Appendix — Effective Repulsive Field and Minimal FLRW Reduction in the HGR Framework

Laurent Besson (original idea)
Appendix writing: ChatGPT

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Purpose of this appendix

The purpose of this appendix is to fix a minimal, coherent, and workable formulation of the *effective repulsive field* in the framework of Hypercomplex General Relativity (HGR), and then to provide its homogeneous and isotropic cosmological reduction. The central idea is *not* to introduce a fundamental ghost field nor a "negative mass" in the naive sense, but rather to represent gravitational repulsion as an *effective geometric effect*, activated in certain curvature regimes and carried by sectors already present in the theory:

- the Weyl sector ϕ_μ,
- the internal hypercomplex connection A_μ,
- the mixed symplectic sector $\Omega_{\rm mix}$,
- the crossed terms arising from the base–fiber coupling.

1 Covariant definition of the effective repulsive sector

We define the activated effective repulsive tensor by

$$\Theta^{\rm rep}_{\mu\nu} := \mathcal{A}\!\left(\frac{W}{W^{\rm eff}_\star}\right) \Theta_{\mu\nu}, \tag{1}$$

where:

- W denotes a curvature invariant acting as the activation variable;
- $W^{\rm eff}_\star$ is the effective activation threshold;
- $\mathcal{A}(z)$ is a smooth function such that $0 \le \mathcal{A}(z) \le 1$, with $\mathcal{A}(z) \to 0$ below threshold and $\mathcal{A}(z) \to 1$ above threshold;
- $\Theta_{\mu\nu}$ collects the contributions from the internal and geometric sectors.

The minimal effective gravitational equation then reads

$$G_{\mu\nu} = 8\pi G\, T^{(m)}_{\mu\nu} + \Theta^{\rm rep}_{\mu\nu}. \tag{2}$$

A more explicit expression for the threshold is

$$W^{\rm eff}_\star = \kappa_X \langle \hat{X}^2 \rangle, \tag{3}$$

where $\hat{X}$ is the quantized internal observable built from the invariants of the hypercomplex sector.

Covariant criterion for the repulsive regime

For an observer of four-velocity u^μ, we introduce the scalar

$$\Xi_{\text{rep}}(u) := -\left(\Theta^{\text{rep}}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\Theta^{\text{rep}}\right)u^\mu u^\nu, \qquad \Theta^{\text{rep}} := g^{\mu\nu}\Theta^{\text{rep}}_{\mu\nu}. \tag{4}$$

The effective repulsive regime corresponds to

$$\Xi_{\text{rep}}(u) > 0. \tag{5}$$

In other words, the geometric correction acts against the usual focusing tendency of congruences.

2 Minimal decomposition of $\Theta_{\mu\nu}$

The most natural minimal closure is

$$\Theta_{\mu\nu} = \Theta^{(\phi)}_{\mu\nu} + \Theta^{(A)}_{\mu\nu} + \Theta^{(\Omega_{\text{mix}})}_{\mu\nu} + \Theta^{(\text{cross})}_{\mu\nu}. \tag{6}$$

2.1 Weyl block

We start from the Maxwell-type term

$$S_\phi = -\frac{1}{4}\int F^{(\phi)}_{\mu\nu}F^{(\phi)\mu\nu}\sqrt{-g}\,\mathrm{d}^4x, \qquad F^{(\phi)}_{\mu\nu} = \partial_\mu\phi_\nu - \partial_\nu\phi_\mu. \tag{7}$$

Metric variation gives

$$\Theta^{(\phi)}_{\mu\nu} = F^{(\phi)}_{\mu\alpha}F^{(\phi)\alpha}{}_{\nu} - \frac{1}{4}g_{\mu\nu}F^{(\phi)}_{\alpha\beta}F^{(\phi)\alpha\beta}. \tag{8}$$

2.2 Internal hypercomplex block

The internal sector is taken in Yang–Mills form:

$$S_A = -\frac{1}{4}\int \mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})\sqrt{-g}\,\mathrm{d}^4x, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]. \tag{9}$$

Metric variation yields

$$\Theta^{(A)}_{\mu\nu} = \mathrm{Tr}(F_{\mu\alpha}F_\nu{}^{\alpha}) - \frac{1}{4}g_{\mu\nu}\mathrm{Tr}(F_{\alpha\beta}F^{\alpha\beta}). \tag{10}$$

2.3 Mixed symplectic block

For the dynamical sector of Ω, we consider the term

$$S_{\Omega,\text{dyn}} = \frac{\kappa_\Omega}{2}\int \mathrm{d}\Omega \wedge \star\mathrm{d}\Omega. \tag{11}$$

We project the observable mixed part as a 2-form

$$B_{\mu\nu} := \Pi_{\text{obs}}(\Omega_{\text{mix}})_{\mu\nu}, \qquad H_{\mu\nu\rho} := 3\nabla_{[\mu}B_{\nu\rho]}. \tag{12}$$

The associated minimal stress-energy tensor is then

$$\Theta^{(\Omega_{\text{mix}})}_{\mu\nu} = \frac{1}{2}H_{\mu\alpha\beta}H_\nu{}^{\alpha\beta} - \frac{1}{12}g_{\mu\nu}H_{\alpha\beta\gamma}H^{\alpha\beta\gamma}. \tag{13}$$

2.4 Crossed block

The crossed terms, arising from gravity–Weyl–hypercomplex–symplectic couplings, are defined by

$$\Theta^{(\mathrm{cross})}_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S_{\mathrm{coup}}}{\delta g^{\mu\nu}}, \tag{14}$$

where S_{coup} denotes the total coupling term.

In summary, one obtains the compact formula

$$\boxed{\Theta_{\mu\nu} = \Theta^{(\phi)}_{\mu\nu} + \Theta^{(A)}_{\mu\nu} + \Theta^{(\Omega_{\mathrm{mix}})}_{\mu\nu} + \Theta^{(\mathrm{cross})}_{\mu\nu}.} \tag{15}$$

3 Minimal FLRW reduction

We adopt the homogeneous and isotropic ansatz

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2\gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j, \qquad H := \frac{\dot{a}}{a}, \tag{16}$$

with

$$\phi = \phi_0(t)\,\mathrm{d}t, \qquad A_i{}^a(t) = \psi(t)\,\delta_i{}^a, \qquad A_0{}^a = 0, \tag{17}$$

and

$$\Omega_{\mathrm{ext}} = 0, \qquad \Omega_{\mathrm{int}} = \varpi(t)\sum_a \sigma_a, \qquad \Omega_{\mathrm{mix}} = \chi(t)\sum_i e^i\wedge\sigma_i. \tag{18}$$

3.1 Contribution of the Weyl block

With $\phi = \phi_0(t)\,\mathrm{d}t$, one has

$$F^{(\phi)} = \mathrm{d}(\phi_0(t)\,\mathrm{d}t) = \dot{\phi}_0\,\mathrm{d}t\wedge\mathrm{d}t = 0. \tag{19}$$

Consequently,

$$\Theta^{(\phi)}_{\mu\nu}|_{\mathrm{FLRW}} = 0. \tag{20}$$

The kinetic Weyl block therefore does not contribute directly to the strict FLRW background.

3.2 Contribution of the internal hypercomplex block

Under the isotropic triadic ansatz, the internal Yang–Mills sector behaves as an effective radiation-like fluid:

$$\rho_A(a) = \frac{C_A}{\lambda^2}\frac{Q_A^2}{a^4}, \qquad p_A(a) = \frac{1}{3}\rho_A(a). \tag{21}$$

The associated coefficient is

$$\beta_A = \frac{C_A}{\lambda^2}Q_A^2. \tag{22}$$

3.3 Contribution of the mixed symplectic block

For the mixed part, one also obtains a radiation-like scaling law:

$$\rho_\Omega(a) = \frac{C_\Omega}{\kappa}\frac{Q_\Omega^2}{a^4}, \qquad p_\Omega(a) = \frac{1}{3}\rho_\Omega(a), \tag{23}$$

with

$$\beta_\Omega = \frac{C_\Omega}{\kappa}Q_\Omega^2. \tag{24}$$

3.4 Contribution of the crossed terms

At this minimal stage, we absorb the additional cosmological couplings into an effective coefficient

$$\beta_{\text{cross}}, \tag{25}$$

so that

$$\beta = \beta_A + \beta_\Omega + \beta_{\text{cross}}. \tag{26}$$

4 Identification of the α/a^2 and β/a^4 terms

In the presently adopted minimal version, one identifies

$$\alpha = \alpha_\theta = \frac{3}{8\pi G}\,\alpha_W H_0^2, \tag{27}$$

where α_W is the late-time parameter of the θ sector.

The total effective density then takes the form

$$\rho_{\text{eff}}(a) = \frac{\alpha}{a^2} - \frac{\beta}{a^4}, \qquad \alpha > 0, \quad \beta > 0. \tag{28}$$

The associated effective pressure is

$$p_{\text{eff}}(a) = -\frac{\alpha}{3a^2} - \frac{\beta}{3a^4}. \tag{29}$$

The effective equation-of-state parameter is

$$w_{\text{eff}}(a) = \frac{p_{\text{eff}}}{\rho_{\text{eff}}} = -\frac{\alpha a^2 + \beta}{3(\alpha a^2 - \beta)}. \tag{30}$$

5 Effective cosmological equations and bounce condition

The modified Friedmann equation reads

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\left(\rho_{\text{mat}} + \frac{\alpha}{a^2} - \frac{\beta}{a^4}\right). \tag{31}$$

In the flat case $k = 0$, neglecting ordinary matter near the bounce, one obtains

$$H^2 = \frac{8\pi G}{3}\left(\frac{\alpha}{a^2} - \frac{\beta}{a^4}\right) = \frac{8\pi G}{3a^4}(\alpha a^2 - \beta). \tag{32}$$

The condition $H^2 \geq 0$ imposes

$$a^2 \geq \frac{\beta}{\alpha}, \tag{33}$$

and the minimum is reached for

$$a_{\min} = \sqrt{\frac{\beta}{\alpha}}. \tag{34}$$

The cosmological bounce then corresponds to the conditions

$$H(t_b) = 0, \qquad \dot{H}(t_b) > 0, \qquad a(t_b) = a_{\min} > 0. \tag{35}$$

Near the bounce, the acceleration condition is satisfied since

$$w_{\text{eff}} < -\frac{1}{3}. \tag{36}$$

6 Threshold-activated version

By incorporating the curvature activation mechanism, we define the activated repulsive density:

$$\rho_{\mathrm{rep}}(a) = \mathcal{A}\!\left(\frac{W}{W_\star^{\mathrm{eff}}}\right)\left(\frac{\alpha}{a^2} - \frac{\beta}{a^4}\right), \tag{37}$$

and the activated pressure:

$$p_{\mathrm{rep}}(a) = \mathcal{A}\!\left(\frac{W}{W_\star^{\mathrm{eff}}}\right)\left(-\frac{\alpha}{3a^2} - \frac{\beta}{3a^4}\right). \tag{38}$$

The activated gravitational equation then becomes

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}^{(m)} + \mathcal{A}\!\left(\frac{W}{W_\star^{\mathrm{eff}}}\right)\left(\Theta_{\mu\nu}^{(\phi)} + \Theta_{\mu\nu}^{(A)} + \Theta_{\mu\nu}^{(\Omega_{\mathrm{mix}})} + \Theta_{\mu\nu}^{(\mathrm{cross})}\right). \tag{39}$$

7 Synthetic proposal

The minimal structure may be summarized as follows:

$$\boxed{G_{\mu\nu} = 8\pi G\, T_{\mu\nu}^{(m)} + \mathcal{A}\!\left(\frac{W}{\kappa_X\langle \hat{X}^2\rangle}\right)\left[\Theta_{\mu\nu}^{(\phi)} + \Theta_{\mu\nu}^{(A)} + \Theta_{\mu\nu}^{(\Omega_{\mathrm{mix}})} + \Theta_{\mu\nu}^{(\mathrm{cross})}\right].} \tag{40}$$

At the minimal FLRW level, this structure reduces to

$$\boxed{\rho_{\mathrm{eff}}(a) = \frac{\alpha}{a^2} - \frac{\beta}{a^4}, \qquad \alpha = \frac{3}{8\pi G}\alpha_W H_0^2, \qquad \beta = \beta_A + \beta_\Omega + \beta_{\mathrm{cross}}.} \tag{41}$$

Conclusion

The previous minimal formulation provides a mathematically clean status for the *effective repulsive field* in HGR:

- it is not a fundamental ghost field;
- it is an *activated geometric source*;
- the a^{-2} term is carried by the late-time θ sector;
- the a^{-4} term arises from the internal hypercomplex sector, the mixed symplectic sector, and possible couplings absorbed into β;
- repulsion is interpreted as an effective *anti-focusing* effect, compatible with a controlled cosmological bounce.