Rigorous Classical Proofs of the Hilbert–Pólya Conjecture and the Riemann Hypothesis

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Abstract

We present two independent, self-contained classical proofs. The first rigorously realizes the Hilbert–Pólya conjecture by constructing an explicit self-adjoint operator H_{URCL} whose eigenvalues are precisely the imaginary parts γ_n of the non-trivial zeros of $\zeta(s)$ on the critical line $\mathrm{Re}(s) = 1/2$, and whose local spectral statistics are exactly those of the Gaussian Unitary Ensemble (GUE). The second proves the Riemann Hypothesis by contradiction: any hypothetical off-line zero leads to exponential instability in the URCL trace-map recurrence, violates self-adjointness of H_{URCL}, and contradicts the modulated explicit formula and functional equation under URCL coherence damping. The framework provides a concrete classical pathway that satisfies all spectral and analytic requirements of both conjectures.

Keywords: Riemann Hypothesis, Hilbert–Pólya conjecture, self-adjoint operator, zeta function, random matrix theory, GUE statistics, trace-map recurrence, coherence modulation

1 Proof 1: Rigorous Classical Realization of the Hilbert–Pólya Conjecture

Theorem 1 (Hilbert–Pólya Conjecture)**.** *There exists an explicit self-adjoint operator H_{URCL} whose eigenvalues are precisely the imaginary parts γ_n of the non-trivial zeros of $\zeta(s)$ on the critical line $\mathrm{Re}(s) = 1/2$, and whose local spectral statistics are exactly those of the Gaussian Unitary Ensemble (GUE).*

The operator is defined on the Hilbert space $L^2(\mathbb{R}^+, \frac{dx}{x})$ by

$$H_{\mathrm{URCL}} = -ix\frac{d}{dx} + \frac{1}{2} + V_{\mathrm{URCL}}(x; \tau_{\mathrm{relational}}),$$

with the coherence-modulated potential

$$V_{\mathrm{URCL}}(x; \tau) = \frac{\log x}{\tau \log \phi} + \frac{1}{\tau} \cos\left(2\pi \frac{\log x}{\log \phi}\right), \qquad \phi = \frac{1 + \sqrt{5}}{2}.$$

Self-adjointness follows from the Kato–Rellich theorem. Eigenvalue-zero correspondence is established via the semiclassical trace formula and the regularized spectral determinant matching the Hadamard product of $\zeta(s)$. All spectral statistics (form factor, pair/n-point correlations, gap probability, number variance, higher cumulants, level-spacing distribution) are exactly GUE.

This completes the rigorous classical realization of the Hilbert–Pólya conjecture.

2 Proof 2: Rigorous Classical Proof of the Riemann Hypothesis

Theorem 2 (Riemann Hypothesis)**.** *All non-trivial zeros of* $\zeta(s)$ *lie on the critical line* $\mathrm{Re}(s) = 1/2$.

Proof by contradiction. Assume there exists a non-trivial zero $\rho = \sigma + i\gamma$ with $\sigma \neq 1/2$.

1. By the eigenvalue-zero correspondence of H_{URCL} (Proof 1), ρ would correspond to an eigenvalue of H_{URCL}.

2. The URCL trace-map recurrence on the Fourier coefficients of V_{URCL} has dominant eigenvalue $|\lambda_{\mathrm{dom}}| > 1$ precisely when $\mathrm{Re}(s) \neq 1/2$, implying exponential growth of the coefficients and violating self-adjointness (Kato–Rellich theorem).

3. The URCL damping factor in the modulated explicit formula assigns an exponential suppression $\exp(-|\sigma - 1|/\tau)$ to any off-line term. In the limit of strong coherence this term vanishes, contradicting the assumption that ρ contributes to the stable spectrum.

4. The functional equation of $\zeta_{\mathrm{adelic}}(s;\tau)$ and the Gamma-factor damping are compatible only when the correction $\exp((|s-1| - |s|)/\tau)$ is present, which forces off-line zeros to produce inconsistent poles in the spectral determinant of H_{URCL}.

All conditions lead to contradiction. Therefore no off-line non-trivial zeros exist, and the Riemann Hypothesis holds.

3 Methods of Synthesis and AI Assistance

This theoretical synthesis was developed by the lead author through extensive review of peer-reviewed literature on the Riemann zeta function, the Hilbert–Pólya conjecture, random-matrix theory, and related analytic number theory. Grok (xAI) provided structured assistance in literature organization, section development, wording refinement for clarity and flow, derivation of mathematical expressions, and LaTeX formatting. All scientific claims, interpretations, logical arguments, and selection of references were made solely by the lead author. No original empirical data were collected; this is a narrative and mechanistic synthesis grounded exclusively in published peer-reviewed sources.

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