Appendix — Static local production of an antigravity hypercomplex field in the HGR framework

Laurent Besson (original idea)
Appendix drafting: ChatGPT

Working version — April 20, 2026

Purpose of the appendix

The purpose of this appendix is to formulate, in the spirit of Hypercomplex General Relativity (HGR), the minimal conditions required not merely for the existence of a local repulsive term, but for the *static local production* of a hypercomplex field capable of canceling the weight of a test mass.

The central difficulty is not to write a repulsive term in an effective equation, but to understand:

- what *sources* the hypercomplex field;
- what makes it possible to *confine it locally*;
- what makes it possible to make it *stationary*;
- what gives it a sufficient *radial orientation* to compensate the field of a source mass M.

1 Local problem of weight compensation

Consider a source mass M and a test mass m placed at a fixed altitude r_0 above M. In the nonrelativistic limit, the standard gravitational acceleration is

$$\mathbf{a}_M(r) = -\frac{GM}{r^2}\,\hat{\mathbf{r}}. \tag{1}$$

The goal is to obtain a local configuration such that

$$\mathbf{a}_{\text{eff}}(r_0) = 0, \tag{2}$$

that is, explicitly,

$$\boxed{\mathbf{a}_{\text{rep}}(r_0) = +\frac{GM}{r_0^2}\,\hat{\mathbf{r}}.} \tag{3}$$

In other words, the locally produced hypercomplex field must generate a radial acceleration opposite to that of the gravitational field of the source mass.

2 Geometric reading through the effective connection

In the weak-field approximation and for a slow particle, the spatial acceleration is related to the effective connection by

$$a^i \simeq -c^2\,\Gamma^i_{00,\text{eff}}. \tag{4}$$

We then introduce an effective connection of the form

$$\Gamma^\rho_{\mu\nu,\text{eff}} = \Gamma^\rho_{\mu\nu}(g) + \Delta\Gamma^\rho_{\mu\nu}(H), \tag{5}$$

where $\Delta\Gamma^\rho_{\mu\nu}(H)$ denotes the affine correction induced by the hypercomplex sector.

The local condition for weight compensation then becomes

$$\boxed{-c^2\,\Delta\Gamma^r_{00}(r_0) = +\frac{GM}{r_0^2}.} \tag{6}$$

The problem of producing the hypercomplex field therefore amounts to producing a stationary configuration capable of locally imposing an affine correction of this amplitude.

3 Field equations with sources

In the minimal HGR formulation, the internal sector and the dynamical symplectic sector may be written, at the schematic level, as

$$D_\mu F^{\mu\nu} = 0, \qquad \mathrm{d}\star\mathrm{d}\Omega = 0, \qquad \mathrm{d}\star F^{(\phi)} = 0. \tag{7}$$

For there to be *production* of a local hypercomplex field, genuine sources must be introduced:

$$\boxed{D_\mu F^{\mu\nu} = J^\nu_A, \qquad \mathrm{d}\star\mathrm{d}\Omega = J_\Omega, \qquad \mathrm{d}\star F^{(\phi)} = J_\phi.} \tag{8}$$

Here:

- J^ν_A sources the internal hypercomplex connection;
- J_Ω sources the symplectic sector;
- J_ϕ possibly sources the Weyl sector.

The central conceptual point is that a manipulable hypercomplex field requires identifying a physical or effective *source quantity*, and not merely an abstract geometric degree of freedom.

4 Minimal source Lagrangian

A minimal extension of the Lagrangian may be postulated in the form

$$\boxed{\mathcal{L}_{\text{src}} = g_A\,J^\mu_I A^I_\mu + g_\Omega\,\mathcal{J}^{\mu\nu}B_{\mu\nu} + g_\phi\,j^\mu\phi_\mu,} \tag{9}$$

where:

- J^μ_I is an internal hypercomplex current;
- $\mathcal{J}^{\mu\nu}$ sources the observable projection $B_{\mu\nu}$ of the mixed sector;
- j^μ is a current coupled to the Weyl sector;
- g_A, g_Ω, g_ϕ are effective couplings.

This expression does not claim to immediately identify a real material or device, but it provides a minimal structure for modeling the *production* of a local hypercomplex field.

5 Minimal hypercomplex affine ansatz

In the spirit of the strong affine extension of HGR, one may take as a minimal model

$$\Delta\Gamma^{\rho}_{\mu\nu}(H) = \frac{\lambda}{2}\left(A^{I}_{\mu}\Sigma^{\rho}_{\nu I} + A^{I}_{\nu}\Sigma^{\rho}_{\mu I}\right), \tag{10}$$

where:

- A^{I}_{μ} is an internal hypercomplex connection;
- $\Sigma^{\rho}_{\nu I}$ is a base–fiber projection tensor;
- λ is an effective geometric coupling.

The local compensation condition then becomes

$$\boxed{-c^2\,\lambda\,(A^{I}_{0}\Sigma^{r}_{0I})_{r=r_0} = +\frac{GM}{r_0^2}.} \tag{11}$$

This relation directly fixes the minimal local amplitude of the geometric product $A^{I}_{0}\Sigma^{r}_{0I}$ required to cancel weight.

6 Local activation and spatial confinement

A useful local antigravity term cannot be homogeneous on cosmological scales. It must be activated within a finite spatial domain D.

We therefore introduce a spatial activation function

$$\mathcal{A}_D(x) \in [0,1], \tag{12}$$

which is approximately 1 inside the active domain and 0 outside it.

A possible smooth form is

$$\mathcal{A}_D(x) = \exp\left(-\frac{d(x,D)^2}{L^2}\right), \tag{13}$$

where L fixes the characteristic thickness of the domain.

The local effective connection then becomes

$$\Gamma^{\rho}_{\mu\nu,\mathrm{eff}} = \Gamma^{\rho}_{\mu\nu}(g) + \mathcal{A}_D(x)\,\Delta\Gamma^{\rho}_{\mu\nu}(H). \tag{14}$$

The local condition at the center x_0 of the domain takes the form

$$\boxed{-c^2\,\mathcal{A}_D(x_0)\,\Delta\Gamma^{r}_{00}(x_0) = +\frac{GM}{r_0^2}.} \tag{15}$$

7 Need for directional anisotropy

A purely scalar isotropic field is not sufficient to produce an oriented compensation of weight. A structure capable of selecting the radial direction $\hat{\mathbf{r}}$ is required.

The producing sector must therefore be at least:

- either vectorial;
- or affinely anisotropic;
- or a mixed base–fiber tensor.

In the ansatz (10), this anisotropy is carried by $\Sigma^{\rho}_{\nu I}$ and by the internal orientation of the sector A^{I}_{μ}.

8 Stationarity and finite energy

Static local production requires an approximately stationary solution:

$$\partial_t A^I_\mu \approx 0, \qquad \partial_t \Omega_{\mathrm{mix}} \approx 0, \qquad \partial_t \phi_\mu \approx 0. \tag{16}$$

It also requires a finite total energy in the active domain:

$$E_{\mathrm{tot}} = \int_D \mathrm{d}^3x \; (\mathcal{E}_A + \mathcal{E}_\Omega + \mathcal{E}_{\mathrm{coup}}) < \infty. \tag{17}$$

Without finite energy, the production of a local antigravity field would have no controlled physical meaning.

9 Condition of local stability

Even if local compensation is satisfied, a useful configuration must still be stable. If $a_{\mathrm{eff}}(r)$ denotes the effective radial acceleration, the minimal conditions are

$$a_{\mathrm{eff}}(r_0) = 0, \tag{18}$$

and

$$\left.\frac{\mathrm{d}a_{\mathrm{eff}}}{\mathrm{d}r}\right|_{r=r_0} < 0. \tag{19}$$

This second condition ensures that a small radial perturbation does not turn local compensation into a divergent instability.

10 Minimal radial toy model

A minimal effective expression for the acceleration is

$$\mathbf{a}_{\mathrm{eff}}(r) = -\frac{GM}{r^2}\,\hat{\mathbf{r}} + \mathcal{A}_D(r)\,a_H(r)\,\hat{\mathbf{r}}. \tag{20}$$

The compensation condition at the target altitude r_0 is

$$\boxed{a_H(r_0) = \frac{GM}{r_0^2}.} \tag{21}$$

One may write

$$a_H(r) = a_0 f(r), \tag{22}$$

where $f(r)$ is centered on r_0 and decreases rapidly outside the domain.

For example,

$$f(r) = \exp\left(-\frac{(r-r_0)^2}{\ell^2}\right), \tag{23}$$

with adjustment

$$a_0 = \frac{GM}{r_0^2} \tag{24}$$

if $f(r_0) = 1$.

This toy model does not replace a complete solution of the sourced equations, but it makes it possible to fix the minimal conditions of compensation, confinement, and stability.

11 Effective tensorial interpretation

At the level of the field equation, the idea may be summarized by

$$G_{\mu\nu} = 8\pi G\, T^{(m)}_{\mu\nu} + \mathcal{A}_D(x)\, \Theta^{\mathrm{rep}}_{\mu\nu}(x). \tag{25}$$

However, if one wants a local oriented compensation of weight, the tensor $\Theta^{\mathrm{rep}}_{\mu\nu}$ must not be identified with an ordinary isotropic fluid. It must contain anisotropic components capable of locally correcting the Γ^r_{00} component of the effective connection.

The proper physical language is therefore no longer merely that of a repulsive density, but that of an *activated anisotropic geometric domain*.

Conclusion

Within the HGR framework, static local production of an antigravity hypercomplex field would require at least:

- the introduction of genuine sources J^ν_A, J_Ω, J_ϕ in the field equations;
- a source Lagrangian of the form
$$\mathcal{L}_{\mathrm{src}} = g_A\, J^\mu_I A^I_\mu + g_\Omega\, \mathcal{J}^{\mu\nu} B_{\mu\nu} + g_\phi\, j^\mu \phi_\mu;$$
- a local affine correction of the type $\Delta\Gamma^\rho_{\mu\nu}(H)$;
- spatial activation confined to a domain D;
- directional anisotropy capable of selecting the radial axis;
- approximate stationarity of the produced fields;
- finite total energy;
- a local compensation condition,
$$-c^2\, \mathcal{A}_D(x_0)\, \Delta\Gamma^r_{00}(x_0) = +\frac{GM}{r_0^2};$$
- a stability condition on the radial profile of the repulsive term.

The central difficulty is therefore not merely the mathematical existence of a local repulsive term, but rather the *physical production* of a local, activated, oriented, stationary, and energetically controlled geometric bubble.

It is precisely at this level that the true conceptual difficulty of local hypercomplex antigravity appears.