ALGORITHM FOR IDENTIFICATION AND OBSERVATION OF THE STATE OF UNCERTAIN HARMONIC DEVIATIONS IN NONLINEAR OBJECTS

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Abstract. This article examines the problem of observing state variables in nonlinear control objects under conditions of unknown harmonic distortions affecting the output signal. The main goal of the study is to develop an adaptive observation algorithm that allows for the asymptotically accurate estimation of an unmeasured state vector. Unlike existing works, this methodology is based on a two-stage synthesis: identification: amplitude, frequency, and phase indicators of harmonic distortion, auxiliary signals, and adjustable models; observation: a state tracker is constructed based on identified parameters. The approach takes into account that the distortion affects the output signal directly rather than the system input, which further complicates the problem and limits the application of classical tracker methods.

Key words: output perturbation, non-minimum phase systems, amplitude estimation, adaptive observer, nonlinear systems, harmonic perturbation, state variable estimation, frequency identification, asymptotic convergence, observer synthesis.

INTRODUCTION

In modern control systems, the issue of accurate assessment of the state of nonlinear objects is of great scientific and practical importance. In practical conditions, many systems deviate from the ideal model due to external environmental influences, measurement errors, or unknown periodic disturbances. In particular, in the presence of harmonic disturbances affecting the output signal, traditional observer algorithms cannot provide sufficient accuracy. In such a situation, special adaptive approaches are required to reliably and stably assess the state of the system.

Classical state observers usually assume that disturbances affecting the system are transmitted through the input or completely ignore them. However, if there is an unknown harmonic disturbance directly affecting the output signal, the observation error may not approach zero. Therefore, the identification of disturbance parameters for such systems and the synthesis of an observer taking into account their estimates is an urgent scientific issue.

A major contribution to the adaptive and robust design of observers is made by Ioannou and Sun: their monograph "Robust Adaptive Control" combines the fundamentals of adaptive algorithms, identification, and robustness analysis to provide methods for robust control of a system under unknown parameters and external disturbances. These works strengthen the theoretical foundations for the design of adaptive observers and discuss the practical limitations of real-time identification [1,2].

Adaptive control and systematic methods for sinusoidal (harmonic) disturbance rejection were summarized by Sastry and Bodson; their monograph summarizes the main results on the stability, convergence, and limitations of adaptive systems. In particular, the papers by Bodson and colleagues on algorithms for adaptive identification and rejection of sinusoidal disturbances provide a theoretical basis for the approach used in your paper, suggesting effective methods for dealing with sinusoids of unknown frequency and amplitude. [3,4]

In this work, an adaptive observer synthesis for a nonlinear object in the presence of unknown harmonic disturbances at the output is proposed. The approach consists of two stages: first, the disturbance parameters are determined, and then, based on them, the state variable estimates are formed. The proposed algorithm is aimed at ensuring that the observation error asymptotically approaches zero, and it can also be applied to nonlinear and non-minimal phase systems.

The problem of synthesizing an asymptotic observer of a vector of state variables for a nonlinear object in the form is considered.

$$\dot{x}(t) = Ax(t) + bu(t) + d\varphi(y), \quad (1)$$

$$y(t) = c^T x(t) + \delta(t), \quad (2)$$

where $x \in R^n$ – immeasurable state variable vector; $A,\ b,\ d$ и c – known matrices and vectors of constant coefficients with corresponding dimensions; $\delta(t) \in R$ – unknown and inaccessible to direct measurements harmonic perturbation; $u(t) \in R$ – control signal; $\varphi(y)$ – known smooth function; $y(t) \in R$ – measured output [5].

If the A matrix is gurvitsev, then this problem can be solved quite simply by calculating the model of the object (1) on a real time scale. In this case, due to the exponential tendency of the free component to zero, the model will generate an estimate of the state variables vector asymptotically converging to the real values of $x(t)$ (1). Otherwise (i.e., if the matrix A is non-gurvitsev), this scheme is inoperable, and the use of classical state vector observers will not allow for the asymptotic convergence of the observation error due to the presence of perturbation $\delta(t)$.

The set task can be solved using adaptive observation methods [1-7]. However, most of the known works are devoted to the case when the harmonic perturbation is reduced to the input of the system [1-5], and their results cannot be directly applied to the considered case of perturbation acting on the output of the system. In works [6, 7], the case of disturbances in the output signal, but for a limited class of linear minimal phase objects, was considered. Thus, constructing an adaptive observer for a nonlinear object (1) - (2) is a new and relevant task.

METHODS

Consider, in general, a non-minimal phase nonlinear object of the form (1), (2). For simplicity, we will limit ourselves to the study of the case when the perturbation $\delta(t)$ is represented as a harmonic function.

$$\delta(t) = \sigma \sin(\omega t + \beta) \quad (3)$$

with unknowns of amplitude σ, frequency ω and initial phase β. Note that the expansion of the proposed approach to the case of perturbation represented by the sum of several harmonic functions does not lead to fundamental difficulties, but complicates the presentation of the main material of the article [8].

Let’s rewrite the object (1), (2) in the form of an input-output model:

$$y(t) = \frac{b(p)}{a(p)} u(t) + \frac{d(p)}{a(p)} \varphi(y) + \delta(t),$$

where $p = d/dt$ – differentiation operator; $a(p) = p^n + a_{n-1}p^{n-1} + \ldots + a_1 p + a_0$, $b(p) = b_m p^m + \ldots + b_1 p + b_0$ и $d(p) = d_r p^r + \ldots + d_1 p + d_0$ – corresponding polynomials obtained as a result of the transition from the input-state-output model to the input-output model: $\frac{b(p)}{a(p)} = c^T (pI - A)^{-1} b$ и $\frac{d(p)}{a(p)} = c^T (pI - A)^{-1} d$

We will consider the following assumptions regarding the system to be satisfied. (1), (2), (4).

Assumption 1. Only the $y(t)$ and $u(t)$ signals are available for measurement.

Assumption 2. The pair $A,\ b$ is completely controlled, and the pair $A,\ c$ is completely observable.

Assumption 3. The polynomial $a(p)$ has no roots $\pm j\omega$, where ω is the frequency of disturbance.

It is required to construct an asymptotic observer of the state variables $x(t)$ of the object (1), (2) such that

$$\lim_{t\to\infty}\left|x(t)-\hat{x}(t)\right|=0, \qquad (5)$$

where $\hat{x}(t)$ is the estimate of the vector $x(t)$.

We will synthesize the observer for the object (1), (2) in two stages. First, let's solve the problem of synthesizing the disturbing observer $\delta(t)$. Further, using the information about $\delta(t)$, we construct the estimate of the vector.

Note that two approaches can be used to solve the given problem. The first approach involves considering an extended system that includes both the management object itself and the external environment model. Using the obtained frequency estimate w, the classical full-dimensional observer coefficients for the extended system can be calculated. The advantages of this approach include that to solve the problem, it is sufficient to identify only the frequency of the perturbation, but not its amplitude and phase. At the same time, the proposed approach will require real-time recalculation of the observer's coefficients, which increases the complexity of the method and makes its practical implementation difficult [9,10].

The synthesis of the observer for the object (1), (2) will be carried out in two stages. First, let’s solve the problem of synthesizing the disturbing observer $\delta(t)$. Further, using the information about $\delta(t)$, we construct the estimate of the vector $x(t)$.

Note that two approaches can be used to solve the given problem. The first approach involves considering an extended system that includes both the management object itself and the external environment model. Using the obtained frequency estimate ω, the classical full-dimensional observer coefficients for the extended system can be calculated. The advantages of this approach include the fact that to solve the problem, it is sufficient to identify only the frequency of the perturbation, but not its amplitude and phase. At the same time, the proposed approach will require real-time recalculation of the observer's coefficients, which increases the complexity of the method and makes its practical implementation difficult [11,12].

The second possible approach is to construct a perturbation observer based on the identification of all its parameters, and to use the obtained perturbation estimate to calculate the object output, followed by constructing a state variable observer. This method is distinguished by its lower computational complexity, and it will be considered later in the work.

RESULTS.

Observer’s synthesis of perturbing influence. So, let’s synthesize the perturbing observer $\delta(t)=\sigma\sin(\omega t+\beta)$, for which it will be necessary to identify the parameters σ, ω and β. Let’s first construct the parameter identifier. We will use the results of the work [13].

Consider an arbitrary Gurvis polynomial $\gamma(p)$ of degree n. Then equation (4) can be rewritten as

$$y(t)=\frac{a_1(p)}{\gamma(p)}y(t)+\frac{b(p)}{\gamma(p)}u(t)+\frac{d(p)}{\gamma(p)}\varphi(y)+\frac{a(p)}{\gamma(p)}\delta(t), \qquad (5)$$

where $a_1(p)=\gamma(p)-a(p)$.

Forming an auxiliary signal:

$$w(t)=y(t)-\frac{a_1(p)}{\gamma(p)}y(t)-\frac{b(p)}{\gamma(p)}u(t)+\frac{d(p)}{\gamma(p)}\varphi(y), \qquad (6)$$

Considering equation (5) we obtain

$$w(t)=y(t)-\frac{a_1(p)}{\gamma(p)}y(t)-\frac{b(p)}{\gamma(p)}u(t)+\frac{d(p)}{\gamma(p)}\varphi(y)=\frac{a(p)}{\gamma(p)}\delta(t)=$$
$$=\frac{a(p)}{\gamma(p)}\sigma\sin(wt+\beta)=\sigma\frac{a(p)}{\gamma(p)}\sin(wt+\beta) \qquad (7)$$

From (7), it follows that the $w(t)$ signal, due to the $\gamma(p)$ polynomial's gravity and the absence of $\pm j\omega$ roots in the $a(p)$ polynomial, is a harmonic function with a ω frequency. Therefore, the $w(t)$ signal can be considered as the output of the dynamic model of the form

$$\frac{d^2 w(t)}{dt^2} = -\omega^2 w(t) = \theta\, w(t) \tag{8}$$

where $\theta = -\omega^2$ – - constant parameter. Following the results of Lemma 1 from work [9], the $w(t)$ signal can be written in the form

$$w(t) = 2\dot{\varsigma}(t) + \varsigma(t) + \theta\varsigma(t) + \varepsilon_y(t) \tag{9}$$

where $\varepsilon_y(t)$ – the exponentially damped time function defined by non-zero initial conditions, and the function $\varsigma(t)$ is formed as follows

$$\varsigma(t) = \frac{1}{(p+1)^2} w(t) \tag{10}$$

As in [9], for the synthesis of the θ unknown parameter identifier, we introduce a new variable - the measured signal of the form

$$z(t) = \ddot{\varsigma}(t) = w(t) - 2\dot{\varsigma}(t) - \varsigma(t) \tag{11}$$

It can be shown that, due to equations (9) and (10), the equality is valid.

$$z(t) = \theta\varsigma(t).$$

Then it is advisable to form the signal $z(t)$ estimate $\hat{z}(t)$ in the form

$$\hat{z}(t) = \hat{\theta}(t)\varsigma(t), \tag{12}$$

where $\hat{\theta}(t)$ – the adjustable parameter (θ parameter estimate).

Statement 1 [9]. *Let the parameter $\hat{\theta}(t)$ be set as follows:*

$$\hat{\theta}(t) = k\varsigma(t)(z(t) - \hat{z}(t)), \tag{13}$$

where $k > 0$ is the adaptation coefficient, the $\varsigma(t)$, $z(t)$ and $\hat{z}(t)$ signals are formed according to expressions (10), (11) and (12) respectively (at the same time, the $w(t)$ signal is formed according to rule (6)). Then

$$\lim_{t\to\infty} \left|\hat{\theta}(t) - \theta\right| = 0$$

Taking into account statement 1, we will calculate the harmonic perturbation frequency as follows:

$$\hat{\omega}(t) = \sqrt{\left|\hat{\theta}(t)\right|}. \tag{14}$$

To construct the perturbation estimate $\delta(t)$, we replace the signal phase σ and β amplitude identification problem with a simpler two-amplitude identification problem. Indeed, for the harmonic signal $w(t)$, the following is true:

$$w(t) = \frac{a(p)}{\gamma(p)}\sigma\sin(wt+\beta) = \frac{a(p)}{\gamma(p)}\sigma_1\sin\omega t + \sigma_2\cos\omega t = \sigma_1\psi_1(t) + \sigma_2\psi_2(t),$$

where the perturbation $\delta(t) = \sigma\sin(wt+\beta)$ is represented as the sum of two harmonic signals of different amplitudes, but with zero initial phase:

$$\delta(t) = \sigma_1\sin\omega t + \sigma_2\cos\omega t,$$

and the physically realized signals $\psi_1(t)$ and $\psi_2(t)$ are formed according to the rule

$$\psi_1(t) = \frac{a(p)}{\gamma(p)}\sin\omega t,$$

$$\psi_2(t) = \frac{a(p)}{\gamma(p)}\cos\omega t$$

Then the perturbation estimate $\delta(t)$ will be formed in the form

$$\hat{\delta}(t) = \hat{\sigma}_1\sin\hat{\omega}t + \hat{\sigma}_2\cos\hat{\omega}t, \tag{15}$$

where $\hat{\sigma}_1$ and $\hat{\sigma}_2$ are adjustable parameters (estimates of σ_1 and σ_2 parameters).

The righteousness of the following statement can be demonstrated.

Statement 2. *Let the parameters $\hat{\sigma}_1$ and $\hat{\sigma}_2$ be set as follows:*

$$\dot{\hat{\sigma}}_1(t) = k_\sigma\hat{\psi}_1(t)\left(w(t) - \hat{\sigma}_1(t)\hat{\psi}_1(t) - \hat{\sigma}_2(t)\hat{\psi}_2(t)\right), \tag{16}$$

$$\dot{\hat{\sigma}}_2(t) = k_\sigma\hat{\psi}_2(t)\left(w(t) - \hat{\sigma}_1(t)\hat{\psi}_1(t) - \hat{\sigma}_2(t)\hat{\psi}_2(t)\right), \tag{17}$$

where k_σ is the adaptation coefficient, the signal $w(t)$ is determined by expression (6), and the signals $\hat{\psi}_1(t)$ and $\hat{\psi}_2(t)$ are formed according to the rule

$$\hat{\psi}_1(t)=\frac{a(p)}{\gamma(p)}\sin\hat{\omega}t,$$

$$\hat{\psi}_2(t)=\frac{a(p)}{\gamma(p)}\cos\hat{\omega}t \qquad (18)$$

using the harmonic perturbation frequency estimate (13), 14). Then

$$\lim|\sigma(t)-\hat{\sigma}(t)|=0. \qquad (19)$$

Thus, an adaptive perturbation observer containing blocks for the formation of auxiliary signals $w(t)$, $\varsigma(t)$ and $z(t)$, (6), (10) and (11) respectively, configurable models (12) and (15), as well as configuration algorithms (13), (16) and (17), provides asymptotic identification for the object (1), (2) of the previously unknown

perturbations (3). In particular, when perturbation (3) has zero initial phase (i.e., $\beta=0$), its identification scheme can be significantly simplified. That is, instead of the estimate (15) and the two tuning algorithms (16) and (17), the estimate of the form can be used [11].

$$\hat{\delta}(t)=\hat{\sigma}\sin\hat{\omega}t, \qquad (20)$$

where the parameter is set according to the rule

$$\dot{\hat{\sigma}}=k_\sigma\hat{\psi}(t)(w(t)-\hat{w}(t)), \qquad (21)$$

$$\hat{\psi}(t)=\frac{a(p)}{\gamma(p)}\sin\hat{\omega}t. \qquad (22)$$

Now, knowing the perturbation estimate $\delta(t)$, we construct a state variable observer $x(t)$ for the controlled object (1), (2). For this, we will use classical

the results of the synthesis of full-dimensional observers, published, for example, in [10]:

$$\dot{\hat{x}}(t)=A\hat{x}(t)+bu(t)+d\varphi(y)+l(y(t)-\hat{y}(t)), \qquad (23)$$

$$\hat{y}(t)=c^T\hat{x}(t)+\hat{\delta}(t), \qquad (24)$$

where $\hat{x}(t)\in R^n$ is the estimate of the vector $x(t)$, $\hat{\delta}(t)\in R$ is the estimate of the unknown perturbation formed according to the rule (15) (or according to the rule (20)), $\hat{y}(t)\in R$ is the estimate of the variable $y(t)$, and the vector of constant coefficients llll is calculated so that the matrix $\bar{A}=A-lc^T$ is a Gurvits matrix.

Let's introduce the $\varepsilon=x-\hat{x}$ state estimation error. Then, by subtracting (23) from (1), taking into account (2) and (24), we obtain the state estimation error model:

$$\dot{\varepsilon}=\bar{A}\varepsilon+l(\sigma(t)-\hat{\sigma}(t)).$$

From the last expression, taking into account the gravity of matrix A and equality (19), the objective condition (5) follows.

DISCUSSION

To illustrate the proposed observer synthesis scheme for a nonlinear object of the form (1), (2), let's consider an example. Let's consider a nonlinear object of the form

$$\dot{x}_1=x_2,$$

$$\dot{x}_2=u-y^3,$$

$$y=x_1+\sigma,$$

where the input signal is $u(t)=1$. For definiteness, we will assume that the unknown perturbation has the form $\sigma(t)=3\sin 4t$. Let's construct an observer using the expressions (6), (10), (14), (20) - (24):

$$w=y-\frac{20p+100}{p^2+20p+100}y-\frac{1}{p^2+20p+100}u+\frac{1}{p^2+20p+100}y^3,$$

$$\varsigma=\frac{1}{(p+1)^2}w,$$

$$\dot{\hat{\theta}}=10^5\varsigma(w-2\dot{\varsigma}-\varsigma-\hat{\theta}\varsigma),$$

$$\hat{\omega}=\sqrt{|\hat{\theta}|},$$

$$\psi = \frac{p^2}{p^2 + 20p + 100}\sin \hat{\omega} t$$

$$\hat{w} = \hat{\sigma}\psi,$$

$$\dot{\hat{\sigma}} = 10^3 \psi (w - \hat{w}),$$

$$\hat{\delta}(t) = \hat{\sigma}\sin \hat{\omega} t,$$

$$\dot{\hat{x}}_1 = \hat{x}_2 + 20(y - \hat{y}),$$

$$\dot{\hat{x}}_2 = u - y^3 + 100(y - \hat{y})$$

$$\hat{y} = \hat{x}_1 + \hat{\delta}$$

The results of computer modeling of the identification of the frequency ω, the amplitude $\hat{\sigma}$, and the graphs of inconsistencies $x_1 - \hat{x}_1$ and $x_2 - \hat{x}_2$ are presented in Figures 1-4, respectively, and demonstrate an asymptotically accurate estimation of the perturbation frequency (Figure 1), perturbation amplitude (Figure 2), and the fulfillment of the objective condition (5) (Figures 3 and 4).

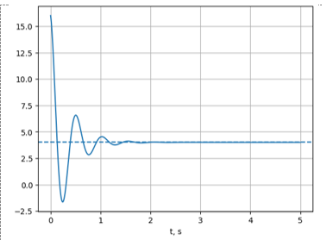

Figure 1.Frequency identification $\hat{\omega}$.

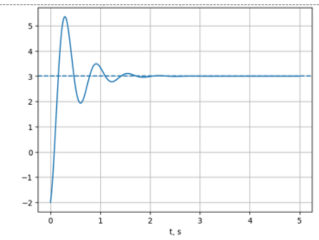

Figure 2. Amplitude identification $\hat{\sigma}$

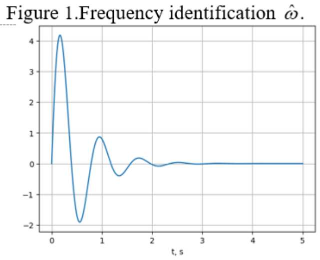

Figure 3. Inconsistency $x_1 - \hat{x}_1$

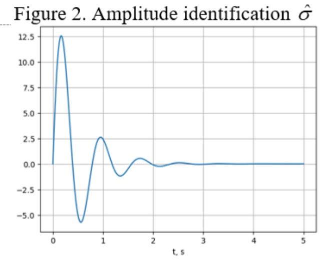

Figure 4. Inconsistency $x_2 - \hat{x}_2$

CONCLUSION

The article proposes an alternative to [6, 7] algorithm for the synthesis of the observer (23), (24) for the nonlinear control object (1), (2). The results of computer modeling, illustrating the functionality of the proposed algorithm, are presented.

In conclusion, this paper considers the problem of tracking state variables in the presence of unknown harmonic disturbances affecting the output signal of nonlinear systems. The proposed approach is based on a two-stage adaptive observer synthesis, which first identifies the disturbance parameters (amplitude, frequency, and phase) and then asymptotically estimates the state variables based on the disturbance estimate. This approach can be applied to nonlinear systems and systems with non-minimum phase.

The advantage of the algorithm presented in this paper is that, unlike classical observation methods, it ensures that the tracking error asymptotically approaches zero even in the presence of harmonic disturbances at the output. Using the adaptive identification mechanism, the disturbance frequency and amplitude are determined, and a full-scale state observer is constructed based on their estimates. This ensures system stability and observation accuracy.

Computer simulation results confirm the effectiveness of the proposed algorithm: the disturbance frequency and amplitude are accurately identified, and the state variable estimates are close to the true values. Thus, the developed method represents an effective and promising solution for use in practical control systems with unknown harmonic effects.

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