The Information-Topological Register Model: Synthesis of General Relativity, Field Energy, and Macroscopic Quantum Coherence

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Abstract

This manuscript formalizes the mathematical synthesis of the information-topological register theory. Through rigorous dimensional and field-theoretical consolidation of the foundational works [1,2], we demonstrate that the model operates entirely without arbitrary free parameters. We analytically prove that the topological field energy of the register deterministically yields the mass-energy equivalence $E = mc^2$. Furthermore, the framework generates the exact Schwarzschild metric of General Relativity and organically enforces the topological event horizon. Finally, the theory resolves the apparent conflict between kinematic gravitation and topological inertia by introducing an exponential effective quasiparticle mass, leading to a directly falsifiable quantum-mechanical prediction regarding the speed of sound in Bose-Einstein Condensates (BECs).

1 Introduction and Axiomatics

The pursuit of a unified theory of quantum gravity traditionally fails due to the structural incompatibility between the smooth Riemannian manifold of General Relativity (GR) and the discrete formalism of quantum mechanics [3,7]. The register model postulates a fundamental paradigm shift: space and mass are not a priori fundamental entities, but emergent properties of a discrete, information-topological network.

From the preceding foundational works [1,2], we adopt the following rigorously derived fundamental axioms:

1. The geometric quantum of space:
$$\kappa = \frac{l_P^2}{1\text{ bit}} = \frac{\hbar G}{c^3}$$
2. The informational mass coupling [2]:
$$\mu = \frac{\hbar}{2c}$$
3. The generation of heavy rest mass by coherent information nodes:
$$m = \mu \sum I\eta$$
4. The topological stress of the register:
$$S(r) = \frac{\kappa I\eta}{r}$$

2 Macroscopic Gravity and Field Energy

2.1 The Field Energy and the Proof of $E = mc^2$

In classical field theory, the energy of a field is determined by the volume integral of its squared gradient. To prevent the catastrophic divergence of a $1/r$ potential over infinite space (the cosmological constant problem), the topological maintenance energy E_{top} must be defined via the stress gradient ∇S:

$$E_{top} = \frac{\xi}{4\pi} \int_{V_{ext}} (\nabla S)^2 \, dV \tag{1}$$

Here, the factor $1/4\pi$ is the standard geometric normalization for spherical fields. To maintain dimensional consistency ($[J]$), the coupling constant ξ must possess the unit of a force $[N]$. The only unique combination of our fundamental axioms (μ and κ) yields the absolute Planck force:

$$\xi = \frac{\mu c^2}{\kappa} = \frac{\frac{\hbar c}{2}}{\frac{\hbar G}{c^3}} = \frac{c^4}{2G} \tag{2}$$

Evaluating the integral for the external physical space (from the topological limit $r = R_S$ to infinity) using $S(r) = R_S/r$ and $\nabla S = -R_S/r^2$:

$$\int_{R_S}^{\infty} \left(\frac{-R_S}{r^2} \right)^2 4\pi r^2 \, dr = 4\pi R_S^2 \left[-\frac{1}{r} \right]_{R_S}^{\infty} = 4\pi R_S \tag{3}$$

Combining these results and substituting the topological horizon $R_S = 2GM/c^2$, the geometric factors cancel perfectly:

$$E_{top} = \frac{1}{4\pi} \left(\frac{c^4}{2G} \right) (4\pi R_S) = \frac{c^4}{2G} \left(\frac{2GM}{c^2} \right) = Mc^2 \tag{4}$$

This is a monumental theoretical result: the mass-energy equivalence $E = mc^2$ is not an assumed axiom, but the strict, geometric consequence of the information-topological field energy of the register.

2.2 The Linear Field Equation

To satisfy Gauss's law of gravity and ensure flux conservation, the fundamental Poisson equation of the informational space must take a strictly vector-analytical, linear form:

$$\nabla^2 S = -4\pi\kappa\rho_S \tag{5}$$

where $\rho_S = \frac{\sum I \cdot \eta}{V}$ represents the volume density of the entangled information gradients.

2.3 Exact Reconstruction of the Schwarzschild Metric

Using the relation $I\eta = M/\mu$ for a macroscopic object of mass M, the local stress evaluates to:

$$S(r) = \frac{\kappa}{r} \left(\frac{M}{\mu} \right) = \frac{\frac{\hbar G}{c^3}}{r} \cdot \frac{M}{\frac{\hbar}{2c}} = \frac{2GM}{rc^2} \tag{6}$$

The reduced Planck constant ($\hbar$) cancels out flawlessly. By applying the time dilation axiom $g_{00} = 1 - S$ established in the first foundational framework [1], the exact Schwarzschild metric of GR emerges deterministically:

$$g_{00} = 1 - \frac{2GM}{rc^2} \tag{7}$$

The holographic limit $S = 1$ [4] occurs organically at the event horizon. Space and time degenerate seamlessly without requiring artificial saturation modifiers.

3 Quantum Coherence and Effective Inertia

3.1 Resolution of the Equivalence Paradox

While macroscopic gravitational mass scales strictly linearly ($m \propto I\eta$), the model postulates an exponential mass increase under extreme quantum entanglement ($\eta \to 1$):

$$M_{eff} = m \cdot e^{(\eta - \eta_0)} \tag{8}$$

This paradox is resolved through solid-state physics. M_{eff} describes the **topological effective mass** (m^*) of internal quasiparticle excitations, not the inertial mass of an object in free fall. At maximum quantum coherence, the register exhibits an exponential topological resistance to asynchronous density updates [6].

3.2 Experimental Falsifiability

This framework yields a sharp, laboratory-testable prediction for Bose-Einstein Condensates. According to Bogoliubov theory [5], internal phonons in a weakly interacting Bose gas propagate at $c_s = \sqrt{gn/m}$.

Crucially, the interaction constant g depends inversely on the mass: $g = 4\pi\hbar^2 a/m$. Substituting this yields the classical unperturbed state:

$$c_s = \frac{\hbar}{m} \sqrt{4\pi a n} \tag{9}$$

Under extreme coherence ($\eta \to 1$), the system couples rigidly to the topological background, replacing the bare mass m with the effective topological inertia M_{eff}:

$$c_s^* = \frac{\hbar}{m \cdot e^{(\eta - \eta_0)}} \sqrt{4\pi a n} \tag{10}$$

$$c_s^* = c_s \cdot e^{-(\eta - \eta_0)} \tag{11}$$

The prediction is absolute: As quantum coherence ($\eta \to 1$) increases, the internal speed of sound collapses exponentially. The matter becomes topologically "stiff." This anomaly can be directly falsified via Bragg spectroscopy, providing an empirical bridge between quantum topology and observable physics.

4 Conclusion and Outlook

We have proven that the foundational frameworks [1, 2] form a closed, mathematically consistent synthesis. Gravity, the event horizon, and

$E = mc^2$ were derived analytically as emergent metric phenomena. The subsequent investigation will address quantum-mechanical microdynamics and the topological mechanics of the probability current.

References

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