A Parameter-Free Einstein–Hilbert Coefficient from a Sink-Current Endurance Ledger

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Abstract

We derive the Einstein–Hilbert action coefficient parameter-free from the Quantum Traction Theory (QTT) endurance ledger. The result has two layers. The QTT-native theorem is the sink-current derivation: axioms A1/A2/A6/A7 fix the space-quantum volume $V_{\rm SQ} = 4\pi\tilde{\ell}^3$, tick $\tilde{t} = \tilde{\ell}/c$ and mass quantum $\tilde{m} = \hbar/(c\tilde{\ell})$, and force the no-knob constitutive projection $\mathbf{g}_{\rm N} = (c/\tilde{\ell})J_{\rm end}$ from endurance flux to acceleration. Combined with the A2 sink continuity $\nabla\cdot J_{\rm end} = -(V_{\rm SQ}/\tilde{m}\tilde{t})\rho$ this gives $\nabla\cdot \mathbf{g}_{\rm N} = -4\pi G\rho$ with $G = \tilde{\ell}^2c^3/\hbar$. The continuum-geometric layer is stated separately: in a coarse-grained, manifoldlike, distinguishing limit, A1 causal order determines the conformal Lorentzian metric and the QTT four-volume count fixes its conformal factor; locality, metricity, capacity regularity and second-order infrared truncation then allow Lovelock uniqueness to select the Einstein tensor plus a metric term. Matching to the QTT Newton–Poisson limit fixes the bulk coefficient

$$\lambda = \frac{c^3}{16\pi G} = \frac{\hbar}{16\pi\tilde{\ell}^2},$$

so that the local Einstein-gauge action is $S_{\rm EH} = \lambda\int d^4x\sqrt{-g}\,R$. We do not claim global equivalence to vanilla general relativity: QTT retains an absolute background clock, an integrable-Weyl lapse, and a creation sector outside the local Einstein gauge. The paper separates QTT-native content from imported continuum mathematics throughout, and states five falsifiers tied to a one-ruler consistency requirement on $\tilde{\ell}$.

1. Introduction

The Einstein–Hilbert action

$$S_{\rm EH} = \frac{c^3}{16\pi G}\int_{\mathcal{M}} d^4x\sqrt{-g}\,R \tag{1}$$

is the compact variational expression behind the Einstein field equations. In standard general relativity one begins with a Lorentzian metric $g_{\mu\nu}$, assumes local covariance, takes R as the simplest scalar curvature, and fixes the coefficient by the Newtonian limit. That route is mathematically clean. It does not, by itself, explain why the gravitational coupling has its observed value.

Quantum Traction Theory (QTT) starts from a different ledger. Its primitive gravitational object is not a curvature scalar. It is an endurance accounting rule: rest mass must be renewed by a per-tick consumption of space quanta. In QTT's notation the primitive micro-units are

$$\tilde{t} = \frac{\tilde{\ell}}{c}, \qquad \tilde{m} = \frac{\hbar}{c\tilde{\ell}}, \qquad V_{\rm SQ} = 4\pi\tilde{\ell}^3, \qquad V_4 = 4\pi\tilde{\ell}^4. \tag{2}$$

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The QTT framework manuscript already states the A2 sink law, the constitutive projection $\mathbf{g}_{\mathrm{N}} = (c/\tilde{\ell})J_{\mathrm{end}}$, and the resulting $G = \tilde{\ell}^2 c^3/\hbar$ [1]. The present paper strengthens that derivation in three ways.

First, it isolates a *QTT-native constitutive projection lemma*: if the endurance current is a speed-like local flux, if local kinematics responds linearly and isotropically, if no new dimensionless knob is allowed, and if a capacity-saturated current maps to the A1/A6 maximum one-tick acceleration, then the coefficient $c/\tilde{\ell} = 1/\tilde{t}$ is forced. The map is therefore not adjusted after seeing Newton's law; it is the canonical A2 projection from flux units to acceleration units.

Second, it does not pretend that a Newtonian vector field alone produces a metric. The metric bridge is stated with its exact status. QTT natively supplies causal order and four-volume counting; standard causal reconstruction supplies the conformal metric; the volume measure fixes the conformal factor. This is a continuum bridge, not a theorem about arbitrary discrete sets.

Third, it treats the Einstein–Hilbert action as an infrared result. A6 capacity regularity does not require every higher-curvature term to vanish. It requires the low-curvature expansion to be organized in powers of $\tilde{\ell}^2 R$, with R as the leading nontrivial covariant scalar in the local Einstein gauge.

The resulting claim is sharp:

> QTT natively fixes the Newtonian coupling; Einstein–Hilbert is the unique leading local covariant continuum completion in the Einstein gauge.

This is stronger than a consistency check, but weaker than the overclaim that QTT derives global GR without importing continuum-geometric mathematics.

Conventions. We use the metric signature $(-,+,+,+)$, Riemann sign convention of Wald [13], and $x^0 = ct$ so that d^4x has dimensions L^4. Indices run $\mu,\nu \in \{0,1,2,3\}$. Throughout, $\tilde{\ell}$ denotes the QTT primitive micro-length identified with the Planck length only after the gravitational coupling is derived; until that identification, $\tilde{\ell}$ is a free QTT scale.

2. Scope and theorem statement

The paper uses five registers.

Register	Content
QTT-native theorem	The A2 sink law plus A1/A6/A7 capacity units and the no-knob constitutive projection fix the Newton–Poisson coefficient $G = \tilde{\ell}^2 c^3/\hbar$.
QTT-native normalization	The 4π in $V_{\mathrm{SQ}} = 4\pi\tilde{\ell}^3$ is the isotropic Gauss capacity normalizer; the four-cell is $V_4 = 4\pi\tilde{\ell}^4$.
Standard mathematical bridge	Causal reconstruction of conformal geometry, volume-fixing of the conformal factor, variation of the Einstein–Hilbert action, and Lovelock uniqueness.
Continuum regularity bridge	A coarse-grained manifoldlike limit with distinguishing causal order, smooth volume measure and curvature below the A6 capacity scale.
Not claimed	Global equivalence to vanilla GR, derivation of the ABC clock away, exact absence of higher-curvature corrections, or an independent Regge proof of GR.

The paper's main theorem is conditional but parameter-free:

Theorem 1 (QTT Einstein-gauge infrared action). *Assume the QTT A1/A2/A6/A7 endurance ledger, the no-knob constitutive projection lemma below, and a regular coarse-grained Einstein-gauge continuum in which the metric is reconstructed from causal order plus volume. If the leading infrared metric equations are local, covariant, stress-energy conserving and second order, then the bulk gravitational action is*

$$S_{\mathrm{EH}} = \frac{\hbar}{16\pi\tilde{\ell}^2}\int \mathrm{d}^4x\,\sqrt{-g}\,R = \frac{c^3}{16\pi G}\int \mathrm{d}^4x\,\sqrt{-g}\,R,$$

with $G = \tilde{\ell}^2 c^3/\hbar$.

The QTT part of the theorem is the coefficient and the Newtonian sink law. The metric and Lovelock steps are standard continuum mathematics applied to QTT's coarse-grained data.

3. The native QTT endurance ledger

QTT A2 defines a space-quantum sink current $J_{\mathrm{end}}(r;T)$. For mass density $\rho(r,T)$,

$$\nabla\cdot J_{\mathrm{end}}(r;T) = -\frac{V_{\mathrm{SQ}}}{\tilde{m}\,\tilde{t}}\rho(r,T). \tag{3}$$

The minus sign says that mass is a sink of space-quantum renewal. Integrating over a closed surface S,

$$\oint_S J_{\mathrm{end}}\cdot \mathrm{d}A = -\frac{V_{\mathrm{SQ}}}{\tilde{m}\,\tilde{t}}M_{\mathrm{enc}}. \tag{4}$$

For a steady, isotropic point source,

$$J_{\mathrm{end}}(r) = -\frac{1}{4\pi r^2}\frac{M}{\tilde{m}}\frac{V_{\mathrm{SQ}}}{\tilde{t}}\,\hat{r}. \tag{5}$$

The dimensions are important. Since $[V_{\mathrm{SQ}}\rho/(\tilde{m}\tilde{t})] = T^{-1}$, the current has dimensions

$$[J_{\mathrm{end}}] = L/T.$$

It is a speed-like flux of space-quantum renewal, not yet a Newtonian acceleration. The remaining step is the constitutive projection from this speed-like flux to the laboratory acceleration field.

4. The no-knob constitutive projection lemma

Earlier presentations wrote

$$\mathbf{g}_{\mathrm{N}} = \frac{c}{\tilde{\ell}}J_{\mathrm{end}}$$

and then derived Newton's law. That is correct, but it leaves a natural question: why the coefficient $c/\tilde{\ell}$?

The answer is not that any coefficient would do. The answer is that, inside QTT, a local linear projection from a speed-like current to acceleration has only one parameter-free normalization.

Lemma 1 (No-knob endurance-to-acceleration projection). *Let J_{end} be the A2 endurance current with dimensions L/T. Assume:*

(C1) locality and isotropy in the Einstein-gauge laboratory sector;

(C2) linear response at weak field, $\mathbf{g}_{\mathrm{N}} = \lambda J_{\mathrm{end}}$;

(C3) no additional length, time, mass or dimensionless coupling beyond the A1/A6/A7 capacity units;

(C4) A1/A6 saturation: a capacity-saturated local flux has $|J_{\mathrm{end}}| = c$, and its largest one-tick kinematic response is the A1 acceleration ceiling

$$a_W = \frac{c}{\tilde{t}} = \frac{c^2}{\tilde{\ell}}.$$

Then

$$\lambda = \frac{1}{\tilde{t}} = \frac{c}{\tilde{\ell}}, \qquad \mathbf{g}_{\mathrm{N}} = \frac{c}{\tilde{\ell}} J_{\mathrm{end}}.$$

Proof. By dimensional analysis, $[\lambda] = T^{-1}$. The primitive QTT clock supplies the only parameter-free local inverse time,

$$\tilde{t}^{-1} = \frac{c}{\tilde{\ell}}.$$

Thus $\lambda = \chi/\tilde{t}$ for some dimensionless χ. Condition (C3) forbids an unexplained dimensionless knob. Equivalently, condition (C4) fixes the remaining normalization: at saturation $|J_{\mathrm{end}}| = c$, the response must be $a_W = c/\tilde{t}$. Therefore

$$a_W = \lambda c \quad \Rightarrow \quad \lambda = \frac{1}{\tilde{t}} = \frac{c}{\tilde{\ell}}.$$

□

Remark on saturation. Condition (C4) is the QTT-internal definition of capacity saturation, fixed by axioms A1/A6/A7: the carrier-speed ceiling c is A1, the one-tick kinematic ceiling $c/\tilde{t}$ is the A1 acceleration limit at A6 capacity, and the no-knob structure between them is A7's modular bundle closure. The lemma's force is therefore conditional on accepting this saturation structure as physically meaningful. It is not externally imposed. The lemma's content is that, *once* A2 is read as a local kinematic response law, the parameter-free coefficient is forced; the alternative is to abandon A1/A6/A7 saturation, which the framework cannot do without losing its capacity scales.

This lemma is the strongest QTT-native answer to the "ad hoc constitutive map" objection. The *existence* of a local flux-to-kinematics projection is part of A2's meaning: A2 is a law of endurance dynamics, not merely a bookkeeping continuity equation. But once that projection is admitted, its coefficient is not tunable. If one artificially allowed a dimensionless χ, the result would be

$$G_\chi = \chi \frac{\tilde{\ell}^2 c^3}{\hbar}.$$

QTT's A6/A7 no-knob saturation is precisely the statement $\chi = 1$.

5. Native derivation of Newton–Poisson and G

With Lemma 1,

$$\mathbf{g}_{\mathrm{N}}(r;T) = \frac{c}{\tilde{\ell}} J_{\mathrm{end}}(r;T). \tag{6}$$

Taking the divergence and using (3),

$$\nabla\cdot \mathbf{g}_{\mathrm{N}} = \frac{c}{\tilde{\ell}} \nabla\cdot J_{\mathrm{end}} = -\frac{c}{\tilde{\ell}} \frac{V_{\mathrm{SQ}}}{\tilde{m}\tilde{t}} \rho = -\frac{V_{\mathrm{SQ}}}{\tilde{m}\tilde{t}^2} \rho. \tag{7}$$

Newton–Poisson requires

$$\nabla\cdot \mathbf{g}_{\mathrm{N}} = -4\pi G \rho.$$

Therefore

$$G = \frac{V_{\mathrm{SQ}}}{4\pi \tilde{m}\tilde{t}^2}. \tag{8}$$

Substitute the QTT capacity units (2):

$$G = \frac{4\pi\tilde{\ell}^3}{4\pi\left(\frac{\hbar}{c\tilde{\ell}}\right)\left(\frac{\tilde{\ell}^2}{c^2}\right)} = \frac{\tilde{\ell}^2 c^3}{\hbar}. \tag{9}$$

For a point mass, (5) and (6) give

$$\mathbf{g}_{\mathrm{N}}(r) = -\frac{GM}{r^2}\hat{r}. \tag{10}$$

In the curl-free static sector $\mathbf{g}_{\mathrm{N}} = -\nabla\Phi$, this is

$$\nabla^2\Phi = 4\pi G\rho. \tag{11}$$

This is the QTT-native gravitational coupling theorem:

$$\boxed{\text{A2 sink} + \text{A1/A6/A7 capacity units} + \text{no-knob projection} \Rightarrow G = \frac{\tilde{\ell}^2 c^3}{\hbar}.}$$

Only after this step may one identify $\tilde{\ell}$ with the Planck length:

$$\tilde{\ell} = \sqrt{\frac{\hbar G}{c^3}}.$$

In this order, Planck units are consequences of the endurance normalization, not inputs.

6. Metric reconstruction: what QTT supplies and what the continuum bridge supplies

A Newtonian acceleration field does not itself force a Lorentzian metric. The present paper therefore states the metric bridge as a separate, auditable step.

6.1. QTT-native continuum data

QTT supplies two pieces of geometric data before any Einstein–Hilbert action is written.

First, A1 gives a causal carrier:

$$c = \frac{\tilde{\ell}}{\tilde{t}}, \qquad \mathrm{d}\tau = N(x,v)\,\mathrm{d}T, \qquad N(x,v) \simeq e^{\phi(x)}\sqrt{1-\frac{v^2}{c^2}}. \tag{12}$$

Null propagation is lapse-invariant in the local lab sector. Thus QTT supplies a causal order: which events can be connected by luminal or timelike carrier histories.

Second, A5/A6/A7 supply volume capacity. A world-cell carries the four-volume quantum

$$V_4 = 4\pi\tilde{\ell}^4,$$

and a compact region Ω has a coarse-grained volume count

$$\int_\Omega \mathrm{d}^4x\,\sqrt{-g} = N_\Omega\, 4\pi\tilde{\ell}^4, \qquad N_\Omega \in \mathbb{Z} \tag{13}$$

up to boundary/coarse-graining error.

6.2. Continuum reconstruction bridge

The standard Hawking–King–McCarthy and Malament reconstruction results state, under appropriate distinguishing/causality hypotheses in continuum spacetime, that causal order determines the conformal class of the Lorentzian metric [6, 7]. Volume information then fixes the conformal factor, a logic also familiar from causal-set reconstruction [8, 9].

QTT uses that mathematical bridge as follows:

$$\text{A1 causal order} \Rightarrow [g_{\mu\nu}], \qquad \text{A5/A6/A7 volume count} \Rightarrow g_{\mu\nu}.$$

This is not a claim that every discrete QTT address set automatically becomes a smooth manifold. It is a controlled low-energy assumption.

Bridge hypothesis 1 (Manifoldlike QTT coarse-graining)**.** *In the infrared sector considered in this paper, the QTT address ledger admits a regular coarse-graining to a four-dimensional distinguishing Lorentzian manifold with smooth volume measure and curvature radii much larger than $\tilde{\ell}$.*

This assumption is the exact place where the causal-set-style regularity problem lives. QTT natively gives causal order and volume quanta; the existence and uniqueness of a smooth continuum approximation is a bridge condition, not yet a fully proven theorem for arbitrary microstates.

7. Einstein gauge, ABC time and the integrable-Weyl scalar

QTT has an absolute background clock T. General relativity, in its standard form, has no preferred global clock. The correct statement is therefore local:

$$\boxed{\text{QTT recovers the Einstein–Hilbert sector in the local Einstein gauge;}}$$

it does not erase the ABC clock globally.

The QTT lapse is represented by an integrable-Weyl scalar ϕ:

$$g^{(\phi)}_{\mu\nu} = e^{2\phi} g_{\mu\nu}.$$

The Einstein-gauge local sector is

$$\phi = 0, \qquad C_0(T) = 0, \tag{14}$$

where $C_0(T)$ is the homogeneous creation source. In this sector local rods and clocks see the usual Lorentzian metric and local covariance. In cosmology or creation-dominated settings, ϕ and $C_0(T)$ cannot generally be gauged away without losing QTT's physical time bookkeeping.

Thus the recovery is:

$$\text{QTT continuum} \longrightarrow \text{Einstein-gauge local metric sector} \longrightarrow \text{Einstein–Hilbert infrared action.}$$

It is not:

$$\text{QTT} \longrightarrow \text{global vanilla GR with no preferred clock.}$$

8. Why the leading scalar is R

A6 capacity regularity does not need to prove that all higher-curvature terms vanish. It needs to organize the infrared expansion. With $x^0 = ct$, d^4x has dimension L^4. The most general local metric action compatible with the QTT length $\tilde{\ell}$ has the EFT form

$$S_g = \hbar \int \mathrm{d}^4x \sqrt{-g} \left[a_0 \tilde{\ell}^{-4} + a_1 \tilde{\ell}^{-2} R + a_2 R^2 + a_3 \tilde{\ell}^2 R^3 + \cdots \right], \tag{15}$$

where the a_k are dimensionless. The a_0 term is the homogeneous creation/vacuum sector. The R term is the leading nontrivial curvature term. The higher terms are suppressed when

$$\tilde{\ell}^2|R| \ll 1.$$

The second-order statement must be formulated precisely. Lovelock's theorem says that, in four dimensions, the only local, symmetric, divergence-free metric field tensor with second-order equations is a linear combination of the Einstein tensor and the metric tensor; the Gauss–Bonnet density is topological in four dimensions and does not change the local bulk equations [5, 13]. Therefore, in the A6 infrared sector, the leading local covariant field equation must be

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa_g T_{\mu\nu}, \tag{16}$$

where κ_g is fixed by the QTT Newtonian limit.

This is not circular. Lovelock supplies the unique covariant tensor once metricity, second-order infrared dynamics and stress-energy conservation are admitted. QTT supplies the value of κ_g.

9. Fixing the Einstein–Hilbert coefficient

Let

$$S_g = \lambda \int \mathrm{d}^4x \sqrt{-g}\, R. \tag{17}$$

For a well-posed boundary-value problem one adds the Gibbons–Hawking–York term [11, 12]. For compact-support variations, or after adding that boundary term,

$$\delta \int \mathrm{d}^4x \sqrt{-g}\, R = \int \mathrm{d}^4x \sqrt{-g}\, G_{\mu\nu}\delta g^{\mu\nu}. \tag{18}$$

Use the matter convention

$$S_m = \frac{1}{c} \int \mathrm{d}^4x \sqrt{-g}\, \mathcal{L}_m, \tag{19}$$

with

$$\delta S_m = -\frac{1}{2c} \int \mathrm{d}^4x \sqrt{-g}\, T_{\mu\nu}\delta g^{\mu\nu}. \tag{20}$$

Then $\delta(S_g + S_m) = 0$ gives

$$G_{\mu\nu} = \frac{1}{2c\lambda} T_{\mu\nu}. \tag{21}$$

The QTT endurance ledger has already fixed the Newtonian limit:

$$\nabla^2\Phi = 4\pi G\rho, \qquad G = \frac{\tilde{\ell}^2 c^3}{\hbar}.$$

The covariant field equation must therefore reduce to

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

in the usual weak, static, slow-motion limit. Hence

$$\frac{1}{2c\lambda} = \frac{8\pi G}{c^4}, \qquad \lambda = \frac{c^3}{16\pi G}. \tag{22}$$

Substituting the QTT endurance value of G,

$$\lambda = \frac{c^3}{16\pi(\tilde{\ell}^2 c^3/\hbar)} = \frac{\hbar}{16\pi\tilde{\ell}^2}. \tag{23}$$

Therefore

$$\boxed{S_{\mathrm{EH}} = \frac{\hbar}{16\pi\tilde{\ell}^2} \int \mathrm{d}^4x \sqrt{-g}\, R = \frac{c^3}{16\pi G} \int \mathrm{d}^4x \sqrt{-g}\, R.} \tag{24}$$

The coefficient has dimensions of action per area. QTT reads it directly as

$$\frac{\hbar}{16\pi\tilde{\ell}^2}: \quad \text{one action quantum distributed over the covariant curvature-area unit.}$$

10. Full QTT continuum and Einstein-gauge limit

The local Einstein–Hilbert term is not the whole QTT continuum. QTT's two-clock structure retains ϕ, and A3 contributes creation. The natural low-energy QTT gravitational action has the form

$$S_G^{\mathrm{QTT}} = \frac{c^3}{16\pi G}\int \mathrm{d}^4x\,\sqrt{-g}\,\big[R(g) - 6(\nabla\phi)^2 - 2\Lambda_{\mathrm{eff}}\big] + S_{\mathrm{matter}}. \tag{25}$$

The coefficient $-6(\nabla\phi)^2$ is the standard four-dimensional conformal-frame kinetic term that arises from the Weyl rescaling $g^{(\phi)}_{\mu\nu} = e^{2\phi}g_{\mu\nu}$ of the curvature scalar [13, 14]: $\sqrt{-g^{(\phi)}}\,R^{(\phi)} = e^{2\phi}\sqrt{-g}\big[R - 6(\nabla\phi)^2 - 6\,\Box\phi\big]$, where the total-derivative $\Box\phi$ term integrates to a boundary contribution. In the Newtonian creation sector,

$$\nabla^2\Phi = 4\pi G\rho - \Lambda_{\mathrm{eff}}c^2, \qquad \Lambda_{\mathrm{eff}}c^2 = \frac{c}{\tilde{\ell}}C_0(T). \tag{26}$$

The Einstein-gauge local sector is obtained by setting

$$\phi = 0, \qquad \Lambda_{\mathrm{eff}} = 0,$$

which reduces (25) to (24) plus matter. The full QTT theory therefore contains a discriminating cosmological layer absent from vanilla local GR: the ABC clock, integrable-Weyl lapse and creation flow may reappear in homogeneous or large-scale settings.

11. Regge/capacity cell: normalization check, not independent proof

Regge calculus represents curvature on a simplicial complex through hinge areas and deficit angles:

$$S_{\mathrm{Regge}} = \frac{c^3}{8\pi G}\sum_h A_h\epsilon_h.$$

Because Regge calculus is constructed to reproduce the Einstein–Hilbert action in the continuum limit [10], it cannot be used as an independent proof that QTT must become GR. It can, however, check the QTT normalization.

If one capacity curvature blip carries

$$\Delta S_G = \hbar,$$

then using $\lambda = \hbar/(16\pi\tilde{\ell}^2)$ gives

$$\Delta\left(\int \mathrm{d}^4x\,\sqrt{-g}\,R\right) = \frac{\Delta S_G}{\lambda} = 16\pi\tilde{\ell}^2. \tag{27}$$

Over the four-cell $V_4 = 4\pi\tilde{\ell}^4$, this corresponds to an average cell curvature

$$\bar{R}_{\mathrm{cell}} = \frac{16\pi\tilde{\ell}^2}{4\pi\tilde{\ell}^4} = \frac{4}{\tilde{\ell}^2}. \tag{28}$$

This confirms mutual consistency between the QTT four-cell normalization and the Einstein–Hilbert coefficient. It is not independent evidence, and the paper does not treat it as such.

12. What is now totally QTT-native

This paper separates the strongest axiom-loyal results from the continuum bridge.

Statement	Status
$V_{\mathrm{SQ}} = 4\pi\tilde{\ell}^3,\ V_4 = 4\pi\tilde{\ell}^4$	QTT capacity geometry and isotropic Gauss normalization.
$\tilde{t} = \tilde{\ell}/c,\ \tilde{m} = \hbar/(c\tilde{\ell})$	QTT A1/A6 primitive unit ledger.

Statement	Status
$\nabla\cdot J_{\rm end} = -(V_{\rm SQ}/\tilde{m}\tilde{t})\rho$	QTT A2 sink continuity law.
$\mathbf{g}_{\rm N} = (c/\tilde{\ell})J_{\rm end}$	QTT-native no-knob constitutive projection from speed-like flux to acceleration (Lemma 1).
$G = V_{\rm SQ}/(4\pi\tilde{m}\tilde{t}^2) = \tilde{\ell}^2c^3/\hbar$	QTT-native consequence of A2 plus the constitutive projection.
$E_{\rm cap} = \hbar c/\tilde{\ell},\ p_{\rm cap} = \hbar/\tilde{\ell},\ m_{\rm cap} = \hbar/(c\tilde{\ell})$	QTT A6 capacity scales; the same $\tilde{\ell}$ must govern UV capacity tests.
Metric $g_{\mu\nu}$ from causal order plus volume	Standard causal/volume reconstruction applied to QTT data; conditional on manifoldlike coarse-graining.
$G_{\mu\nu} + \Lambda g_{\mu\nu}$ as leading second-order field tensor	Lovelock theorem plus the A6 infrared truncation; imported mathematics.
$\lambda = c^3/(16\pi G) = \hbar/(16\pi\tilde{\ell}^2)$	QTT coefficient after matching the Lovelock-allowed covariant equation to the QTT-native Newtonian limit.
Global equivalence to vanilla GR	Not claimed. QTT retains ABC time, ϕ and creation outside the local Einstein gauge.

The important upgrade is the fourth row. The earlier version treated $\mathbf{g}_{\rm N} = (c/\tilde{\ell})J_{\rm end}$ as a declared constitutive map. The constitutive projection lemma shows that, once A2 is understood as a local kinematic response law, $c/\tilde{\ell}$ is the unique parameter-free normalization compatible with A1/A6/A7 saturation.

13. Cross-domain one-ruler audit

The gravitational coefficient is now no longer a separate parameter in QTT:

$$G = \frac{\tilde{\ell}^2c^3}{\hbar}.$$

This turns $\tilde{\ell}$ into a cross-domain ruler. The same length must appear in every A6/A7 capacity prediction. This is important because it prevents the Einstein–Hilbert recovery from being an isolated dimensional trick.

Examples include:

$$E_{\rm cap} = \frac{\hbar c}{\tilde{\ell}}, \qquad p_{\rm cap} = \frac{\hbar}{\tilde{\ell}}, \qquad m_{\rm cap} = \frac{\hbar}{c\tilde{\ell}}, \tag{29}$$

$$U_{\rm self} \le \frac{n^2\alpha\hbar c}{2\tilde{\ell}}, \qquad V_4 = 4\pi\tilde{\ell}^4. \tag{30}$$

The QTT framework manuscript also formulates parameter-free checks such as the ballistic noise–heat identity

$$(\partial_T S_V(0))\left(\frac{\kappa}{T}\right) = \frac{4\pi^2}{3}\frac{k_B^3}{e^2}, \tag{31}$$

and a single-kernel, no-retune QED precision test after fixing α once from the electron anomaly [1, §25].

These are not used to derive (24). They are the independent audit demanded by a serious foundational theory:

> The same $\tilde{\ell}$ that fixes G must also control UV capacity, QED self-energy bounds and transport endpoints.

Failure of this one-ruler consistency would disfavor QTT even if the local Einstein-gauge recovery remained algebraically correct.

14. Falsifiability and empirical scope

Because (24) is the same leading action as GR, the local low-energy sector inherits the standard successful tests of GR: gravitational redshift, light bending, Shapiro delay, perihelion precession, gravitational waves with speed c, and the Newtonian inverse-square law. These are recovery checks, not new predictions.

The discriminating QTT content lies in five directions:

1. **Constitutive normalization.** Any empirical or theoretical necessity for $\mathbf{g}_{\mathrm{N}} = \chi(c/\tilde{\ell})J_{\mathrm{end}}$ with $\chi \neq 1$ would falsify the no-knob A2 projection.
2. **One-ruler consistency.** The $\tilde{\ell}$ inferred from G must also govern A6 capacity endpoints: UV self-energy bounds, capacity-Hadamard regularity and transport limits.
3. **Higher-curvature suppression.** Local tests should see no unsuppressed deviations from GR when $\tilde{\ell}^2|R| \ll 1$. Deviations, if present, must scale as powers of $\tilde{\ell}^2 R$ or with the QTT lapse/creation sector.
4. **Einstein-gauge locality.** In local laboratory regimes where $\phi \approx 0$ and $C_0 \approx 0$, QTT must reproduce the Einstein–Hilbert predictions already tested to high precision.
5. **Cosmological non-equivalence.** In homogeneous cosmology, where ABC time, F_{drift}, ϕ and creation cannot all be removed, QTT predicts possible departures from vanilla GR/ΛCDM interpretations [2].

A sharp negative result would be a clean local-gravity experiment requiring an additional dimensionless gravitational normalization not reducible to the A2 saturation factor, or a cross-domain capacity measurement showing that the $\tilde{\ell}$ controlling gravity is not the $\tilde{\ell}$ controlling the A6 endpoint.

15. Discussion

Comparison with related programs. Several quantum-gravity programs recover Newton's constant parameter-free from a microstructure: causal-set theory derives the Einstein–Hilbert action from counting causal arrangements [9, 8]; loop quantum gravity recovers G from spin-network coherent-state limits; Verlinde's entropic gravity expresses G via a Bekenstein-area accounting on a holographic screen. What distinguishes the present derivation is the route: a sink-current continuity law (A2) plus a no-knob constitutive projection from speed-like flux to acceleration, with the coefficient $c/\tilde{\ell}$ forced by capacity saturation. The endurance ledger is non-geometric in origin — it is a per-tick consumption rule on space quanta — and only acquires geometric content through the causal + volume reconstruction bridge of Section 6. Whether this is a deeper microstructure than causal sets or an equivalent reformulation is a question this paper does not settle; it is for empirical and structural comparison elsewhere.

Strongest honest conclusion.

The gravitational coefficient is QTT-native; the Einstein–Hilbert form is the standard leading metric continuum completion.

This is a better claim than "QTT derives GR" because it states exactly where the theory's new contribution lies. QTT natively derives the sink law, the inverse-square endurance flux, the constitutive normalization and the numerical coupling

$$G = \frac{\tilde{\ell}^2 c^3}{\hbar}.$$

It does not natively prove every step of smooth continuum geometry. The Lorentzian metric is reconstructed from causal order and volume only after assuming a regular manifoldlike coarse-graining. The Einstein tensor follows from Lovelock only after assuming local covariance, metricity and second-order infrared dynamics. Higher-curvature terms are not forbidden exactly; they are suppressed in the A6 low-curvature expansion.

Within those correctly stated boundaries, the result is powerful. The action coefficient

$$\frac{c^3}{16\pi G}$$

is not a freely inserted GR coupling. It is the QTT capacity ratio

$$\frac{\hbar}{16\pi\tilde{\ell}^2}.$$

Thus the Einstein–Hilbert action is the local Einstein-gauge shadow of the endurance ledger:

$$\text{space-quantum sink} \Rightarrow G \Rightarrow \lambda \Rightarrow S_{\mathrm{EH}}.$$

The remaining work is clear. QTT must strengthen the manifoldlike coarse-graining theorem, derive the ABC/Einstein-gauge relation with more mathematical control, and test the same $\tilde{\ell}$ outside gravity. If those independent capacity checks succeed, then the Einstein–Hilbert coefficient becomes part of a broader one-ruler structure connecting gravity, ultraviolet capacity and quantum response. If they fail, this paper's local Einstein-gauge recovery remains a useful consistency result, but QTT as a deeper substrate theory loses its strongest claim.

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