Appendix — Unit Conventions and Dimensional Normalizations in HGR

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1 Purpose of this appendix

This appendix fixes the unit conventions used in the numerical applications of the HGR/RGH framework. Its purpose is to avoid any ambiguous mixing between geometric units, where one often sets $c = 1$, and strict SI units, which are required as soon as numerical values are given in $\mathrm{kg\,m^{-3}}$, $\mathrm{m^{-2}}$, $\mathrm{m^{-4}}$, seconds, or light-years.

In every numerical table, unless explicitly stated otherwise, strict SI units are used. Therefore, all necessary factors of c must be kept whenever they are required by dimensional consistency.

2 Mass density, energy density, and pressure

We distinguish three quantities:

$$\rho_m \quad : \text{ mass density, in } \mathrm{kg\,m^{-3}}, \tag{1}$$

$$u \quad : \text{ energy density, in } \mathrm{J\,m^{-3}}, \tag{2}$$

$$p \quad : \text{ pressure, in } \mathrm{Pa} = \mathrm{J\,m^{-3}}. \tag{3}$$

They are related by

$$u = \rho_m c^2. \tag{4}$$

For a perfect fluid with constant equation-of-state parameter w,

$$p = wu = w\rho_m c^2. \tag{5}$$

In the numerical HGR documents, whenever the symbol ρ appears in a Friedmann equation with a factor $8\pi G/3$, it denotes by default a mass density ρ_m.

3 Friedmann equation in strict SI units

For a FLRW metric, with scale factor $a(t)$ and normalized spatial curvature $k \in \{-1, 0, +1\}$, we use the convention

$$H^2 + \frac{kc^2}{a^2} = \frac{8\pi G}{3}\rho_m + \frac{\Lambda c^2}{3}, \tag{6}$$

with

$$H = \frac{\dot{a}}{a}. \tag{7}$$

If one prefers to write the same equation in terms of the energy density u, then

$$H^2 + \frac{kc^2}{a^2} = \frac{8\pi G}{3c^2}u + \frac{\Lambda c^2}{3}. \tag{8}$$

The two forms are equivalent if $u = \rho_m c^2$.

4 Effective density and H^2-normalized contribution

We strictly distinguish:

$$\rho_{\text{eff}} \quad : \text{effective mass density, in kg m}^{-3}, \tag{9}$$

$$\mathcal{H}_{\text{eff}} \quad : \text{contribution already normalized in the Friedmann equation, in s}^{-2}. \tag{10}$$

The relation between them is

$$\mathcal{H}_{\text{eff}} \equiv \frac{8\pi G}{3}\rho_{\text{eff}}. \tag{11}$$

Thus, the following two expressions are equivalent but must not be confused:

$$H^2 + \frac{kc^2}{a^2} = \frac{8\pi G}{3}\left(\rho_m + \rho_{\text{eff}}\right), \tag{12}$$

or

$$H^2 + \frac{kc^2}{a^2} = \frac{8\pi G}{3}\rho_m + \mathcal{H}_{\text{eff}}. \tag{13}$$

In numerical applications, the symbol ρ_{eff} is reserved for a physical or effective density expressed in $\mathrm{kg\,m^{-3}}$. Terms added directly to H^2 must be denoted by $\mathcal{H}_{\text{eff}}$, or by an equivalent symbol with dimension s^{-2}.

5 Scaling law for a perfect fluid

For a perfect fluid with constant equation-of-state parameter w, standard conservation gives

$$\dot{\rho}_m + 3H(1+w)\rho_m = 0, \tag{14}$$

that is,

$$\rho_m(a) = \rho_{m,0}\left(\frac{a}{a_0}\right)^{-3(1+w)}. \tag{15}$$

The usual cases are:

$$w = 0 \quad \Rightarrow \quad \rho_m \propto a^{-3} \quad \text{non-relativistic matter}, \tag{16}$$

$$w = \frac{1}{3} \quad \Rightarrow \quad \rho_m \propto a^{-4} \quad \text{radiation}, \tag{17}$$

$$w = 1 \quad \Rightarrow \quad \rho_m \propto a^{-6} \quad \text{stiff fluid}. \tag{18}$$

When tables use the same reference value ρ_0 to compare radiation and a stiff fluid, the stiff columns must be understood as comparative. A physical prediction for a stiff sector requires fixing its own present-day density, denoted $\rho_{w=1,0}$.

6 Ricci scalar in strict SI units

With the sign and signature convention fixed in the main text, the Ricci scalar is related to the trace of the energy-momentum tensor. At the FLRW order-of-magnitude level, for a perfect fluid, we retain the dimensional relation

$$|R| \sim \frac{8\pi G}{c^2}\,|1-3w|\,\rho_m. \tag{19}$$

This immediately shows that, for a perfect radiation fluid in strict FLRW,

$$w = \frac{1}{3} \quad \Rightarrow \quad R = 0. \tag{20}$$

A Ricci-threshold trigger $R_\star$ therefore cannot trigger a bounce in pure standard FLRW radiation, unless one introduces an effective deviation $w_{\mathrm{eff}} \neq 1/3$, a non-radiative component, or a geometric modification specific to the HGR sector.

The equivalent critical density associated with a threshold $R_\star$ reads, in strict SI units,

$$\rho_\star(R) \sim \frac{c^2 R_\star}{8\pi G|1-3w|}. \tag{21}$$

7 Kretschmann invariant in strict SI units

The Kretschmann invariant is defined by

$$K \equiv R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}, \tag{22}$$

with dimension m^{-4}.

In FLRW, for a perfect fluid, we use the order-of-magnitude calibration

$$K \sim \frac{G^2}{c^4}\,\rho_m^2 \times \mathcal{C}_K(w), \tag{23}$$

where $\mathcal{C}_K(w)$ is a numerical coefficient depending on the equation of state and on the exact convention used.

If the detailed form

$$K \approx \frac{64\pi^2 G^2}{3c^4}\,\rho_m^2 \left(9w^2+12w+7\right), \tag{24}$$

is used, then the equivalent critical density associated with a threshold $K_\star$ is

$$\rho_\star(K) \approx \frac{c^2}{G}\sqrt{\frac{3K_\star}{64\pi^2\,(9w^2+12w+7)}}. \tag{25}$$

Any use of a formula without the factor c^{-4} implicitly corresponds to geometric units with $c=1$ and must not be mixed directly with densities expressed in $\mathrm{kg\,m^{-3}}$.

8 Conformal Weyl invariant

The conformal Weyl invariant is defined by

$$W \equiv C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma}. \tag{26}$$

It has dimension m^{-4}.

In strict FLRW,

$$C_{\mu\nu\rho\sigma} = 0, \qquad W = 0. \tag{27}$$

A trigger based on $W \to W_\star$ therefore requires at least anisotropy, inhomogeneity, shear, a domain structure, or a Weyl branch that is not strictly FLRW.

In toy models where tidal effects are parameterized by a dimensionless scalar Σ, one uses the dimensional ansatz

$$W \sim \kappa\Sigma^2 H^4. \tag{28}$$

This ansatz is useful as a scaling law, but its use in strict SI units requires an explicit dimensional calibration, because H^4 has dimension s^{-4} whereas W has dimension m^{-4}.

To avoid ambiguity in the numerical tables, the equivalent critical density should be written in the calibrated form

$$\rho_\star(W,\Sigma) = \frac{3}{8\pi G}\sqrt{\frac{W_\star}{\kappa\Sigma^2}}\,\mathcal{U}_W, \tag{29}$$

where $\mathcal{U}_W$ is the chosen dimensional conversion factor. In geometric units $c=1$, $\mathcal{U}_W = 1$. In strict SI units, $\mathcal{U}_W$ must be fixed explicitly by the convention relating H^4 to the invariant W in m^{-4}.

9 Radii, distances, and light-years

The numerical applications often use the reference scale

$$R_0 = 96\,\mathrm{Gly}. \tag{30}$$

This quantity is a physical scale associated with a reference comoving volume. It must not be interpreted as a rigid material radius of the Universe.

The conversion used is

$$1\,\mathrm{ly} = 9.460730472 \times 10^{15}\,\mathrm{m}. \tag{31}$$

Thus,

$$96\,\mathrm{Gly} = 96 \times 10^9\,\mathrm{ly} \simeq 9.082 \times 10^{26}\,\mathrm{m}. \tag{32}$$

If $x = a_b/a_0$, then

$$R_b = R_0 x. \tag{33}$$

10 Reading rule for numerical tables

In every HGR/RGH numerical table:

1. densities given in $\mathrm{kg\,m^{-3}}$ are mass densities;
2. energy densities must be given in $\mathrm{J\,m^{-3}}$;
3. terms added directly to H^2 must be given in $\mathrm{s^{-2}}$;
4. the thresholds $R_\star$, $K_\star$, and $W_\star$ must respectively carry dimensions $\mathrm{m^{-2}}$, $\mathrm{m^{-4}}$, and $\mathrm{m^{-4}}$;
5. any formula derived in a $c = 1$ convention must be converted before being inserted into a SI table;
6. stiff-fluid columns using the same normalization as radiation are comparisons of scaling exponents, not autonomous physical predictions.

11 Operational summary

The recommended convention for numerical applications is:

$$H^2 = \frac{8\pi G}{3}\rho_m, \tag{34}$$

$$u = \rho_m c^2, \tag{35}$$

$$p = w\rho_m c^2, \tag{36}$$

$$R \sim \frac{8\pi G}{c^2}\rho_m, \tag{37}$$

$$K \sim \frac{G^2}{c^4}\rho_m^2, \tag{38}$$

$$W \sim \frac{G^2}{c^4}\rho_m^2 \quad \text{outside strict FLRW or with anisotropies.} \tag{39}$$

This appendix serves as a common reference for the numerical applications. Any deliberate deviation, for instance switching to geometric units with $c = 1$, must be explicitly announced before the formulas and must not be mixed with SI values in the same table.