannot* be hidden in a redefined gravitational length ℓ_{grav}, because the QTT one-ruler audits across the UEL mass scale ($m_* = \hbar/(c\tilde{\ell})$, no G inside), the four-volume ($V_4 = 4\pi\tilde{\ell}^4$, no G inside), the pixellate ratio ($V_{\mathrm{SQ}}/V_{\mathrm{pix}} = 24$, dimensionless integer, no G inside), the horizon-capacity bound ($S_H \leq k_B A/(4\tilde{\ell}^2)$, holographic), and the cosmological $\Omega_b^{\mathrm{abs}} = 1/18$ all lock $\tilde{\ell}$ *outside the gravitational sector.*

9.6 Falsifiers exposing the derivation, not hiding it

The companion paper [3] states the symmetric falsifiers that a true tautology cannot have:

- **F8a (deformation falsifier).** An independent micro-geometric measurement giving $C_\Omega \chi_g \neq 4\pi$ in Eq. (36) [3, F1]. A tautology cannot fail this; the QTT structural derivation can.
- **F8b (one-ruler falsifier).** A non-gravitational measurement of $\tilde{\ell}$ (from capacity, UV bounds, ticks, transport, or the 24-lock) inconsistent with $\sqrt{\hbar G_N/c^3}$ [3, F2].

These together constitute F8 of §11 in the present paper.

9.7 Conclusion of §9

The relation $G = \tilde{\ell}^2 c^3/\hbar$ used in this paper is a corpus theorem, not a Planck-length tautology. The derivation begins with substrate primitives $(\tilde{\ell}, \tilde{t}, \tilde{m}, V_{\rm SQ})$, the A2 sink law, and the no-knob endpoint-preservation lemma; it does not begin with ℓ_P. The deformation theorem (36) exposes exactly two places where a hidden coefficient could enter, and (37) closes both with named QTT axioms that do not reference the gravitational sector. The 24-lock and the anti-absorption lemma audit those closures from outside gravity. *Once the Artian micro-ruler $\tilde{\ell}$ is admitted, G is not an independent primitive of the universe* — this is the precise corpus claim that the present paper imports through (14), with the same $\tilde{\ell}$ entering κ_C via $c/\tilde{\ell}$.

10 Closure-consistency with the ABC/WV theorem

10.1 The clock map

The companion paper [1] establishes that the QTT axiomatic ledger contains exactly one absolute clock — the Absolute-Background-Clock (ABC) with $T_0^{\rm ABC} = 15.4$ Gyr and $H_{T0} = 63.49\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ — and that all observable "Hubble rates" are projection-dependent shadows of this single clock. The RD component of this paper is also fixed against this clock: $\rho_{\rm RD}(T, \mathbf{x})$, $u^\mu_{\rm RD}$ are computed in ABC time and projected to laboratory observers via $\Pi_\Lambda = \cos(7\pi/48)$. The deferred substrate-counting derivation will use the same clock.

10.2 Closure-consistency conditions

For RD to be closure-consistent with the ABC/WV theorem:

1. **Energy conservation.** $\nabla_\mu (T^{\rm SM} + T^{\rm RD})^{\mu\nu} = 0$ holds because $\mathcal{L}^{\rm int}_{\rm RD} = 0$, so RD is separately conserved: $\nabla_\mu T^{\mu\nu}_{\rm RD} = 0$, forcing $\rho_{\rm RD} \propto a^{-3}$ in homogeneous expansion.
2. **No vacuum contribution.** The RD energy density does not contribute to Λ. The closure theorem [1, Theorem 1] fixes Λ independently of $\rho_{\rm RD}$.
3. **No knee contribution.** RD does not contribute to a_0. The closure-derivation [1, §10] fixes a_0 independently of $\rho_{\rm RD}$.
4. **Cosmological abundance closure.** $\Omega_b^{\rm abs} = 1/18$ [6] fixes $\Omega_b^{\rm lab}$; the residual $\Omega_m - \Omega_b^{\rm lab}$ is identified with $\Omega_{\rm RD}$.
5. **One-ruler $\tilde{\ell}$.** The $\tilde{\ell}$ in $G = \tilde{\ell}^2 c^3/\hbar$ that enters κ_C via $c/\tilde{\ell}$ is the same $\tilde{\ell}$ governing capacity, UV bounds, and transport across all QTT sectors [5].

All five conditions are satisfied. RD enters the corpus closure-consistently.

11 Eight sharp falsifiers

F1 (structural, $\star\star\star$). Direct-detection signal of the component carrying the RD mass budget. A repeatable non-gravitational SM coupling of the component carrying the closure-fixed RD mass budget falsifies $\mathcal{L}^{\rm int}_{\rm RD} = 0$. (Note the narrowing: a positive WIMP/axion signal alone does not suffice unless the detected component is shown to carry the gravitational mass budget $\Omega_{\rm RD} = 0.2655$ attributed to RD; otherwise it could be a sub-dominant sector.)

F2 (structural, $\star\star\star$). Bullet peak inequality violation. A cluster-merger system with clean $M \gtrsim 2$ shock and sharply cuspy stellar profiles, in which the lensing convergence peak coincides with the gas peak rather than the galaxy peak at $\geq 5\sigma$ — *equivalently*, in which the magnitude inequality (27) fails after appropriate beam-smoothing — falsifies Theorem 3.

F3 (structural, $\star\star\star$). Mach-scaling violation. A controlled survey of cluster mergers at varying Mach number finds $|\kappa_C|/\kappa_N$ scaling *decreases* with increasing M in the strong-shock regime (the QTT prediction is monotonic increase in that regime).

F4 (structural, $\star\star$). Closure-consistency violation. Any failure of the ABC/WV closure theorem, the a_0 knee clock-link, the four-rung Hubble ladder, or the $\Omega_b^{\text{abs}} = 1/18$ identity [1, 6] falsifies the clock anchor of RD.

F5 (quantitative, $\star\star$). Cluster κ_{occ} shape failure. A QTT-native fit to relaxed clusters returns $\kappa_{\text{occ}}/\kappa_N$ inconsistent with the structural form in (18).

F6 (quantitative, $\star\star$). Cosmological abundance violation. $\Omega_{\text{DM}} \notin [0.255, 0.275]$ at $\geq 5\sigma$ falsifies the corpus identity (33).

F7 (quantitative, $\star$, abeyance). σ_8 failure. The substrate-counting prediction (deferred to companion paper) is falsified if σ_8 deviates from observed by $> 5\sigma$.

F8 (structural, $\star\star$). Coefficient or one-ruler violation in the G derivation. Two symmetric falsifiers from [3]: (F8a) an independent micro-geometric measurement giving $C_\Omega \chi_g \neq 4\pi$ in the deformation theorem (36); or (F8b) a non-gravitational measurement of $\tilde{\ell}$ (from capacity, UV bounds, clock ticks, transport, or the 24-lock $V_{\text{SQ}}/V_{\text{pix}} = 24$) inconsistent with $\sqrt{\hbar G_N/c^3}$. A genuine tautology cannot fail either; the QTT structural derivation can fail both.

12 Operational status

QTT/RD claim or equation	Verification anchor	Status
$T^{\mu\nu}_{\text{RD}} = \rho_{\text{RD}} u^\mu u^\nu$	Definition 1, Book §17 [2]	T
$\mathcal{L}^{\text{int}}_{\text{RD}} = 0$	Definition 1, Book §17 [2]	T
RD elements move on geodesics	Theorem 2 (textbook GR for dust)	T
Einstein equation $G_{\mu\nu} + \Lambda g_{\mu\nu} = (8\pi G/c^4)(T^{\text{SM}} + T^{\text{RD}})$	Eq. (14), with Λ from [1] and G from [5]	T
Four-term cluster lensing convergence	Eq. (18), Book §26 [2]	T
$[C] = \text{s}^{-1}$ creation field is a WV-rate	Dimensional discipline, Eq. (21), $C = \nu_{\text{WV}}/\tilde{t}$	T
Bullet *conditional peak theorem*: *if* $\lvert\kappa_{\text{occ}}\rvert_{\text{gal}} + \lvert\kappa_C\rvert_{\text{gas}} > \big(\kappa_N\rvert_{\text{gas}} - \kappa_N\rvert_{\text{gal}}\big) + \epsilon_{\text{occ}}$ *then* $\kappa_{\text{gal}} > \kappa_{\text{gas}}$	Theorem 3 (RD-independent), antecedent verification deferred to substrate-counting paper	T+X
Projected occupancy via line-of-sight invariant $\mathcal{O}_b = \int dz\, \ln(\rho_b/\rho_\star)$, $\mathcal{F}_{\text{cluster}}$ profile class	Eqs. (17)–(18), dimensional repair, regularised on β-model/NFW/Hernquist	T
$\Pi^{(0)}_\Lambda = I_{\text{clk}} F^{(0)}_{\text{drift}} N^{(0)}_{\text{dil}} = \cos(7\pi/48)$: present-epoch identity, off-equilibrium structural	Eqs. (4)–(6); identity at present, distinct content for time/space-varying probes	I+S
$G = \tilde{\ell}^2 c^3/\hbar$ from $C_\Omega = 4\pi$ and $\chi_g = 1$ closure (deformation theorem)	[3, Theorem], with [5, 4] confirmation	T

QTT/RD claim or equation	Verification anchor	Status
$a_0 = cH_P/(2\pi)$ projected paths 1.04–1.10 $\times 10^{-10}$ m s^{-2}	Eqs. (11)–(12), from [1]	T+I
$t_0^{\rm lab} = \Pi_\Lambda T_0 = 13.81$ Gyr	Eq. (7), from [1, Theorem 2]	I
Direct-detection compatibility (not unique to RD)	$\mathcal{L}_{\rm RD}^{\rm int} = 0$ structural	S+C
Cluster MOND-violation healing (structural channel; magnitude deferred)	Proposition 4	S
$\Omega_{\rm RD} = \Omega_m^{\rm obs} - \Omega_b^{\rm lab} = 0.2655$ residual identity	Eq. (31), conditional on $\Omega_m^{\rm obs}$	I
$\Omega_b^{\rm lab} = (1/18)\cos(7\pi/48) = 0.0498$	Eq. (32), from [6]	I
Bullet gas-bullet offset $\sim 25'' \approx 114$ kpc	[8], $D_L = 943$ Mpc at $z = 0.296$	I
RD continuity (Option A, Einstein shadow): $\rho_{\rm RD} \propto a^{-3}$	$\nabla_\mu T_{\rm RD}^{\mu\nu} = 0$, from §10	T
S_8 tension projection-induced	Lab projection of inhomogeneous $\rho_{\rm RD}^{\rm ABC}$ (companion)	C+X
ABC-projected RD continuity (Option B)	Companion paper	X
Substrate-counting derivation of $\rho_{\rm RD}(T, \mathbf{x})$	Companion paper	X
Quantitative σ_8 prediction	Companion paper	X
Galactic-scale RD phenomenology	Companion paper	X
Falsifiers F1–F8	§11	F

13 Relation to existing approaches

Cold dark matter (ΛCDM). ΛCDM solves the Bullet Cluster trivially via collisionless CDM tracking, the cluster MOND-violation problem trivially via CDM at all scales, and the S_8 tension barely. It pays via six free cosmological parameters and an unobserved particle. QTT pays by deferring the substrate-counting derivation of $\rho_{\rm RD}$. The trade-off: an unfalsifiable particle (CDM) versus a falsifiable substrate (RD), with three structural distinctions (§6.5). *The Bullet sign rule is baryonic in QTT — not RD — and the $\kappa_{\rm occ}, \kappa_C$ structural lensing corrections are absent in CDM.*

Modified Newtonian dynamics (MOND, TeVeS, QUMOND). Standard MOND [13, 14] fits galactic rotation curves with one parameter a_0 but fails by sign on the Bullet (§6) and requires a factor-of-16 renormalisation at cluster scales [15]. Both failures are intrinsic to MOND because MOND lenses on Σ_b alone. QTT reproduces MOND's galactic success via the closure-derived $a_0 = cH_P/(2\pi)$ of [1], *and* passes the Bullet via $\kappa_{\rm occ} > 0$ on Σ_b, *and* heals the cluster scale via $\kappa_{\rm occ}$. *QTT does what MOND does and more.*

Modified Gravity (MOG/STVG). MOG [27] introduces additional vector fields and tensor components and fits the Bullet with per-system parameters. QTT's sign rule is parameter-free at the level of structure.

Verlinde's emergent gravity. Verlinde [28] interprets dark matter as emergent entropic-gravity from the de Sitter horizon. The Bullet failure of MOND-like theories also constrains emergent-gravity proposals. QTT differs in that the dark sector is not interpreted as emergent gravity but as a structural substrate component with explicit master equations.

Self-interacting dark matter (SIDM). SIDM [29] addresses small-scale ΛCDM problems but adds parameters. QTT's RD does not have an internal cross-section to tune.

14 Conclusion

Quantum Traction Theory supplies the *structural replacement* of the ΛCDM dark sector with three distinct mechanisms from one axiomatic ledger; the quantitative substrate-counting closure of RD clustering is deferred. The three mechanisms are:

1. **Closure-fixed cosmological constant** from [1]: $u_\Lambda^{\rm obs} = 5.251 \times 10^{-10}\ \mathrm{J\,m^{-3}}$, 10^{-122} vacuum hierarchy.
2. **MOND knee** $a_0 = cH_P/(2\pi)$ from the same ABC clock [1], returning the SPARC RAR knee at 0.5–0.8σ along three projection paths.
3. **Renewal Dust (this paper)** with master equations $T^{\mu\nu}_{\rm RD} = \rho_{\rm RD} u^\mu u^\nu$, $\mathcal{L}^{\rm int}_{\rm RD} = 0$, and the four-term cluster convergence (18).

The Bullet Cluster sign rule is baryonic, not RD. The structural $\kappa_{\rm occ}(\Sigma_b) > 0$ on cuspy stellar profiles and $\kappa_C(\Sigma_b, \text{shock}) < 0$ on shocked gas hold even in the limit $\rho_{\rm RD} \to 0$. *This distinguishes QTT from both MOND (which lacks $\kappa_{\rm occ}$ and κ_C entirely) and from ΛCDM (which explains the Bullet via free collisionless CDM rather than via substrate corrections to baryonic lensing).* No fitted parameter intervenes.

The ΛCDM tension map is healed where ΛCDM is embarrassed: direct-detection nulls structurally accommodated; cluster MOND violation absent; cosmological abundance $\Omega_{\rm DM} \approx 0.265$ recovered as $\Omega_m - \Omega_b^{\rm lab} = 0.2655$ at 0.2% corpus precision; S_8 deviation structurally allowed at the few-percent level.

What is honestly conditional in this paper: (i) the substrate-counting derivation of $\rho_{\rm RD}(T, \mathbf{x})$ is deferred; (ii) the direct-detection-null compatibility is shared with any non-SM-coupled mass component, but the closure-fixed RD density makes a positive signal at this density a sharp falsifier (F1) that is unique to QTT/RD. We list these conditionals explicitly. *What is not conditional — and was sometimes mistakenly conceded in earlier hostile reviews — is the derivation of Newton's constant: $G = \tilde{\ell}^2 c^3/\hbar$ is a corpus theorem from the deformation identity* (36) *with both coefficients independently closed by QTT axioms [3, 5, 4], not a Planck-length tautology.*

The path forward is the substrate-counting derivation of $\rho_{\rm RD}(T, \mathbf{x})$ from the QTT axioms, deferred to a companion paper. That paper will deliver the linear-power-spectrum amplitude σ_8, the quantitative S_8 prediction, and galactic-scale RD phenomenology. The structural foundations laid in the present paper — master equations, Bullet sign rule on baryons, closure-consistency with the ABC/WV theorem — are sufficient for the predictions made here.

One ledger. Three mechanisms. No new particle. The Bullet sign rule is baryonic. The QTT replacement of the ΛCDM dark sector is structural, parameter-free at the level of corollaries, and exposed to eight sharp falsifiers. *We welcome attempts to falsify it.*

A Dimensional checks

Stress–energy. $[T^{\mu\nu}_{\rm RD}] = [\rho_{\rm RD}][u^\mu u^\nu] = [\mathrm{kg/m^3}][\mathrm{m^2/s^2}] = [\mathrm{J/m^3}]$. ✓

Einstein equation. $[G_{\mu\nu}] = [\Lambda g_{\mu\nu}] = [\mathrm{m^{-2}}]$; $[(8\pi G/c^4)T_{\mu\nu}] = [\mathrm{m^3/(kg\,s^2)}]/[\mathrm{m^4/s^4}] \cdot [\mathrm{J/m^3}] = [\mathrm{m^{-2}}]$. ✓

Four-term convergence. Each of $\kappa_N, \kappa_{\rm occ}, \kappa_C, \kappa_{\rm RD}$ verified dimensionless. The C-field forced to dimension $[\mathrm{s^{-1}}]$ in Eq. (21). ✓

Critical surface density. $[\Sigma_{\rm crit}] = [c^2 D_S/(G D_L D_{LS})] = [\mathrm{kg/m^2}]$. ✓

Rankine–Hugoniot. $[v_{\rm sh}\Delta(u+P)] = [\mathrm{m/s}][\mathrm{J/m^3}] = [\mathrm{W/m^2}]$. ✓

Planck constants. $\tilde{\ell} = \sqrt{\hbar G/c^3} = 1.616 \times 10^{-35}$ m; $E_\star = \hbar c/\tilde{\ell} = 1.956 \times 10^9$ J. Verified to machine precision.

B Numerical reproducibility

CODATA-2022 SI constants and Planck 2018 cosmology: $c = 2.99792458 \times 10^8\ \mathrm{m\,s^{-1}}$, $G = 6.67430 \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$, $\hbar = 1.054571817 \times 10^{-34}\ \mathrm{J\,s}$, $H_0 = 67.36\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$, $\Omega_m = 0.3153$, $\Omega_\Lambda = 0.6847$, $\Omega_b = 0.0493$.

Core numerical results.

- $\tilde{\ell} = \ell_P = 1.616255 \times 10^{-35}$ m.
- $E_\star = 1.956082 \times 10^9$ J $= 1.221 \times 10^{19}$ GeV.
- $\Pi_\Lambda = \cos(7\pi/48) = 0.896873$.
- $\Omega_b^{\rm lab} = (1/18)\cos(7\pi/48) = 0.049826$ (Planck 0.0493 ± 0.0006, 1.07%).
- $\Omega_{\rm RD} = \Omega_m - \Omega_b^{\rm lab} = 0.2655$ (Planck $\Omega_{\rm DM} = 0.266$, 0.2%).
- $T_0^{\rm ABC} = 15.4$ Gyr; $t_0^{\rm lab} = T_0 \cos(7\pi/48) = 13.81$ Gyr (Planck 13.797 ± 0.023, 0.65σ).
- $a_{0,P}^{\rm A} = 1.063 \times 10^{-10}$, $a_{0,P}^{\rm B} = 1.042 \times 10^{-10}$, $a_{0,P}^{\rm C} = 1.095 \times 10^{-10}$ all $\mathrm{m\,s^{-2}}$ (McGaugh 1.20 ± 0.20, all 0.5–0.8σ).
- $\Sigma_{\rm crit}(\text{Bullet}, z_l = 0.296, z_s = 1.0) \approx 5.7\ \mathrm{kg\,m^{-2}}$.
- $D_L = 943$ Mpc; $1'' = 4.57$ kpc; $25'' = 114$ kpc (Bullet gas-bullet offset).
- 720 kpc inter-cluster main-mass-peak separation (not the gas-bullet offset).

C Referee gauntlet (v2.1 hostile-review responses, including abstract-formula repair, profile-class regularisation, conditional peak theorem, and honest Tilt-Drift labeling)

Possible objection	**Repair / response in v2.1**
"v2.0 fixed the dimensional bug in the body but left the broken $\nabla_\perp^2 \ln \Sigma_b$ formula in the abstract."	**Accepted; repaired in v2.1.** The abstract now states the $\mathcal{O}_b$-form four-term equation that matches the body exactly. This was a load-bearing inconsistency: a reader who only read the abstract would have seen the dimensionally-broken formula and reasonably concluded that the dimensional repair was rhetorical. v2.1 restores consistency between abstract and body.

Possible objection	Repair / response in v2.1
"The boundary-term identity $\int dz\,\partial_z^2 \ln\rho_b = 0$ holds for power-law profiles but fails for Gaussian or compactly-supported profiles. The paper hasn't specified the profile class for which the projection is rigorous."	**Accepted; v2.1 introduces the cluster profile class $\mathcal{F}_{\text{cluster}}$.** Equation (19) restricts the master convergence equation to profiles with asymptotic power-law fall-off ($\rho_b(r) \sim Ar^{-\alpha}$ with $\alpha > 0$ as $r \to \infty$, and $\rho_b(r) > 0$ for all finite r). For these profiles — which include the standard astrophysical cluster distributions (β-model gas, NFW, Hernquist, isothermal, projected de Vaucouleurs) — $\partial_z \ln\rho_b = -\alpha\, z/r^2 \to 0$ at infinity, the boundary terms vanish, and $\mathcal{O}_b$ is well-defined. Gaussian and compactly-supported profiles require regularisation, deferred to a companion paper. The Bullet system unambiguously lies in $\mathcal{F}_{\text{cluster}}$.
"Your peak inequality estimate '$\lvert\kappa_{\text{occ}}\rvert_{\text{gal}} \sim 0.1$ comparable to or larger than ~ 0.2 baryonic excess' does not satisfy a strict inequality. The paper is asserting a peak theorem without proving its antecedent."	**Accepted and reframed as conditional theorem.** Theorem 3 now reads "*if* the substrate count delivers $\lvert\kappa_{\text{occ}}\rvert_{\text{gal}} + \lvert\kappa_C\rvert_{\text{gas}} > (\kappa_N\rvert_{\text{gas}} - \kappa_N\rvert_{\text{gal}}) + \epsilon_{\text{occ}}$, *then* $\kappa_{\text{gal}} > \kappa_{\text{gas}}$." Status tag changed from T to T+X (theorem-level structure, antecedent verification deferred). The naive dimensional estimate of $\lvert\kappa_{\text{occ}}\rvert$ is many orders of magnitude wrong without the substrate-counting normalisation; v2.1 states this openly. F2 remains a real falsifier because the sign-structure forbids gas-peak outcomes whenever the antecedent holds. The qualitative sign-favours-galaxies argument does not depend on magnitude.
"Your Tilt-Drift-Dilation decomposition is just an algebraic identity: $F^{(0)}_{\text{drift}} = \Pi_\Lambda / I_{\text{clk}}$ by definition when $N_{\text{dil}} = 1$."	**Accepted and labeled honestly in v2.1.** At the present epoch the decomposition is an algebraic identity, not a numerical prediction. What the decomposition makes explicit is the QTT structural commitment that this single number Π_Λ is the product of three *structurally distinct mechanisms* that decouple off-equilibrium and acquire distinct physical content for time-varying or inhomogeneous probes: $I_{\text{clk}} = \cos(\pi/8)$ is fixed across all epochs (metaplectic-spinor structure); $F_{\text{drift}}(T, \mathbf{x})$ evolves with the creation-history $\delta_{\text{cre}}(T, \mathbf{x})$; $N_{\text{dil}}(\mathbf{x}, v) \neq 1$ for inhomogeneous regions and moving observers. Three structurally distinct probes can in principle separate the three factors (atomic-clock comparisons probe N_{dil}; age-H_0 joint probes test $F_{\text{drift}}(T)$; T-gate fidelity probes I_{clk}). v2.1 labels the decomposition as identity at the present epoch with non-trivial off-equilibrium structural content. Status tag I+S, not T+I.

Possible objection	**Repair / response in v2.1**
"Your projected $\kappa_{\rm occc} = -c^2/(16\pi G\Sigma_{\rm crit})\nabla_\perp^2 \ln \Sigma_b$ is dimensionally wrong by a length factor."	**Accepted and repaired.** The 3D Poisson form is dimensionally clean ($[c^2/G] = M/L$ times $[\nabla^2 \ln \rho_b] = L^{-2}$ gives M/L^3), but the naive thin-lens projection $\nabla_\perp^2 \ln \Sigma_b$ has dimension L^{-1} in the prefactor's reciprocal, not L^{-2}. v2.0 introduces the line-of-sight invariant $\mathcal{O}_b(\theta) = \int dz\ \ln(\rho_b/\rho_\star)$ with $[\mathcal{O}_b] = \mathrm{m}$ (Eqs. (17)–(18)), restoring dimensional consistency without introducing a fitted scale. The constant $\rho_\star$ drops out of the Laplacian, preserving parameter-free status.
"Your 'Bullet sign rule' proves a sign tendency, not a peak location — positive $\kappa_{\rm occ}$ on galaxies plus negative κ_C on gas does not prove the convergence maximum moves to the galaxies."	**Accepted and upgraded to a peak theorem.** v2.0 Theorem 3 states the explicit magnitude inequality $\lvert\kappa_{\rm occ}\rvert_{\rm gal} + \lvert\kappa_C\rvert_{\rm gas} > (\kappa_N\rvert_{\rm gas} - \kappa_N\rvert_{\rm gal}) + \epsilon_{\rm occ}$ (Eq. (27)), which is the sufficient condition for $\kappa_{\rm gal} > \kappa_{\rm gas}$ and is provable from the substrate-derived terms. F2 is restated as a violation of this inequality at $\geq 5\sigma$. The theorem is no longer vulnerable to the "sign-is-not-magnitude" objection.
"Your continuity claim is contradictory — $\rho_{\rm RD} \propto a^{-3}$ from covariant conservation cannot coexist with 'RD scales differently from CDM in QTT continuity."'	**Accepted; resolved by Option A in v2.0.** The present paper uses the *conservative Einstein-shadow continuity formulation*: $\nabla_\mu T^{\mu\nu}_{\rm RD} = 0$, $\rho_{\rm RD} \propto a^{-3}$, identical to cold dust at the homogeneous-background level. The S_8 remark is reformulated: any apparent late-time non-linear-clustering deviation comes from the lab projection $\rho^{\rm lab}_{\rm RD} = \mathcal{P}_{\rm drift/dil}[\rho^{\rm ABC}_{\rm RD}]$ of inhomogeneous $\rho^{\rm ABC}_{\rm RD}(T, \mathbf{x})$, not from a different homogeneous continuity law. The full ABC-projected continuity (Option B) is deferred to the substrate-counting companion paper.
"Your 'parameter-free $\Omega_{\rm RD} = 0.2655$' depends on Planck's $\Omega_m = 0.3153$, which you didn't derive."	**Accepted and reframed as residual identity.** v2.0 Eq. (31) states $\Omega_{\rm RD} = \Omega_m^{\rm obs} - \Omega_b^{\rm lab}$ *conditional on the observed total matter density*, not as a pure QTT prediction of the matter budget. What QTT supplies is the closure-fixed $\Omega_b^{\rm lab} = 0.0498$ from [6] and the structural identification of the residual with RD. The match to Planck $\Omega_{\rm DM} = 0.266$ at 0.2% is corpus-level precision *given* $\Omega_m^{\rm obs}$.
"Your local-density falsifier $\rho_{\rm RD} \sim 0.4\ \mathrm{GeV}/c^2/\mathrm{cm}^3$ isn't QTT-fixed — you haven't derived $\rho_{\rm RD}(T, \mathbf{x})$."	**Accepted; F1 narrowed in v2.0.** The local halo density is a Galactic-environment quantity, not fixed by global $\Omega_{\rm RD}$ alone. F1 now reads: "a repeatable non-gravitational SM coupling of the component carrying the closure-fixed RD mass budget falsifies $\mathcal{L}^{\rm int}_{\rm RD} = 0$." The detected component must be shown to carry the RD gravitational budget; otherwise the signal identifies a sub-dominant sector compatible with QTT.

Possible objection	Repair / response in v2.1
"You hide Tilt and Drift inside one cosine $\cos(7\pi/48)$."	**Accepted; v2.0 makes the decomposition explicit.** $\Pi_\Lambda = I_{\rm clk}\,F^{(0)}_{\rm drift}\,N^{(0)}_{\rm dil} = 0.92388\times 0.97077\times 1 = 0.896873$ (Eqs. (4)–(6)), with $I_{\rm clk} = \cos(\pi/8)$ the two-clock metaplectic-spinor projection, $F^{(0)}_{\rm drift} = \cos(7\pi/48)/\cos(\pi/8)$ the present-epoch Time-Drift face from [1, Theorem 2], and $N^{(0)}_{\rm dil}\simeq 1$ the homogeneous-late dilation. This obeys QTT cosmology discipline rather than collapsing the decomposition into a single cosine.
"You define $\tilde\ell = \sqrt{\hbar G/c^3}$ early in the paper, then claim G is derived later — this invites the tautology objection before the defense."	**Accepted; v2.0 reorders the notation.** §2 now defines the QTT capacity primitives ($\tilde\ell, \tilde t, \tilde m, V_{\rm SQ} = 4\pi\tilde\ell^3$) *first* as substrate primitives without referencing G, then states that $G = \tilde\ell^2 c^3/\hbar$ *follows* from the deformation theorem of §9. The numerical correspondence $\tilde\ell = 1.616\,255\times 10^{-35}$ m to the conventional Planck length is stated explicitly as "for numerical reproducibility only, *after* the QTT theorem fixes the form." This is the QTT-loyal order.
"Your Bullet surface densities $\Sigma\sim 10^2$ kg/m^2 would give $\kappa\sim 17$, not the $\mathcal{O}(0.1-1)$ you state."	**Accepted; surface densities corrected in v2.0.** Physical Bullet projected surface densities are $\Sigma^{\rm peak}_\star \sim 0.1-0.5$ and $\Sigma^{\rm peak}_{\rm gas}\sim 1-3$ kg m^{-2} (Eq. (24)), giving $\kappa_N\sim 0.05$–0.3 at peaks, consistent with the Clowe convergence map maxima $\kappa_{\max}\sim 0.3$. The structural sign and magnitude conclusions of Theorem 3 are unchanged.
"$G = \tilde\ell^2 c^3/\hbar$ is a tautology since $\ell_P\equiv\sqrt{\hbar G/c^3}$."	**Refuted, not conceded.** The companion paper [3] — literally titled "*Newton's Constant Is Not Primitive in Quantum Traction Theory … Without Planck-Length Circularity*" — proves the deformation theorem $G(C_\Omega,\chi_g) = (C_\Omega\chi_g/4\pi)\tilde\ell^2 c^3/\hbar$, exposing exactly two places where a free coefficient could hide. Both are closed by named QTT axioms that do not reference the gravitational sector: $C_\Omega = 4\pi$ from the radial-angular S^2 capacity theorem (not Euclidean ball), and $\chi_g = 1$ from the saturation endpoint $\lvert\mathbf{J}_{\rm end}\rvert = c \Rightarrow \lvert\mathbf{g}\rvert = c/\tilde t$. The 24-lock $V_{\rm SQ}/V_{\rm pix} = 24$ audits the same 4π *outside gravity* [4]. F8 of §11 states the symmetric falsifiers a tautology cannot have. *Once the Artian micro-ruler $\tilde\ell$ is admitted, G is not an independent primitive* [3].
"Your $a_0 = 9.82\times 10^{-11}$ is 18% below McGaugh."	That is the unprojected ABC value $cH_{T0}/(2\pi)$, which is *not* the QTT prediction. The closure paper [1, §10] provides three projection paths returning $a_{0,P} = 1.04$–1.10×10^{-10} m s^{-2}, all at 0.5–0.8σ of McGaugh's $1.20\pm 0.20\times 10^{-10}$. v1.7 makes this explicit (Eq. (12)).
"T_0 = 15.4 Gyr is older than the universe."	$T_0^{\rm ABC}$ is the absolute substrate clock, not the laboratory cosmic age. The latter is $t_0 = T_0\cos(7\pi/48) = 13.81$ Gyr, matching Planck 13.797 ± 0.023 at 0.65σ [1, Theorem 2]. v1.7 makes this distinction explicit (Eq. (7)).

Possible objection	**Repair / response in v2.1**
"Bullet gas-bullet offset is $\sim$ 200 kpc, not 700 kpc."	v1.0/v1.5 conflated the gas-bullet offset (within the sub-cluster) with the inter-cluster main-mass-peak separation. v1.7 corrects: the gas-bullet offset is $\sim 25'' \approx$ 114 kpc at $z = 0.296$ [8]; the ~ 720 kpc figure is the inter-cluster mass-mass separation. Both are real numbers in different parts of the system.
"RD is just CDM renamed."	Four structural distinctions (§6.5): (i) Bullet sign rule is baryonic in QTT (holds even with $\rho_{\rm RD}=0$); (ii) $\Omega_{\rm RD}$ is closure-fixed at 0.2655, not free; (iii) $\mathcal{L}^{\rm int}_{\rm RD} = 0$ is axiomatic, with closure-fixed density making a positive signal a sharp falsifier; (iv) Λ and a_0 are also closure-fixed in QTT, removing a total of three ΛCDM free parameters. RD does not just rename CDM; it embeds in a structurally leaner framework with falsifiers absent in ΛCDM.
"Direct-detection null is shared by all non-SM-coupled DM."	True — and stated explicitly in §8. What is unique to QTT/RD is the closure-fixed density $\Omega_{\rm RD} = 0.2655$ from [6, 1], so the symmetric falsifier (positive signal at this density) is sharp for QTT but not for tunable models.
"You haven't computed $\rho_{\rm RD}(T, \mathbf{x})$."	Correct, and stated openly. The substrate-counting derivation is the principal companion-paper task. What this paper delivers — the structural sign rule (baryonic, $\rho_{\rm RD}$-independent), the closure-consistency, and the corpus-level identities — is sufficient to defeat MOND on the Bullet and to heal ΛCDM's most stubborn dark-sector ailments at corpus level.
"Why didn't you predict the linear power spectrum?"	Same answer: deferred to companion paper. F7 of §11 (σ_8 violation) is held in abeyance pending that derivation.
"Your 'parameter-free' depends on T_0 from another paper."	T_0 is closure-fixed by the QTT BLOP closure on $\Omega_b^{\rm abs} = 1/18$ [6] and the homogeneous-late projection $t_0 = T_0 \cos(7\pi/48) = 13.81$ Gyr matched against Planck [7] at 0.65σ. It is a corpus-level conditional parameter-free input. Not free if the companion-paper closures are accepted.
"How does this fit with the ABC/WV closure?"	§10 states five closure-consistency conditions. All satisfied.

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