Interaction Between a Hypercomplex Field and a Weyl Field

Weak or Vanishing Coupling Regime in the RGH Framework

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Abstract

We study, within the spirit of Hypercomplex General Relativity (RGH) reformulated in symplectic form, the case in which the direct coupling between a Weyl field and a hypercomplex field is very weak, or even vanishing. The goal is to clarify what disappears, what remains, and which geometric and cosmological consequences follow. We show that, in this limit, the two sectors decouple at the level of direct mixed terms, while remaining indirectly correlated through the emergent metric and through the effective gravitational field equation. This yields a minimal, economical, and potentially more stable framework in which the Weyl sector plays mainly a conformal role, whereas the hypercomplex sector may carry the dominant effective gravitational corrections.

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1 Statement of the problem

Within the RGH framework enriched by a Weyl structure and by an internal hypercomplex sector, the total connection may be viewed as the sum of three components:

$$D_\mu = \partial_\mu + \Gamma_\mu + W_\mu + A_\mu, \tag{1}$$

where:

- Γ_μ denotes the usual gravitational/spinorial part;
- W_μ is the Weyl field, associated with dilatational structure or conformal compatibility;
- A_μ is the internal hypercomplex connection (quaternionic or related).

The total curvature then takes the schematic form

$$[D_\mu, D_\nu] = R^{(\Gamma)}_{\mu\nu} + F^{(W)}_{\mu\nu} + F^{(H)}_{\mu\nu} + F^{(\text{mix})}_{\mu\nu}, \tag{2}$$

with

$$F^{(W)}_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu, \tag{3}$$

$$F^{(H)}_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]. \tag{4}$$

The question addressed here is the following:

> *What becomes of the theory if the direct coupling between the Weyl sector and the hypercomplex sector is very weak, or even zero?*

2 Hypotheses

We assume the following.

2.1 Hypothesis H1: decomposed action

The total action can be written in the form

$$S = S_{\text{grav}} + S_W + S_H + \varepsilon\, S_{\text{mix}} + S_{\text{mat}}, \tag{5}$$

where:

- S_{grav} denotes the emergent gravitational sector;

- S_W the Weyl sector;
- S_H the hypercomplex sector;
- S_{mix} the direct coupling terms between Weyl and hypercomplex sectors;
- ε is a coupling parameter.

2.2 Hypothesis H2: weak regime

The regime under consideration is

$$0 \leq \varepsilon \ll 1, \tag{6}$$

and the strict decoupling limit is

$$\varepsilon = 0. \tag{7}$$

2.3 Hypothesis H3: common observable geometry

Even when $\varepsilon = 0$, both sectors live on the same effective geometry $g_{\mu\nu}$, or more precisely both participate in its emergent dynamics.

3 Physical intuition

One must clearly distinguish two levels of interaction.

3.1 Direct interaction

It arises from explicitly mixed terms, for example

$$L_{\text{mix}} = \zeta\, \Phi_I\, F^{(W)}_{\mu\nu} F_I^{(H)\,\mu\nu} + \eta\, W_\mu J^\mu_H + \xi\, R\, A^I_\mu A^\mu_I. \tag{8}$$

If ε is very small, these terms become perturbative; if $\varepsilon = 0$, they disappear.

3.2 Indirect interaction

Even if the terms in (8) vanish, the Weyl field and the hypercomplex field remain indirectly coupled through their common contribution to the effective energy–momentum tensor and hence to the observable geometry.

In other words:

> *the absence of direct coupling does not mean the absence of any dialogue; it means only the absence of explicit local mixing between the two sectors.*

4 Minimal model

4.1 Weyl sector

We adopt a Maxwell-type action

$$S_W = -\frac{1}{4} \int d^4x\, \sqrt{-g}\, F^{(W)}_{\mu\nu} F^{(W)\,\mu\nu}. \tag{9}$$

Variation with respect to W_μ yields

$$\nabla_\mu F^{(W)\,\mu\nu} = J^\nu_{(W)}. \tag{10}$$

4.2 Hypercomplex sector

For the internal sector we take a Yang–Mills-type action:

$$S_H = -\frac{1}{4} \int d^4x \sqrt{-g}\, \mathrm{Tr}\Big(F^{(H)}_{\mu\nu} F^{(H)\,\mu\nu} \Big) \,. \tag{11}$$

Variation with respect to A_μ gives

$$\nabla_\mu F^{(H)\,\mu\nu} + [A_\mu, F^{(H)\,\mu\nu}] = J^\nu_{(H)}. \tag{12}$$

4.3 Mixed sector

We introduce a generic mixed term

$$S_{\mathrm{mix}} = \int d^4x \sqrt{-g}\, L_{\mathrm{mix}}, \tag{13}$$

where L_{mix} is given by (8) as an indicative form.

The purpose of the present document is precisely to examine what happens when this term is negligible or absent.

5 Formal results in the weak-coupling limit

5.1 Decoupling of mixed curvature

When $\varepsilon \to 0$, the mixed blocks become negligible:

$$F^{(\mathrm{mix})}_{\mu\nu} \simeq 0. \tag{14}$$

At the level of the global decomposition,

$$[D_\mu, D_\nu] \simeq R^{(\Gamma)}_{\mu\nu} + F^{(W)}_{\mu\nu} + F^{(H)}_{\mu\nu}. \tag{15}$$

5.2 Almost separated equations

The field equations then become, at leading order,

$$\nabla_\mu F^{(W)\,\mu\nu} = J^\nu_{(W)} + O(\varepsilon), \tag{16}$$

$$\nabla_\mu F^{(H)\,\mu\nu} + [A_\mu, F^{(H)\,\mu\nu}] = J^\nu_{(H)} + O(\varepsilon). \tag{17}$$

In the strictly decoupled case $\varepsilon = 0$, one obtains

$$\nabla_\mu F^{(W)\,\mu\nu} = J^\nu_{(W)}, \tag{18}$$

$$\nabla_\mu F^{(H)\,\mu\nu} + [A_\mu, F^{(H)\,\mu\nu}] = J^\nu_{(H)}. \tag{19}$$

5.3 Effective energy–momentum tensor

The effective gravitational dynamics nevertheless remains sensitive to both sectors:

$$G_{\mu\nu} = \kappa \left(T^{(\mathrm{mat})}_{\mu\nu} + T^{(W)}_{\mu\nu} + T^{(H)}_{\mu\nu} + \varepsilon\, T^{(\mathrm{mix})}_{\mu\nu} \right). \tag{20}$$

In the decoupled limit,

$$G_{\mu\nu} = \kappa \left(T^{(\mathrm{mat})}_{\mu\nu} + T^{(W)}_{\mu\nu} + T^{(H)}_{\mu\nu} \right), \tag{21}$$

so the two sectors remain linked *gravitationally* even without direct coupling.

6 Conceptual interpretation

6.1 What disappears

When the direct coupling is very weak or vanishing, one loses:

- direct transfers of energy or structure between Weyl and hypercomplex sectors;
- dominant interference terms;
- significant crossed sources in the equations of motion.

6.2 What remains

By contrast, one retains:

- an autonomous Weyl field;
- an autonomous hypercomplex sector;
- the sum of their contributions in the effective gravitational equation;
- the possibility that the hypercomplex sector alone generates an observable additional gravitation.

6.3 Physical reading

In this limit, the Weyl field may be interpreted as a conformal or scale-gauge sector, relatively discreet, whereas the hypercomplex field may support:

- geometric halos;
- effective corrections to cosmological equations;
- bounce, saturation, or invisible-gravity-type contributions.

7 FLRW cosmological reduction

7.1 Effective Friedmann equation

Assume an effective FLRW geometry,

$$ds^2 = -dt^2 + a(t)^2 \gamma_{ij} dx^i dx^j, \tag{22}$$

and a modified Friedmann equation

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3} (\rho_{\mathrm{mat}} + \rho_{\mathrm{eff}}), \tag{23}$$

with

$$\rho_{\mathrm{eff}} = \rho_W + \rho_H + \varepsilon\, \rho_{\mathrm{mix}}. \tag{24}$$

In the weak-coupling regime,

$$\rho_{\mathrm{eff}} \simeq \rho_W + \rho_H. \tag{25}$$

7.2 Useful minimal ansatz

In the spirit of the minimal RGH–Weyl model, one may parametrize

$$\rho_W(a) \sim \frac{\alpha}{a^2}, \qquad \rho_H(a) \sim -\frac{\beta}{a^4}, \qquad \alpha > 0,\ \beta > 0. \tag{26}$$

This gives

$$\rho_{\text{eff}}(a) = \frac{\alpha}{a^2} - \frac{\beta}{a^4}, \tag{27}$$

without requiring a Weyl–hypercomplex interference term at leading order.

7.3 Bounce condition

In a spatially flat universe, neglecting ordinary matter near the bounce, the Friedmann equation becomes

$$H^2 = \frac{8\pi G}{3}\left(\frac{\alpha}{a^2} - \frac{\beta}{a^4}\right). \tag{28}$$

The condition $H = 0$ imposes

$$a_{\min} = \sqrt{\frac{\beta}{\alpha}}. \tag{29}$$

Thus, even in the absence of strong direct Weyl–hypercomplex coupling, an effective bounce may exist provided the hypercomplex sector contributes with the appropriate sign and strength in (27).

7.4 Essential cosmological message

The key conceptual point is the following:

> *the bounce does not need to be carried by a strong dialogue between Weyl and hypercomplex sectors; it may already emerge from the indirect gravitational combination of two almost separate sectors.*

8 Particular case: exactly vanishing coupling

Let us now take the radical case

$$\varepsilon = 0. \tag{30}$$

Then:

1. the mixed terms disappear from the action;
2. the Weyl and hypercomplex equations are dynamically separated;
3. the only remaining interaction is gravitational, through the effective metric.

This regime may be summarized by the scheme

$$\text{emergent GR} \quad + \quad \text{autonomous Weyl} \quad + \quad \text{autonomous hypercomplex sector}. \tag{31}$$

One therefore obtains a more minimal, more readable, and often more defensible phenomenological model.

9 Methodological discussion

9.1 Why this regime is interesting

The weak-coupling regime has several advantages:

- it limits the proliferation of free parameters;
- it reduces the risk of dynamical pathologies associated with overly aggressive mixing;
- it makes it easier to attribute an observed effect to a given sector;
- it provides a cleaner basis for numerical or observational tests.

9.2 What cannot yet be concluded

However, without specifying the model further, one still cannot determine unambiguously:

- the exact sign of all effective terms;
- the full perturbative stability;
- the exact form of the observables in fine structure, lensing, or cosmological spectra;
- the actual value of the parameter ε in nature.

10 Conclusion

The case of a very weak, or even vanishing, coupling between a hypercomplex field and a Weyl field does not empty the theory of substance; it clarifies it.

One then obtains the following picture:

1. the direct mixed terms are negligible or absent;
2. the Weyl and hypercomplex sectors evolve almost separately;
3. they nevertheless remain linked through their common contribution to effective gravitation;
4. the hypercomplex sector may remain the main carrier of additional gravitation;
5. the Weyl sector preserves a more discreet conformal and structural role.

This regime therefore appears as a minimalist, clean, and potentially more robust version of the RGH–Weyl framework. It provides a natural basis for a more technical continuation: minimal ansatz, perturbations, stability, and then observational confrontation.

Synthetic formula. If

$$S = S_{\text{grav}} + S_W + S_H + \varepsilon S_{\text{mix}}, \qquad \varepsilon \ll 1, \tag{32}$$

then, at leading order,

$$\boxed{\text{weak direct interaction} \implies \text{almost complete local decoupling but common gravitational backreaction.}} \tag{33}$$