First-Principles Proofs of Four IHC Theorems: T3, T4, T5, and T8

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Abstract

This paper provides complete first-principles proofs of four theorems in Inverted Hypersphere Cosmology (IHC), expanding on derivations established in the Prequel [1] and companion papers.

T8 (Section 1): The $N = 33$ shell count follows from a vacuum self-consistency condition, not from dynamical stability. On $\mathbb{RP}^4$ with a φ-scaled vacuum, a harmonic degree l is self-consistent if and only if its multiplicity $d(S^4, l)$ is a Fibonacci number. This is proved from the Binet formula and the Hurwitz theorem alone. No KAM theory is required or relevant.

T4 (Section 2): The cohesion field Ψ must satisfy $\Psi(-x) = -\Psi(x)$ on $\mathbb{RP}^4$. This follows because $\mathbb{RP}^4$ is non-orientable and the vacuum has $L_{\text{net}} = -\frac{1}{2}$, which forces the field into the twisted scalar sector of the orientation bundle. Conformal coupling $\xi = 1/6$ follows from the conformal flatness of S^4.

T3 (Section 2): The UV–IR seesaw $\rho_\Lambda = \sqrt{\rho_{UV}|\rho_{IR}|}$ is independent of regularisation scheme because $Z^{\text{reg}}(-1) = -631/30$ is an exact rational number fixed by the analytic continuation of a spectral zeta function. Analytic continuation is unique; there is no scheme to choose.

T5 (Section 3): $\bar{\theta}_{\text{QCD}} = 0$ exactly on $\mathbb{RP}^4$. The antipodal map reverses the orientation of S^4, which forces $\int_{S^4} \text{Tr}(F \wedge F) = 0$ for every equivariant gauge bundle. There is only one topological sector. The strong-CP problem is not resolved — it is dissolved. No Atiyah–Patodi–Singer theory is needed.

Keywords: Fibonacci self-consistency; Binet formula; Hurwitz theorem; scheme independence; spectral zeta function; anti-periodic boundary conditions; orientation bundle; strong-CP; $\mathbb{RP}^4$; inverted hypersphere cosmology

Introduction

The IHC series [1–3] derives cosmological and particle physics parameters from a single axiom: the pre-geometric state is a non-preferential void. This paper collects and completes the proofs of four theorems whose demonstrations were abbreviated in earlier papers. Each section is self-contained and may be read independently, with one exception: T4 is used inside the T3 proof, so we present T4 before T3 in Section 2.

The T8 proof in particular benefits from a clarification of framing. The stability of the $N = 33$ shell structure is sometimes discussed in dynamical terms, but the pre-geometric vacuum has no time and no trajectories. The correct question is not whether an orbit is stable but whether a mode count is algebraically compatible with the φ-scaled structure of the vacuum — a self-consistency condition, not a stability condition, and provable from nineteenth-century number theory.

1 T8: $N = 33$ from Vacuum Self-Consistency

1.1 The Problem

IHC predicts $N = 33$ nested toroidal shells from the condition that the harmonic multiplicity $d(S^4, l)$ at degree l is a Fibonacci number. The Hurwitz theorem motivates this, but the precise logical bridge from Hurwitz to Fibonacci degeneracy was not laid out in the Prequel. We provide it here.

The key is recognising that the correct frame is not stability but self-consistency. The pre-geometric vacuum does not evolve: there is no Hamiltonian, no trajectory, no time in which perturbations could grow. Asking whether the vacuum is stable is a category error. The right question is whether a mode at degree l is compatible with the φ-scaling structure of the vacuum — a purely algebraic condition. We call this *self-consistency*, and we prove that it is satisfied if and only if $d(S^4, l)$ is a Fibonacci number.

1.2 Setting

The IHC cohesion field Ψ_0 on $\mathbb{RP}^4 = S^4/\mathbb{Z}_2$ has φ-scaled amplitudes: $\Psi_0(k) \propto \varphi^{-k}$ at shell k [1]. In the harmonic basis on S^4 it decomposes as

$$\Psi_0 = \sum_{l\,\text{even}} \sum_{m=1}^{d(S^4,l)} c_{l,m}\, Y_l^m, \qquad d(S^4, l) = \frac{(2l+3)(l+1)(l+2)}{6}, \tag{1}$$

where only even l survive the $\mathbb{Z}_2$ antipodal projection. The self-consistency problem is: which values of l allow coefficients $c_{l,m}$ that are genuinely compatible with the φ-scaling of the vacuum?

1.3 The Binet–Hurwitz Lemma

The proof rests on a number-theoretic fact that the φ-scaling structure of the vacuum forces: at each scale φ^m, the natural integer count of modes is precisely the Fibonacci number F_m. This is not an assumption; it is a theorem.

Lemma 1.1 (Binet–Hurwitz)**.** *Let F_m denote the Fibonacci numbers and let $\varphi = (1 + \sqrt{5})/2$, $\psi = -\varphi^{-1}$.*

(i) ***Binet formula:*** *$F_m = (\varphi^m - \psi^m)/\sqrt{5}$ for all $m \geq 1$ [4].*
(ii) ***Uniqueness:*** *F_m is the unique integer satisfying $|F_m - \varphi^m/\sqrt{5}| < \frac{1}{2}$.*
(iii) ***Gap:*** *Any integer $n \neq F_m$ satisfies $|n - \varphi^m/\sqrt{5}| \geq 1 - |\psi|^m/\sqrt{5} > \frac{1}{2}$ for all $m \geq 1$.*

Proof. Part (i) is the standard Binet formula. For part (ii): by (i), $|F_m - \varphi^m/\sqrt{5}| = |\psi|^m/\sqrt{5} = \varphi^{-m}/\sqrt{5}$, which equals 0.276 at $m = 1$ and decreases monotonically thereafter. Uniqueness follows because consecutive Fibonacci numbers differ by $F_{m+1} - F_m = F_{m-1} \geq 1$ for $m \geq 2$, so any integer other than F_m is at distance at least $1 - \varphi^{-m}/\sqrt{5} > \frac{1}{2}$ from $\varphi^m/\sqrt{5}$. Part (iii) is then immediate. □ □

The quantitative content of this lemma is the Hurwitz theorem [5]: $\varphi = [1; 1, 1, 1, \ldots]$ has the slowest possible continued-fraction convergence of any irrational number, meaning it is the hardest irrational to approximate by rationals. Its best integer approximants to φ^m are exactly the Fibonacci numbers $\sqrt{5}\, F_m$. This is a theorem about number theory; we are applying it to the mode counting of a quantum field.

1.4 Self-Consistency and the Fibonacci Condition

Definition 1.2 (Phi-Vacuum Self-Consistency). *The degree-l harmonic sector is φ-vacuum self-consistent if there exists a positive integer m such that*

$$\left| d(S^4, l) - \frac{\varphi^m}{\sqrt{5}} \right| < \frac{1}{2}. \tag{2}$$

The physical meaning is direct. The φ-vacuum carries spectral weight proportional to φ^m at the m-th φ-scale. By the Binet formula, the natural integer count matching this weight is $F_m = \mathrm{round}(\varphi^m/\sqrt{5})$. A degree-$l$ mode is self-consistent when its multiplicity $d(S^4, l)$ matches one of these natural counts to within the Binet rounding error. If it does not, the sector has a fractional remainder that cannot be absorbed into the φ-structure and is excluded from the vacuum.

Theorem 1.3 (Fibonacci Self-Consistency, T8). *The degree-l harmonic sector is φ-vacuum self-consistent if and only if $d(S^4, l)$ is a Fibonacci number:*

$$\textit{self-consistent} \iff d(S^4, l) \in \{1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, \ldots\}. \tag{3}$$

Proof. If $d(S^4, l) = F_m$ for some m, then by Lemma 1.1(ii) condition (2) holds. Conversely, if (2) holds for some m, then $d(S^4, l)$ is an integer within $\frac{1}{2}$ of $\varphi^m/\sqrt{5}$; by Lemma 1.1(ii), F_m is the unique such integer, so $d(S^4, l) = F_m$. □ □

1.5 Identifying the Unique Accessible Mode on $\mathbb{RP}^4$

The $\mathbb{Z}_2$ antipodal projection eliminates all odd-l harmonics. Among the remaining even-l modes, we now identify which ones are self-consistent.

Theorem 1.4 (Unique Accessible Self-Consistent Mode). *On $\mathbb{RP}^4$, the unique self-consistent even-l mode with $l \geq 2$ is $l = 4$, with $d(S^4, 4) = 55 = F_{10}$.*

Proof. Direct computation of the minimum distance to the nearest $\varphi^m/\sqrt{5}$:

l	$d(S^4, l)$	Nearest $\varphi^m/\sqrt{5}$	Distance	Self-consistent?
2	14	$m = 7$: 12.985	1.015	No
4	**55**	$\boldsymbol{m = 10}$: 55.004	0.004	**Yes**
6	140	$m = 12$: 143.999	4.001	No
8	285	$m = 13$: 232.999	52.001	No
10	506	$m = 15$: 609.999	104.000	No
12	819	$m = 16$: 986.999	168.000	No

For $l \geq 6$: $d(S^4, l)$ grows as $l^3/3$ while Fibonacci numbers grow exponentially. The gap between consecutive Fibonacci numbers $F_{m+1} - F_m = F_{m-1}$ also grows exponentially, so $d(S^4, l)$ eventually falls permanently between consecutive Fibonacci numbers with irreducible distance exceeding $\frac{1}{2}$. The minimum distance increases monotonically from $l = 6$ onward, confirming that $l = 4$ is the unique solution. □ □

1.6 Deriving $N = 33$

With $l = 4$ identified as the unique self-consistent mode, the shell count follows from the non-preferential condition that drove the IHC construction in the first place.

Corollary 1.5 ($N = 33$)**.** *The non-preferential condition — no direction of $\mathbb{R}^5$ is geometrically privileged — distributes the 55 self-consistent modes equally across the 5 embedding directions of $\mathbb{R}^5$. This gives:*

$$M = \frac{d(S^4, 4)}{d(S^4, 1)} = \frac{F_{10}}{F_5} = \frac{55}{5} = 11 = L_5, \qquad N = 3M = \boxed{33}, \tag{4}$$

where $L_5 = 11$ is the fifth Lucas number (from the Fibonacci identity $F_{2n}/F_n = L_n$ at $n = 5$), and $N = 3M$ follows from $\mathbb{Z}_3$ triality [6].

The Hurwitz theorem enters this proof not as a stability criterion but as a uniqueness theorem: it identifies φ as the unique irrational whose power sequence $\varphi^m/\sqrt{5}$ lies within $\frac{1}{2}$ of integers for every m. Those integers are the Fibonacci numbers. Self-consistency simply requires mode counts to lie on this sequence.

2 T4 and T3: Anti-Periodic Boundary Conditions and Scheme Independence

We prove T4 first because T3 depends on it.

2.1 T4: Why the Field Must Be Anti-Periodic

The cohesion field Ψ on $\mathbb{RP}^4$ satisfies either $\Psi(-x) = +\Psi(x)$ or $\Psi(-x) = -\Psi(x)$. The claim is the latter. The Prequel stated this and invoked $L_{\text{net}} = -\frac{1}{2}$; here we make the bundle-theoretic content explicit. The argument requires knowing what kind of manifold $\mathbb{RP}^4$ is and what types of scalar field it admits.

2.1.1 $\mathbb{RP}^4$ is Non-Orientable

Lemma 2.1. *$\mathbb{RP}^4 = S^4/\mathbb{Z}_2$ is non-orientable.*

Proof. The antipodal map $A : S^n \to S^n$ is the composition of $n+1$ coordinate reflections of $\mathbb{R}^{n+1}$, each of degree -1, so its degree is $(-1)^{n+1}$. At $n = 4$: $\deg(A) = (-1)^5 = -1$. Since A reverses orientation, the quotient $S^4/\langle A\rangle$ is non-orientable. □ □

Non-orientability is the key. On an orientable manifold, scalars live in the trivial real line bundle, and boundary conditions around any loop return the field to its original value. On a non-orientable manifold there are two choices: the trivial bundle, giving $\Psi(-x) = +\Psi(x)$, and the orientation bundle $\mathcal{O}$, the non-trivial $\mathbb{Z}_2$ line bundle with holonomy -1 around orientation-reversing loops. The antipodal identification $x \sim -x$ generates exactly such a loop. A field in the orientation bundle satisfies $\Psi(-x) = -\Psi(x)$.

2.1.2 The Measurement Operator Selects the Twisted Sector

Definition 2.2 (Measurement Operator)**.** $\hat{M} = \text{Vol}(S^4)^{-1} \int_{S^4} |x\rangle\langle -x| \, d\sigma(x)$.

Lemma 2.3. *$\hat{M}\, Y_l^m = (-1)^l\, Y_l^m$.*

Proof. $\hat{M}$ maps the state at x to the state at $-x$. Hyperspherical harmonics satisfy $Y_l^m(-x) = (-1)^l Y_l^m(x)$. □ □

Theorem 2.4 (Anti-Periodic Boundary Condition, T4)**.** *The cohesion field Ψ on $\mathbb{RP}^4$ satisfies $\Psi(-x) = -\Psi(x)$.*

Proof. The $\mathbb{RP}^4$ identification is physical: the vacuum is self-referential, and the measurement operator $\hat{M}$ must act consistently on the field that defines it. The spin-statistics constraint on the antipodal involution A requires the vacuum $|\Omega\rangle$ to satisfy $A|\Omega\rangle = e^{2\pi i L_{\text{net}}}|\Omega\rangle$. Since $A^2 = 1$, it follows that $L_{\text{net}} = n/2$ for integer n. For the antipodal identification to be non-trivial — to actually distinguish $\mathbb{RP}^4$ from S^4 — n must be odd. The IHC vacuum has $L_{\text{net}} = -\frac{1}{2}$ [1], giving $A|\Omega\rangle = -|\Omega\rangle$. The vacuum wavefunction is odd under the antipodal map, which means Ψ is a section of the orientation bundle $\mathcal{O}$: $\Psi(-x) = -\Psi(x)$. □ □

Theorem 2.5 (Conformal Coupling)**.** *The conformal coupling is uniquely $\xi = 1/6$.*

Proof. In four dimensions, local Weyl invariance of the scalar field action uniquely fixes $\xi = (n-2)/(4(n-1))\big|_{n=4} = 1/6$ [7]. S^4 is conformally flat (stereographic projection maps it to $\mathbb{R}^4$), so conformal invariance of the field equation is self-consistent, and $\xi = 1/6$ is uniquely selected. □ □

Remark 2.6 (Fermions and the Pin Structure). $\mathbb{RP}^4$ does not admit a standard spin structure because it is non-orientable, but it does admit a pin structure [8]. Fermions on $\mathbb{RP}^4$ are sections of the $\mathrm{Pin}^-(4)$ bundle; their anti-periodicity is the spin-$\frac{1}{2}$ counterpart of the twisted scalar condition proved above. The detailed pin-structure analysis of the fermion content is reserved for a subsequent paper.

2.2 T3: The UV–IR Seesaw is Scheme-Independent

2.2.1 Why This is Normally Hard

Vacuum energy density in quantum field theory is notorious for being scheme-dependent: the finite part of the renormalised vacuum energy shifts when you change regularisation prescription. What makes the IHC case different is that the relevant quantity is not the vacuum energy itself but the analytic continuation of a spectral zeta function at a negative integer. Analytic continuation is unique by the Identity Theorem for holomorphic functions. There is no scheme to choose; the answer is what it is.

2.2.2 The Spectral Zeta Function on $\mathbb{RP}^3$

The infrared Casimir energy of the IHC vacuum is determined by the Dirac spectral zeta function on $\mathbb{RP}^3$. With the anti-periodic spinor boundary condition imposed by T4 (Theorem 2.4, Section 2), only the odd-k modes survive the antipodal projection. Substituting $k = 2m+1$ gives eigenvalue $(2m+\frac{5}{2})$ and degeneracy $d_{2m+1} = 4(m+1)(2m+3)$ for $m = 0, 1, 2, \ldots$ [9, 10].

Theorem 2.7 (Exact Rational Spectral Invariant, T3)**.** *The zeta-regularised Casimir spectral sum on $\mathbb{RP}^3$ is the exact rational number:*

$$Z^{\mathrm{reg}}(-1) = -\frac{631}{30}. \tag{5}$$

This value is independent of any regularisation scheme.

Proof. The spectral zeta function is:

$$Z(s) = \sum_{m=0}^{\infty} 4(m+1)(2m+3)\left(2m+\tfrac{5}{2}\right)^{-s}. \tag{6}$$

Expanding $(m+1)(2m+3) = 2m^2+5m+3$ decomposes the sum into a linear combination of Hurwitz zeta functions at the argument $m + \frac{5}{4}$. Analytic continuation to $s = -1$ uses the standard values $\zeta(-3) = \frac{1}{120}$, $\zeta(-1) = -\frac{1}{12}$, $\zeta(0) = -\frac{1}{2}$ [11]:

$$Z^{\mathrm{reg}}(-1) = 4\left[\frac{4}{120} + 0 - \frac{37}{24} - \frac{15}{4}\right] = -\frac{631}{30}. \tag{7}$$

Scheme independence is immediate. The value $-631/30$ is a rational number arising from the analytic continuation of a Dirichlet series; it is the unique answer that the Identity Theorem assigns to this function at $s = -1$. Dimensional regularisation, Pauli–Villars, zeta regularisation — all are methods for evaluating the same spectral sum by analytic continuation, and they all give the same result. The key distinction from typical renormalisation is that ordinary vacuum energy has a scheme-dependent additive constant; here the relevant quantity is a value at a negative integer, which is fixed by algebra. □ □

Theorem 2.8 (Scheme Independence of the Seesaw). *The UV–IR seesaw $\rho_\Lambda = \sqrt{\rho_{UV}|\rho_{IR}|}$ is independent of any $\mathbb{RP}^4$-symmetric regularisation scheme.*

Proof. The UV energy density $\rho_{UV} = \hbar c/l_P^4 = c^7/(\hbar G^2)$ is fixed by Newton's constant G and fundamental constants; it is a physical quantity, not a regularisation artefact. The IR energy density $\rho_{IR} = Z^{\mathrm{reg}}(-1) \cdot \hbar c/(\pi^2 R_H^4)$ is determined by $Z^{\mathrm{reg}}(-1) = -631/30$, which is unique by Theorem 2.7. Neither factor has scheme dependence; their product and square root do not either. □ □

The scheme-independence here is qualitatively stronger than what is normally meant in renormalisation theory. In standard QFT, proving scheme-independence means showing that physical observables are invariant under changes of renormalisation prescription. Here, the IR Casimir energy is not renormalised at all — it is the analytic continuation of a convergent spectral sum, a purely mathematical operation with a unique answer.

3 T5: The Strong-CP Problem is Dissolved by $\mathbb{RP}^4$ Topology

3.1 The Claim

IHC predicts $\bar{\theta}_{\text{QCD}} = 0$ exactly, without an axion or any new field [3]. The intuition was stated there: on $\mathbb{RP}^4$, the antipodal map acts as charge conjugation on the gauge bundle, pairing every instanton sector n with sector $-n$; the θ-term therefore vanishes. Here we give the formal proof.

The result is stronger than the intuition suggests. The reason the strong-CP problem is dissolved on $\mathbb{RP}^4$ is not merely that sectors cancel pairwise — it is that there is only one topological sector, the trivial one. When there is only one sector there is no θ to relax, and no anomalous phase can appear between sectors that do not exist. Atiyah–Patodi–Singer index theory, which addresses phase differences between distinct sectors, is simply not needed.

3.2 Gauge Bundles on $\mathbb{RP}^4$ Must Be Equivariant

A gauge bundle over $\mathbb{RP}^4 = S^4/\mathbb{Z}_2$ is equivalently a principal G-bundle over S^4 that is equivariant under the antipodal map. Concretely: there must exist a bundle isomorphism $\phi : A^*\tilde{P} \to \tilde{P}$ covering A. Gauge fields on $\mathbb{RP}^4$ pull back to equivariant gauge fields on S^4.

Lemma 3.1 (Orientation reversal)**.** *The antipodal map $A : S^4 \to S^4$ is orientation-reversing.*

Proof. A is the composition of 5 coordinate reflections on $\mathbb{R}^5$, each of degree -1, giving $\deg(A) = (-1)^5 = -1$. □ □

Lemma 3.2 (Pontryagin density negation)**.** *For any equivariant gauge bundle on S^4: $A^*\mathrm{Tr}(F \wedge F) = -\mathrm{Tr}(F \wedge F)$.*

Proof. The equivariance isomorphism ϕ relates A^*F to F through the gauge bundle structure. The Pontryagin density $\mathrm{Tr}(F \wedge F)$ is a 4-form. Under the pullback of an orientation-reversing diffeomorphism, any 4-form ω on a 4-manifold satisfies $A^*\omega = \deg(A) \cdot \omega$ — specifically, $A^*\mathrm{dvol} = -\mathrm{dvol}$ since $\deg(A) = -1$. The equivariance of the bundle means the Pontryagin density transforms as a pure 4-form under A, giving $A^*\mathrm{Tr}(F \wedge F) = -\mathrm{Tr}(F \wedge F)$. □ □

3.3 There is Only One Topological Sector

Theorem 3.3 (Vanishing Instanton Number)**.** *For every equivariant $SU(N)$ gauge bundle on S^4:*

$$n = \frac{1}{8\pi^2} \int_{S^4} \mathrm{Tr}(F \wedge F) = 0. \tag{8}$$

Proof. Write $S^4 = D_+ \cup D_-$ where D_+ and D_- are the upper and lower hemispheres and

$A(D_+) = D_-$. Then:

$$\begin{aligned}\int_{S^4} \mathrm{Tr}(F \wedge F) &= \int_{D_+} \mathrm{Tr}(F \wedge F) + \int_{D_-} \mathrm{Tr}(F \wedge F) \\ &= \int_{D_+} \mathrm{Tr}(F \wedge F) + \int_{A(D_+)} \mathrm{Tr}(F \wedge F) \\ &= \int_{D_+} \mathrm{Tr}(F \wedge F) + \int_{D_+} A^*\mathrm{Tr}(F \wedge F) \\ &= \int_{D_+} \mathrm{Tr}(F \wedge F) - \int_{D_+} \mathrm{Tr}(F \wedge F) \quad \text{(Lemma 3.2)} \\ &= 0.\end{aligned}$$

□

Theorem 3.4 (Trivial Chern Class)**.** *Every principal $SU(N)$ bundle over $\mathbb{RP}^4$ has $c_2 = 0 \in H^4(\mathbb{RP}^4; \mathbb{Z}_2)$.*

Proof. For a bundle $P \to \mathbb{RP}^4$, the pullback $\pi^*P \to S^4$ via the double-covering map π satisfies:

$$\int_{S^4} c_2(\pi^*P) = \deg(\pi) \cdot \int_{\mathbb{RP}^4} c_2(P) = 2 \int_{\mathbb{RP}^4} c_2(P). \tag{9}$$

By Theorem 3.3, the left side is zero, so $\int_{\mathbb{RP}^4} c_2(P) = 0$. Since $H^4(\mathbb{RP}^4; \mathbb{Z}_2) = \mathbb{Z}_2$, this forces $c_2(P) = 0$. □ □

Theorem 3.5 (Strong-CP Resolution, T5)**.** *$\bar{\theta}_{\mathrm{QCD}} = 0$ exactly on $\mathbb{RP}^4$. No axion or new symmetry is required. The predicted neutron electric dipole moment is $d_n = 0$ exactly.*

Proof. Every $SU(3)$ gauge bundle on $\mathbb{RP}^4$ has $n = 0$ (Theorem 3.3) and $c_2 = 0$ (Theorem 3.4). There is exactly one topological sector. The θ-term contributes $e^{i\bar{\theta}n} = 1$ to every configuration; the parameter $\bar{\theta}$ does not appear in any physical amplitude. It is not merely zero: it is undefined as a physical parameter. □ □

The Peccei–Quinn mechanism introduces a dynamical field to relax $\bar{\theta}$ toward zero. On $\mathbb{RP}^4$ there is nothing to relax. The problem is dissolved at the topological level, not solved at the dynamical one.

The question of whether the fermion determinant could introduce a $\bar{\theta}$-dependent phase between topological sectors is vacuous here. There are no distinct sectors between which such a phase could appear.

Summary: All Eight Theorems Now Proved

	Theorem	Content	Status
T1	$\mathbb{RP}^4$ topology	Killing–Hopf + centre of $O(5)$ [1]	Proved
T2	UV–IR pairing	Antipodal maps $k \to -k$ [2]	Proved
T3	Scheme independence	$Z^{\mathrm{reg}}(-1) = -631/30$ exact (Section 2)	**This paper**
T4	Anti-periodic BC	Orientation bundle; $\xi = 1/6$ (Section 2)	**This paper**
T5	Strong-CP	One topological sector; no axion (Section 3)	**This paper**
T6	Golden ratio	$q^2 + q = 1$ fixed point [1]	Proved
T7	Equal energy	Equipartition; $\rho_k \propto R_k^{-3}$ [1]	Proved
T8	$N = 33$	Fibonacci self-consistency (Section 1)	**This paper**

All eight IHC theorems are now proved from first principles.

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