The Interior Observer Cosmological Framework

Paper 21 — The AC1 Derivation, the SU(2) Geometric Weight, and the Radiation Response Problem

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Version 1.5 - May 2026

Version History

v1.5 (May 2026): PRyMordial output index correction (YPBBN -> YPCMB) and amplitude framework alignment to Paper 22 v1.3 / Paper 24 v2.2 standard (epsilon_w = 0.012301, epsilon_n = 0.023842). Paper 21 v1.4 BBN scorecard reflected an older amplitude branch and read PRyMordial output index 4 (YPBBN) instead of index 3 (YPCMB). Corrected scorecard: Y_p = 0.24782 (+0.70 sigma), D/H = 2.510 x 10^-5 (-0.55 sigma), Li-7/H = 5.36 x 10^-10 (+12.20 sigma), chi^2(D/H + Y_p) = 0.80. Part II's nineteen-route radiation investigation is superseded by Paper 22 v1.3's rate-dressing paradigm: the v1.4 D/H tension at -3.9 sigma (chi^2 approximately 15.6) that motivated the F_abs = 0.36879 / DeltaN_eff approximately 0.37 hunt shrinks to -0.55 sigma (chi^2 = 0.80) under the corrected wrapper plus modern amplitudes, removing the phenomenological need for ΔN_eff approximately 0.37. Paper 22 v1.3, Theorem 22.23, derives the active resolution via the j=1 weak channel and j=2 nuclear channel rate-dressing construction (epsilon_w = K_gauge x L_1, epsilon_n = (langle K rangle / 10) x L_2) using exactly the puncture loads L_1 = 0.22417 and L_2 = 0.13805 derived in Paper 21. The nineteen no-go results in Part II remain as documented research provenance constraining the search space; the L_1 and L_2 puncture-load derivations remain load-bearing inputs to Paper 22's rate-dressing construction. Observational denominators per IO Framework Observational Conventions v1 (https://dfife.github.io/data/observational_conventions_v1.md). Paper 21's distinctive results (operator algebra construction, nineteen killed P_resp routes, puncture load derivations L_1 and L_2) are unaffected by this correction.

v1.4 (April 2026): Appendix replaced with Paper 20 v1.5 clean + Paper 21 results. AC1 closed. Open/Closed tracking.

v1.3 (April 2026): Full Schur branch appendix correction. All inherited appendix steps (29, 56, 58, 60, 107, 122) and open-problem Item 16 updated to conditional Schur branch (H_0 = 68.91, Paper 29). Paper 19 intermediate supersession notes updated. Bare branch (H_0 = 61.06) references in §§7, 13, 14 marked SUPERSEDED (Paper 29). Title page standardized. All body theorems (AC1 derivation, SU(2) geometric weight, sky-slot inheritance, optical filtration, F_abs) are branch-independent.

v1.2 (March 2026): Clean Appendix A catalog rebuilt from Paper 17 base with Papers 18–21 results appended (Steps 1–207). Freeze-in confirmed dead. Route count harmonized (19 = 18 P_resp + freeze-in). Reproducibility language harmonized. Optical filtration scope narrowed. Title page reformatted.

v1.1 (March 2026): Cycloid parameterization correction.

v1.0 (March 2026): Publication candidate.

Multi-AI Collaboration: Claude (Anthropic), Codex/ChatGPT (OpenAI), Wolfram/ChatGPT (OpenAI), Gemini (Google DeepMind)

Abstract

Beginning from two premises - (1) the observable universe exists inside a Schwarzschild black hole, and (2) the physics inside the horizon is the same as outside - this paper addresses two foundation problems of the Interior Observer (IO) framework: the acoustic class membership premise AC1, and the structural origin of the D/H radiation deficit DeltaN_eff approximately 0.37 (this second program is retained as documented research provenance but superseded as the active D/H-resolution strategy by Paper 22 v1.3's rate-dressing theorem; see v1.5 version history and Part II supersession note).

Part I derives AC1 from the existing Papers 15–20 stack, upgrading Paper 20's Theorem 20.2 from CONDITIONAL THEOREM to DERIVED/THEOREM. The zero-parameter prediction θ*_pred = 0.599° (Planck: 0.597°, residual 0.429%) is now theorem-grade within the reduced scalar/longitudinal acoustic sector.

Part II investigates the structural origin of the D/H radiation deficit and substantially advances the reduced -> full algebra extension. (Note: in v1.5, Part II is retained as documented research provenance but superseded as the active D/H-resolution strategy by Paper 22 v1.3's rate-dressing theorem; the L_1 and L_2 puncture-load derivations described below remain load-bearing inputs to Paper 22's resolution.) The central discoveries: the one-puncture partition function Z_punc = 1.4221 is a temperature-independent geometric constant; the Optical Filtration Theorem proves the direct reduced RT/BY optical readout of θ* is blind to noncentral SU(2) puncture data; the BBN Branch Assignment Theorem proves BBN evaluates at T_IO, not T_obs; and the Local Nontraciality Theorem proves that global gauge invariance does not imply local traciality, breaking the Paper 20 tracial obstruction.

Part II also discovers the Absolute Local Casimir Load F_abs = Tr[ρ_loc H_punc $C_2/(4+C_2)$] / Tr[ρ_loc H_punc] = 0.36879, matching the D/H target to 0.066% precision with zero fitted parameters. The denominator λ=4 is conditionally unique within the bounded resolvent family, fixed by the 4D history-to-Hamiltonian descent. However, eighteen systematic attempts to close the response map P_resp that delivers this number to BBN observables are documented and killed: including optical filtration (the Casimir operator lives in the puncture fixed-point algebra), conformal insulation (persistent ΔN_eff shifts θ* by −1.56%), conformal a^{-4} sourcing (framework gives a^{-8}, w=5/3), stiff-fluid dilution (correct θ* insulation but wrong BBN time profile), separable weak-rate suppression (destroys Y_p), and twelve others. Notably, the baryon freeze-in direction (Theorem 21.E) reverses under the corrected expanding-phase OS cycloid parameterization, but Codex computation

confirms the route is dead: $\omega_{b,geom} = 0.02087$ is already below the BBN target 0.02237. The precise remaining target is a non-separable, channel-dependent radiation response operator on the local weak-sector/puncture algebra.

This paper is part of the Interior Observer Cosmological Framework research series.

1. Introduction

The Interior Observer (IO) framework models the observable universe as the interior of a Schwarzschild black hole with mass $M_U = 4.50 \times 10^{53}$ kg. Papers 1–21 derived all major cosmological observables from this single measured input plus one external parameter ($\gamma_{BI} = 0.2375$ from Loop Quantum Gravity) and three physical principles — the Conformal Modular Principle (CMP, Paper 10; upgraded to DERIVED/THEOREM, Paper 18), the Baryon Dictionary Principle (BDP, Paper 12; upgraded to DERIVED/THEOREM, Paper 18), and the Gauge Thermal Transfer Principle (GTTP, Paper 13; upgraded to DERIVED/THEOREM, Paper 17).

On the interior geometry: A generic Schwarzschild interior is the Kantowski-Sachs metric — a dynamically collapsing anisotropic geometry — not a homogeneous isotropic FRW spacetime. The IO framework does not use a generic interior. Paper 3 establishes the Oppenheimer-Snyder (OS) dust solution, which produces an exact $k = +1$ closed FRW interior metric with zero free parameters (Oppenheimer & Snyder 1939 [9]). The OS cycloid $a(\eta) = (r_s/2)(1-\cos \eta)$ describes an expanding universe from the Big Bang ($\eta = 0$, $a = 0$) through maximum expansion ($\eta = \pi$, $a = r_s$). The IO framework places the current epoch at $\eta_s = 1.893$, in the expanding phase ($a'(\eta) > 0$) before maximum expansion. The framework predicts a closed, recollapsing universe: a falsifiable prediction that distinguishes it from ΛCDM.

On external parameters: The Barbero-Immirzi parameter $\gamma = 0.2375$ is determined within Loop Quantum Gravity by fitting to the Bekenstein-Hawking black hole entropy formula — an independent gravitational observable, not any cosmological measurement. Its use in the IO framework is analogous to using G or $\hbar$: an external input from an independent theory. When this paper refers to “zero fitted parameters,” it means zero parameters fitted to cosmological data (CMB, BBN, supernovae, BAO). All cosmological predictions follow from M_U and γ without tuning.

Two foundation problems motivate this paper. First, Paper 20’s Theorem 20.2 (Acoustic Phase-Calibration) established $J_\theta = x^{-1/2}\sqrt{1+\gamma^2}$, giving $\theta^*_{pred} = 0.599°$ with zero fitted parameters, but was classified as CONDITIONAL THEOREM conditional on AC1 - the assumption that the observed acoustic ruler belongs to the Paper 16 degree-1 boundary-to-bulk readout class. Second, historically (before the v1.5 YPCMB correction and Paper 22 v1.3 amplitude alignment, which together resolve this tension), the D/H tension at -3.9 sigma with imported $N_{eff} = 3.044$ identified ΔN_{eff} approximately 0.37 as a phenomenological target with no structural origin.

1.1 Claims and Scope

Part I — AC1 Derivation (§§2–6):

1. AC1 is derivable from the Papers 15–20 stack in sharpened composite form.

2. The A4 bridge closes within the reduced scalar/longitudinal acoustic sector via post-lens linearity.

3. Theorem 20.2 upgrades to DERIVED/THEOREM under AH1–AH7 + P1–P6 + RT/BY + B1–B5 (AC1 removed).

Part II — Radiation Investigation (§§7–13):

4. The assembly gap and ΔN_{eff} are decoupled problems.

5. The SU(2) Geometric Weight Theorem: the energy-weighted j=1+2 fraction = 0.3622 from the temperature-independent puncture partition function, within 1.7% of the ΔN_{eff} target.

6. The Optical Filtration Theorem: RT/BY optical readout commutes with SU(2) averaging, rendering noncentral puncture weights invisible to boundary-projected measurements like θ^*.

7. Six dead ends documented with full rigor.

Not claimed: closure of the full ΔN_{eff} derivation (requires the response map P_{resp}); that F_{abs} enters the bulk as permanent radiation (No-go 21.AB1 proves this would shift θ^* by 1.56%); extension to vector/tensor modes; resolution of the assembly gap; resolution of the lithium problem.

1.2 Methodology

This paper was produced through a four-system adversarial collaboration. David Fife (Principal Investigator) directed all investigations and made all final decisions. Claude (Anthropic, Opus 4.6) served as orchestrator. Codex/ChatGPT (OpenAI) performed all computation and theorem construction. Wolfram/ChatGPT (OpenAI, 5.4 Pro Extended) served as primary mathematical validator. Gemini (Google DeepMind, 2.5 Pro) provided key physical insights including the “lens vs light” framing (convergent with Claude), the measurement-dependence paradigm, and the SU(2) geometric freeze-out concept that led to the partition function investigation.

On AI-assisted proofs: The mathematical results in this paper are algebraic and reproducible by any human expert or formal proof assistant. The AI systems served as computational tools and adversarial reviewers, not as authorities. Every theorem statement is self-contained: the hypotheses, logical steps, and conclusions can be verified independently of the AI pipeline that produced them. Computation scripts for all BBN scorecards (using the PRyMordial code [25]), partition function evaluations, and no-go verifications are available upon request and will be published to a public repository (see Appendix A.5).

On status labels: The foundation five-tier classification from Paper 14 (§1.4) — DERIVED/THEOREM, SEMICLASSICAL PRINCIPLE, OBSERVATIONALLY SELECTED CLOSURE, STRUCTURAL OBSERVATION, and DISCUSSION — is used for core results. Papers 15–21 extend the taxonomy with operational labels: DERIVED/NO-GO and COMPUTATIONAL NO-GO for killed routes; CONDITIONAL/THEOREM for results conditional on stated premises; CONSTRUCTED/VERIFIED for algebraic constructions; SCOPE-BOUNDARY for operator-algebra scope limits; STRUCTURALLY SUPPORTED CONSTRUCTION for objects with correct output but open physical delivery; FRONTIER/SPECIFICATION for surviving targets; and DEFINITION, VALIDATED, RECALIBRATED for bookkeeping. Each step states its premises and the specific sense in which the result holds.

Part I: The AC1 Derivation

2. Prior Stack Inputs

The following established results are used in this paper.

From Paper 19:

FC6. The primitive redshift/frequency observable $\omega = -k_\mu u^\mu$ is a one-slot boundary-dressed mixed observable with unique reduced factor $\sqrt{(1+\gamma^2)}$. STATUS: DERIVED/THEOREM (Paper 19, Step 126).

FC5. Universal transport / duplicated bulk dressing is ruled out. STATUS: DERIVED (Paper 19).

From Paper 20:

G1. The background sound horizon r_s is a 1D acoustic-history observable on the observer-side homogeneous comoving-dust sector. STATUS: DERIVED/THEOREM (Theorem 20.1, AH1–AH7).

G2 / AB5. The local plasma computation (Thomson scattering, baryon-photon integral) is gauge-neutral in the reduced-core sense. STATUS: DERIVED.

AB3 / AB4. D_A is a pure transverse S^2 area-distance observable. It is gauge-neutral and not in the local Paper 16 bilinear class. STATUS: DERIVED.

PC1–PC2. Once a one-slot primitive acoustic sky readout is certified, all non-phase channels are exhausted. The surviving slot must calibrate the acoustic ruler in the phase lattice, not in a uniform amplitude channel. STATUS: DERIVED.

From Paper 16:

P1–P6. The Composite Uniqueness Theorem. Within the explicit local boundary-to-bulk bilinear class, degree-1 uniquely gives the $\sqrt{(1+\gamma^2)}$ factor. Degree-2 gives $(1+\gamma^2)$.

P6. Universal transport no-go. Breaks GTTP at 20.15σ. STATUS: DERIVED.

3. Round 1: The Structural Framework

3.1 Theorem P21.A1 (Optical Necessity Theorem for Observed θ*)

For the observed acoustic angle θ*, the relevant numerator is optical by construction. It is not a competing present-day local bulk measurement of the primordial sound horizon.

Proof. No analogous non-optical counterexample exists for the observed θ*. By definition, that quantity is extracted from the observed boundary photon sky: ΔT/T on S^2 → a_ℓm → C_ℓ → ℓ_peak → θ*. So for the actual θ* observable, the numerator is optical by construction. QED.

Status: DERIVED. Validated at reduced scope. The extraction pipeline is optical; the universal "no non-optical access" claim is stronger than the papers establish but is not load-bearing.

3.2 Theorem P21.A2 (Primitive Angular Stage Theorem)

The gauge-sensitive stage relevant to θ* lies at the primitive angular readout stage, not at the degree-2 self-intensity stage.

Proof. By the Paper 20 measurement-chain theorems: a_ℓm is linear, C_ℓ is quadratic, and peak positions are invariant under uniform positive rescaling of C_ℓ. Therefore a pure degree-2 dressing can move peak heights but cannot move peak positions. QED.

Status: DERIVED. Validated.

3.3 Theorem P21.A3 (No Second Source-Side A-Action Theorem)

On the current Papers 15–20 stack, the acoustic readout admits no second natural A-sensitive action on the source/history side.

Proof. Three candidates eliminated: (1) Local plasma microphysics — ruled out by Paper 20 G2/AB5. (2) Transverse denominator — ruled out by Paper 20 AB4. (3) Universal transport / bulk duplication — ruled out by Paper 16 P6 and Paper 19 FC5. QED.

Status: DERIVED. Validated.

3.4 Theorem P21.A4 (Sky-Slot Inheritance Theorem)

The primitive boundary sky-pattern readout inherits exactly one A-sensitive slot from the certified primitive photon datum.

Proof. For each line of sight n, the observed boundary photon datum ω(n) = −k(n)_μ u^μ is one-slot degree-1 by Paper 19 FC6. Collecting these over S^2 yields the primitive sky field. By Theorem P21.A3, no second A-slot exists. By Theorem P21.A2, cannot retype to degree-2. Therefore the primitive angular stage is one-slot / degree-1. QED.

Note: The A4 gap was identified during adversarial validation as requiring a specific bridge theorem. That bridge is proved in §4 (Theorems P21.B2–B5).

3.5 Theorem P21.A5 (AC1 Reduction Theorem, Sharpened Composite Form)

Within the homogeneous observer-side acoustic-history scope, AC1 is replaced by the derived statement: the numerator readout is one-slot / degree-1; the denominator is gauge-neutral.

Proof. By Paper 20 G1: $J_{r,geom} = x^{(1/2)}$. By Paper 20 AB3/AB4: $J_D = x$, gauge-neutral. By Theorem P21.A4: one-slot degree-1. By Paper 20 PC1–PC2: phase lattice. Therefore $J_r = x^{(1/2)}\sqrt{(1+\gamma^2)}$, $J_D = x$, $J_\theta = x^{(-1/2)}\sqrt{(1+\gamma^2)} = 0.83395$. QED.

Status: DERIVED/THEOREM under AH1–AH7 + P1–P6 + RT/BY + B1–B5. Scope: reduced scalar/longitudinal acoustic sector. Validated.

4. Round 2: The A4 Bridge Theorem

4.1 Scope

Explicitly restricted to the reduced scalar/longitudinal acoustic sector: scalar acoustic modes only, no vector/tensor, no lensing, no multipole-mixing beyond bare scalar harmonic decomposition.

4.2 Theorem P21.B0 (Dead End: ΔT/T Is the Wrong Bridge Object)

The GTTP factor cancels in the temperature ratio ΔT/T. The correct carrier is the primitive sky field $\Xi(n)$ and its linear harmonic image $a_{\ell m}$, before same-fiber squaring.

Status: DERIVED. Validated.

4.3 Theorem P21.B2 (Collection Preservation Theorem)

Within the reduced scalar/longitudinal sector, every gauge-neutral collection $C_f[\Omega] = \int_{S^2} f(n)\Omega(n)d\Omega(n)$ of the one-slot primitive ray datum preserves the one-slot / degree-1 typing.

Proof. Collection coefficients f(n) and measure dΩ live in the gauge-neutral sky algebra. Gauge-neutral linear superposition of observables sharing the same single A-sensitive leg type does not create a new gauge-bearing leg. QED.

Status: DERIVED/THEOREM. Validated.

4.4 Theorem P21.B3 (Spherical-Harmonic Preservation Theorem)

The harmonic coefficients $a_{\ell m} = \langle Y_{\ell m}, \Xi\rangle$ preserve one-slot / degree-1 typing.

Proof. $a_{\ell m} = C_{Y^*_{\ell m}}[\Xi]$. By B2, gauge-neutral collection preserves typing. QED.

Status: DERIVED/THEOREM. Validated.

4.5 Theorem P21.B4 (Scalar-Sector No-New-Slot Theorem)

Within the reduced scalar/longitudinal sector, no second A-sensitive slot can appear. Four candidates eliminated: (1) local plasma — gauge-neutral; (2) S^2 geometry — gauge-neutral; (3) vector/tensor — excluded by scope; (4) nonlocal transport — none constructed, P6 blocks duplication.

Status: DERIVED/THEOREM. Validated.

4.6 Theorem P21.B5 (A4 Bridge Theorem)

Within the reduced scalar/longitudinal acoustic sector: direction-resolved collection and spherical decomposition preserve one-slot degree and introduce no second A-sensitive slot. The full primitive angular stage ω(n) → Ξ(n) → a_ℓm remains one-slot / degree-1.

Status: DERIVED/THEOREM. Validated.

4.7 Theorem P21.B6 (Primitive-Stage Inheritance Theorem for θ*)

Ξ and a_ℓm are one-slot / degree-1 (B5). C_ℓ is degree-2. Peak positions are invariant under uniform rescaling of C_ℓ. Therefore θ* inherits gauge content only from the primitive one-slot stage.

Status: DERIVED/THEOREM. Validated.

5. Part I Dead Routes

1. GTTP frequency channel: preserves ratios, cannot move peak lattice. 2. Degree-2 self-intensity: λC_ℓ does not move peak positions. 3. Denominator dressing: D_A gauge-neutral. 4. Local plasma insertion: gauge-neutral. 5. Universal bulk transport: Paper 16 P6. 6. Pure Paper 15 carrier numerology.

6. Part I Summary

AC1 is derived in sharpened composite form. Theorem 20.2 is upgraded to DERIVED/THEOREM under AH1–AH7 + P1–P6 + RT/BY + B1–B5 (AC1 removed). θ*_pred = 0.599° is theorem-grade. The 0.429% residual persists and localizes to J_θ; candidates are perturbation-level effects external to AC1.

Part II: The Radiation Investigation

SUPERSEDED BY PAPER 22 v1.3 - RATE-DRESSING RESOLUTION

Supersession note (v1.5): Part II below was structured around closing a v1.4 D/H tension at -3.9 sigma (chi^2 approximately 15.6) by hunting for a phenomenological excess radiation contribution DeltaN_eff approximately 0.37, with the nineteen-route investigation converging on the Absolute Local Casimir Load F_abs = 0.36879 (matching the target to 0.066%) as the strongest candidate quantity. Under v1.5 corrections (YPBBN -> YPCMB output, modern Paper 22 v1.3 amplitudes epsilon_w = 0.012301, epsilon_n = 0.023842, conventions v1 denominators), the v1.4 tension shrinks to D/H = -0.55 sigma (chi^2 = 0.80), removing the phenomenological need for additional radiation entirely. Paper 22 v1.3 (April 2026), Theorem 22.23, derives the active framework resolution: the j=1 weak channel and j=2 nuclear channel rate-dressing construction (epsilon_w = K_gauge x L_1, epsilon_n = (langle K rangle / 10) x L_2) on the Spatial Hodge Complex, using exactly the puncture-load values L_1 = 0.22417 and L_2 = 0.13805 from Paper 21's puncture-load construction. Paper 22's chi^2(D/H + Y_p) = 1.50 with zero fitted parameters is consistent with the v1.5 corrected scorecard. The architectural correction was that the L_1 and L_2 puncture loads enter BBN through rate dressing (modifying interaction cross-sections) rather than energy-density injection (modifying the Friedmann expansion rate). Part II's nineteen no-go results remain documented as structural constraints on the radiation-sector search space that informed the development of Paper 22's rate-dressing paradigm; the L_1 and L_2 puncture-load derivations remain load-bearing inputs to Paper 22 v1.3. The F_abs / DeltaN_eff hunt itself is superseded.

7. Motivation

Paper 20 identified $\Delta N\_eff \approx 0.37$ as a phenomenological target from the D/H tension (-3.9σ at imported N_eff = 3.044). The assembly gap (H_0 = 61.06, age = 15.07 Gyr on the bare branch [SUPERSEDED: bare branch retired by Paper 29; Schur gives H_0 = 68.91]) was a second open wound. Part II investigates the structural origin of the radiation deficit through systematic probing, discovering a zero-parameter prediction from the unreduced SU(2) boundary algebra.

8. Six Dead Ends

8.1 Assembly Gap Decoupling (Theorem 21.C)

On the torsion-Λ homogeneous branch, the assembly gap does not independently constrain ΔN_eff. Sensitivity $d \ln H_0\_obs / d \ln N\_eff = 4.96\times10^{-5}$. Achieving H_0_obs = 67.4 would require N_eff ≈ 6708. The assembly gap and D/H radiation deficit are decoupled problems.

Status: DERIVED/DIAGNOSTIC.

8.2 H_0 Observable-Class Jacobian (Theorem 21.A)

Within Papers 18–20, no additional observable-class Jacobian for H_0 follows beyond the already-selected optical boundary-Hamiltonian readout H_obs = Δ^(1/4) H_bare. The α-ladder lives on typed baryon-loading observables descending from the BDP transport class; no theorem transfers it to the Friedmann expansion slot.

Status: DERIVED/NO-GO.

8.3 Bogoliubov Dispersion (Theorems 21.Ba, 21.Bb)

On the exact homogeneous OS interior $ds^2 = a(\eta)^2[-d\eta^2 + d\chi^2 + \sin^2\chi\, d\Omega^2]$, source-free Maxwell transport is conformally invariant. The scale factor causes redshift but no mode squeezing.

Theorem 21.Ba (Conformal Transparency). Source-free Maxwell on exact 3+1 FRW/OS is conformally invariant; at the homogeneous level there is no conformal mode-squeezing sourced by the scale factor itself. Status: DERIVED/THEOREM (validated).

Theorem 21.Bb (No Dynamic Scattering). The stronger package $\beta_{dyn}(\omega) = 0$, $\Gamma_{dyn}(\omega) = 1$, $\Delta N_{eff} = 0$ requires the jump from conformal invariance to the full no-scattering conclusion. Paper 18 flags the full 3+1 mode-matching kernel as open. Status: DERIVED/NO-GO (not yet theorem-grade for the bundled claim).

8.4 V(α) Radiation Probe

The reduced gauge center Z_g is one-dimensional (Paper 18). V(α) governs the lens rules but not the bulk particle inventory. The scope-boundary result (Paper 20) holds: V(α) cannot determine N_{eff} internally.

Status: SCOPE-BOUNDARY (reconfirmed).

8.5 Baryon Freeze-In (Theorem 21.E)

Along the dust-path formula $f_b(\eta) = 2\gamma/x(\eta) = \gamma(1-\cos\eta)$ on the expanding OS cycloid, f_b increases forward in time as the universe expands. For any past freeze-in epoch $\eta^* < \eta_s$, $f_b(\eta^*) < f_b(\eta_s)$, yielding $\omega_b(\eta^*) < \omega_{b,geom}$. Since $\omega_{b,geom} = 0.02087$ is already 6.7% below the BBN target $\omega_b = 0.02237$, past freeze-in pushes ω_b further below the target. The sign reversal from the contracting convention is confirmed, but the route is dead: the geometric value is already below the target, so lowering it further worsens the gap. Codex verified: $f_b(z) = f_b(\text{now})/(1+z)$ exactly, so any early-epoch freeze-in suppresses ω_b by orders of magnitude (e.g., at $z_{eq} = 1758$, ω_b drops to $\sim 10^{-5}$).

Status: DERIVED/THEOREM (validated). The sign reversal is confirmed under the expanding convention. However, Codex computation shows the route is DEAD: $\omega_{b,geom} = 0.02087$ is already below the BBN target 0.02237, so past freeze-in (which lowers ω_b further) moves in the wrong direction relative to the target. The route is dead on both conventions: contracting gave the wrong sign; expanding gives the right sign but the geometric value is already too low.

8.6 Tensor Coupling Classification (Theorem 21.F)

In the puncture-resolved horizon algebra, j=2 is the first genuine rank-2 noncentral multiplet. $V_1 \otimes V_1 = V_0 \oplus V_1 \oplus V_2$; $\dim V_2 = 5$ (symmetric traceless rank-2). Under rotation-covariant boundary-to-bulk maps, j=2 classifies tensor/quadrupolar bulk data. After ABCK tracial averaging plus Schur reduction, j=2 data are erased. The only live tensor route is noncentral + nontracial + quadrupolar + nonthermal.

Status: DERIVED/THEOREM for the algebraic classification (validated). Scope: standard SU(2) representation theory; erasure claim is theorem-grade within the reduced central/scalar algebra, not yet a theorem about the full unreduced puncture-resolved IO algebra.

9. Cross-Pattern: What Survives

The six dead ends converge on a sharp conclusion: the reduced stack is exhausted for radiation. Every path that stays on the reduced algebra (Z_g, $M_{th} \otimes Z_g$, $V(\alpha)$) is provably dead. The radiation deficit must originate in the noncentral SU(2) structure — the $j \geq 1$ representations erased by tracial averaging.

10. The SU(2) Geometric Weight

10.1 Theorem 21.G (Temperature-Independence Theorem)

On the standard isolated-horizon puncture Hamiltonian $E_j = a_j c^4/(8\pi G\ell)$ with local Unruh temperature $k_BT = \hbar c/(2\pi\ell)$, the Boltzmann exponent is $\beta E_j = 2\pi\gamma\sqrt{j(j+1)}$, which is temperature-independent (the local scale ℓ cancels exactly). The $j \geq 1$ modes do not undergo thermal freeze-out on this Hamiltonian.

Status: DERIVED/THEOREM (validated). Scope: standard isolated-horizon puncture Hamiltonian + local Unruh temperature pairing.

10.2 Theorem 21.I (SU(2) Geometric Weight Theorem)

The one-puncture Gibbs weights $w_j = (2j+1)\exp[-2\pi\gamma\sqrt{j(j+1)}]$ are temperature-independent geometric constants. The physical puncture spectrum is $j \in \{1/2, 1, 3/2, \ldots\}$; $j=0$ is not a physical puncture state. The partition function $Z_{punc} = \Sigma_{j\geq 1/2} w_j = 1.4221$ is a geometric constant.

The energy-weighted fraction carried by the $j=1$ and $j=2$ noncentral modes:

$$f_E(\{1,2\}) = [\chi_1 \times 3e^{-\chi_1} + \chi_2 \times 5e^{-\chi_2}] / \Sigma_{j\geq 1/2}\, \chi_j(2j+1)e^{-\chi_j} = 0.3622$$

where $\chi_j = 2\pi\gamma\sqrt{j(j+1)}$.

This is within 1.7% of the D/H-derived ΔN_{eff} target 0.3685, with zero fitted parameters.

Adversarial validation clarification: four distinct quantities exist and must not be conflated. Plain partition weight ($j=1+2$) = 0.3466. Noncentral bookkeeping share = 0.5871. $(Z_{punc} - 1)/Z_{punc}$ = 0.2968. Energy-weighted fraction = 0.3622. Only the last is the correct internal energy comparator.

Status: DERIVED/THEOREM for the toy Gibbs spectrum and temperature-independence. STRUCTURAL/SUGGESTIVE for comparison to ΔN_{eff}. Validated.

10.3 Interpretation: Permanent Geometric Feature

The temperature independence is not a bug — it is a required feature of a holographic isolated horizon. The Unruh temperature and the area energy scale are both inversely proportional to the local geometric scale ℓ; their ratio must be purely topological. The 0.3622 extra weight is a permanent geometric feature of the full SU(2) boundary, present at all epochs. Note: this permanence is established for the boundary partition weight itself, not for a cosmological a^{-4} source term. Whether and how the boundary weight enters the bulk as effective radiation remains the open problem addressed in §§12A–12C.

11. The Optical Filtration Theorem

11.1 Theorem 21.J (Optical Filtration Theorem)

On the reduced RT/BY optical readout class, SU(2) averaging is a physical measurement filter.

Proof. Paper 17 constructs M_big (full pre-reduction algebra) and the gauge-averaging map E_g, giving $M_{big}^{SU(2)} = M_{th} \otimes Z_g$. The A-vacuum state satisfies $\omega_A = \omega_A \circ E_g$. For any gauge-neutral optical extension $A_{opt}^{gn} \otimes M_{big}$, with $E_{opt} = id_{A_{opt}^{gn}} \otimes E_g$, any SU(2)-invariant optical state satisfies $\omega(O) = \omega(E_{opt}(O))$. Therefore optical observables in this class depend only on $A_{opt}^{gn} \otimes M_{th} \otimes Z_g$ and are blind to noncentral SU(2) data. QED.

Combined with Paper 21 Part I (A4: sky-slot inheritance), Paper 20 measurement-chain theorem, and Paper 20 phase-calibration theorem: θ* is filtered to the reduced central optical sector and cannot directly see unreduced $j>0$ puncture weights.

Scope boundary: This is a theorem about the reduced RT/BY optical class, not a theorem that every conceivable optical observable belongs to that class.

Status: DERIVED/THEOREM within reduced RT/BY optical class. Validated.

11.2 The Measurement-Dependence Hypothesis (Intermediate — Later Constrained)

An initial working hypothesis: the resolution of the apparent paradox — permanent extra radiation entering BBN but not θ* — is measurement dependence rather than epoch dependence. Under this hypothesis, BBN would feel the full SU(2) boundary partition function while θ* would see only the $j=0$ central sector through optical filtration.

This hypothesis motivated the investigation in §§12A–12C. While the observable-class split between bulk and optical measurements survives as a structural principle (Paper 19), the strong form of measurement dependence — “the extra radiation is always in C_bare but the optical observer simply does not see it” — is later falsified by No-go 21.AB1 (§12B.3): persistent $\Delta N_{eff} \approx 0.37$ in C_bare shifts θ* by −1.56% through the expansion history, regardless of optical filtration. See §12C for the surviving specification.

11A. The BBN Branch Assignment

11A.1 Theorem 21.L (BBN Thermal Branch Assignment)

Under Paper 17 GTTP theorem, Paper 19 observable-class architecture, Paper 21 Theorem 21.J, and the minimal new premise TIO1 (BBN abundance/rate observables are local bulk thermodynamic plasma observables with no primitive RT/BY optical readout leg), GTTP belongs only to the RT/BY optical readout class. Local bulk thermodynamic observables are on the local/bare branch and are immune to the RT/BY optical GTTP correction.

Proof. GTTP was derived in Paper 17 as a theorem about boundary-photon readout, not as a universal replacement of every thermal scale. Papers 19 and 21 localize that machinery to the optical RT/BY class. Once BBN is typed as local bulk thermodynamics rather than optical readout, GTTP has no primitive slot on which to act. The BBN plasma state stays on the base KMS branch T_{IO}, with baryon loading on the Paper 19 $\alpha = 1$ rung. QED.

Corollary 21.L1. For the geometric candidate $N_{eff} = 3.4062$, the exact BBN scorecard evaluated on the T_{IO} branch (historical v1.4 scorecard under the superseded ΔN_{eff} framing; see Part II supersession note): D/H = -0.08 sigma, Y_p = +1.72 sigma, $\chi^2(D/H + Y_p) = 2.97$. On the class-mismatched T_{obs} branch: D/H = +9.83 sigma, $\chi^2 = 99.02$ - confirming T_{obs} insertion is a class error. The branch-mismatch demonstration (T_{IO} vs T_{obs} class distinction) is the load-bearing point of this corollary and is unaffected by the v1.5 supersession.

Scope boundary: This is a theorem about the reduced bulk-vs-optical class architecture. It does not assert that every pre-recombination quantity uses T_{IO}; it classifies BBN specifically as local bulk thermodynamics, not optical readout.

Status: CONDITIONAL/THEOREM at reduced scope. Validated.

11B. Advancing the Unreduced Algebra

11B.1 Theorem 21.O (Local Nontraciality)

On a bipartite puncture Hilbert space $H_A = \bigoplus_J (V_J \otimes M_A^J)$, $H_B = \bigoplus_J (V_J \otimes M_B^J)$, the reduced local state from the normalized projector onto the global invariant subspace is:

$$\rho_A = d_{inv}^{-1} \sum_J [m_B^J/(2J+1)]\, I_{V_J} \otimes I_{M_A^J}$$

with $d_{inv} = \sum_J m_A^J m_B^J$.

This is always SU(2)-invariant but is tracial ONLY if $m_B^J/(2J+1)$ is independent of J. Generically it is not. Therefore: global gauge invariance does NOT imply local traciality.

Physical meaning: The global boundary state satisfies the Gauss constraint (tracial/invariant). But the reduced density matrix at the BBN mean-free-path scale is generically nontracial. This opens a consistent route by which the BBN plasma could couple to nontracial local geometry, while the RT/BY observer sees the global invariant trace. This resolves the obstruction from Paper 20's tracial-state no-go: the GLOBAL state is tracial, but the LOCAL state is not.

Verified example: A = two spin-1/2 punctures, B = two spin-1/2 punctures: $\rho_A = (1/2)I_{V_0} \oplus (1/6)I_{V_1}$, which is invariant but not tracial.

Status: DERIVED/THEOREM for the mathematical result (validated). CONDITIONAL/BRIDGE THEOREM for the physical interpretation (the actual physical partition A|B and coupling to the BBN sector still need specification).

11B.2 Proposition 21.O1 (Gauge-Invariant Isotypic Projectors)

On the local isotypic decomposition, the projector $P_S = \Sigma_{J\in S} P_J$ commutes with SU(2) for any sector set S. Each isotypic block is invariant under the group action, with SU(2) acting irreducibly on V_J and trivially on M_A^J. Both $P_{int} = \Sigma_{J\in\mathbb{Z}\geq 1} P_J$ and $P_{\{1,2\}} = P_1 + P_2$ are positive gauge-invariant operators. The multiplicity operator M_{punc} is mathematically definable.

Status: DERIVED/THEOREM. Validated.

11B.3 No-Go 21.P (Wigner-Eckart Does Not Select {1,2})

Standard Wigner-Eckart theory does not derive $M_{punc} = P_{\{1,2\}}$. For a rank-k tensor operator, $\langle j'm'|T^{(k)}_q|jm\rangle$ vanishes unless $|j-j'| \leq k \leq j+j'$. For diagonal rank-2 ($j'=j$): requires $2j \geq 2$, so $j \geq 1$. This excludes $j=1/2$ but allows $j = 3/2, 5/2, \ldots$. Furthermore, the homogeneous Friedmann radiation slot uses the scalar T_{00} channel ($j=0$ under spatial rotations), not a pure rank-2 tensor. So $T_{\mu\nu}$ does not supply a {1,2} selector for background radiation.

Numerical confirmation: $f_E(\{1,2\}) = 0.3622$, $f_E(\text{integer } j\geq 1) = 0.4552$, $f_E(\text{rank-2 admissible } j\geq 1) = 0.7926$. No representation-theoretic cut reproduces 0.3622.

Status: DERIVED/NO-GO (validated). Kills the specific claim that Wigner-Eckart / tensor-rank projection explains the {1,2} selection.

11B.4 P_multlift Decomposition

The bridge from unreduced puncture partition function into the bare Friedmann radiation slot decomposes as:

$P_{multlift} = P_{locred} + P_{resp}$

P_{locred} (local nontraciality): CLOSED (Theorem 21.O).

P_{resp} (positive response map from local puncture sectors into scalar bare radiation slot): OPEN. Three selection-rule attempts documented and killed: metric tensor-product (No-go 21.M1 — metric is singlet), Wigner-Eckart (No-go 21.P — allows $j=3/2,5/2,\ldots$), Weinberg-

Witten (No-go 21.R — curved spacetime, wrong premises). The {1,2} selection remains STRUCTURAL/SUGGESTIVE.

The frontier is now: P_resp = P_spinmap + P_fieldscope + P_weight, where P_fieldscope is partially constrained (only j=1 and j=2 geometric bulk channels exist on the IO stack) and P_spinmap + P_weight remain open.

12. No-Go 21.K: The Remaining Gap

The full synthesis “BBN sees 0.3622 while θ* does not” is not yet theorem-grade. The optical half (Theorem 21.J) is closed at reduced scope. The bulk-coupling half requires:

P_bulkfull: Local bulk thermodynamic source terms couple linearly to unreduced puncture Gibbs energy before optical reduction, with a gauge-invariant stress-energy map into the bare radiation slot.

This is the reduced → full algebra extension problem in bulk-thermodynamic language. It is narrower in statement than the full extension, but tightly entangled with the same open machinery: full puncture-resolved algebra/state, nontracial puncture state, physical Hamiltonian, and the actual radiation map (adversarial assessment).

Status: DERIVED/NO-GO DIAGNOSTIC. Validated.

12A. The Absolute Local Casimir Load

12A.1 The Optical Filtration Functional (Failed Construction — see 21.Y/21.Z)

An initial construction attempt: define ΔN_eff via the optical filtration difference:

F_filt = Tr[ρ_loc (I − E_opt)(H_punc h(C_2))] / Tr[ρ_loc H_punc]

The motivation: since E_opt kills all J>0 and the physical puncture spectrum starts at J=1/2, this would reduce to the absolute local Casimir load. However, Theorems 21.Y and 21.Z (§12B.1–12B.2) prove this construction fails: H_punc h(C_2) is a spectral function of C_2 and therefore already lies in the puncture fixed-point algebra. The conditional expectation E_punc preserves it identically, so (I − E_punc)(H_punc h(C_2)) = 0. The filtration mechanism cannot distinguish this operator from optically reduced content. The numerical result F_abs = 0.36879 survives as an absolute local load (§12A.3) but not through this filtration route.

12A.2 Theorem 21.X (λ=4 Conditional Uniqueness)

Inside the bounded resolvent family h_λ(C_2) = C_2/(λ+C_2), imposing the small-load scalarization rule h(C) = C/D + O(C^2) for descent into the D-dimensional Friedmann scalar slot forces λ = D. The stack fixes D = 4 through the four-dimensional history-to-Hamiltonian descent (K_geom = 4 ln x). Status: CONDITIONAL/THEOREM. Validated.

12A.3 The Construction Hit

With $h(C_2) = C_2/(4+C_2)$ on the full $j \geq 1/2$ spectrum with local denominator:

$F\_abs = Tr[\rho\_loc\ H\_punc\ C_2/(4+C_2)] / Tr[\rho\_loc\ H\_punc] = 0.36878514007842433$

Target $\Delta N\_eff = 0.368541479723$. Miss: 0.066%.

This is the tightest match in the framework. The {1,2} restriction HURTS (gives 0.1445). The near-hit requires the full spectrum. The bounded Casimir response naturally weights: j=1/2 is suppressed ($C_2=0.75$ → small weight), higher j get larger Casimir weight but exponentially suppressed by the Gibbs factor.

Previous near-hits are shadows of the same family: $\lambda(f\_b + K\_gauge) = 4.025$, $\lambda(f\_E(\{1,2\})) = 4.140$.

Status: STRUCTURALLY SUPPORTED CONSTRUCTION. Validated.

12A.4 BBN Scorecard at F_abs

On the theorem-assigned T_IO = 2.6635 K branch with ω_b = 0.02108: N_eff = 3.044 + 0.36879 = 3.41279. D/H = +0.11σ, Y_p = +1.74σ, Li7 = +12.3σ, χ^2(D/H + Y_p) = 3.05. Status: CONDITIONAL/COMPUTATIONAL SCORECARD (validated).

12B. The Response Map: Nineteen Dead Routes

12B.1 Theorem 21.Y (Puncture Fixed-Point Algebra)

On the local puncture decomposition H_A = ⨁_J (V_J ⊗ M_A^J), the fixed-point algebra is B(H_A)^{SU(2)} = ⨁J (I{V_J} ⊗ B(M_A^J)). Any spectral function of $C_2 = J(J+1)$ lies in the fixed-point algebra. Status: DERIVED/THEOREM. Validated.

12B.2 No-Go 21.Z (Optical-Kill)

E_punc(H_punc $h(C_2)$) = H_punc $h(C_2)$, therefore (I − E_punc)(H_punc $h(C_2)$) = 0. The optical filtration functional F_filt = 0.36879 is not theorem-grade in its (I−E_opt) form. Spectral functions of C_2 already lie in the puncture fixed-point algebra. Status: DERIVED/NO-GO. Validated.

12B.3 No-Go 21.AB1 (θ* Insulation Failure)

If ΔN_eff ≈ 0.37 persists as a permanent addition to C_bare through recombination, θ* shifts by −1.56%, destroying the Part I precision. Conformal transparency of null transport does not insulate θ* once the component enters the background expansion history. Status: DERIVED/NO-GO. Validated.

12B.4 No-Go 21.AD (Conformal Dressing Equation of State)

Conformal geometric dressing does not define a positive a^{-4} radiation slot. Paper 20’s conformal-stress result gives a^{-8}, w = 5/3. Status: DERIVED/NO-GO. Validated.

12B.5 No-Go 21.AF (Broad Admissibility)

Broad admissibility axioms do not uniquely select $h(C_2) = C_2/(4+C_2)$. Counterexamples: $1-\exp(-C_2/4) \to 0.449$, $\tanh(C_2/4) \to 0.511$. Uniqueness holds only within the bounded resolvent family (Corollary 21.AF1). Status: DERIVED/NO-GO. Validated.

12B.6 Theorem 21.AG (Separable Dressing Collapse)

Any separable scalar local reaction dressing Γ_dress = $\Gamma_0 \otimes G$ on H_IO^loc collapses under the local state to a uniform weak-rate factor Γ_eff = s × Γ_0. Status: CONDITIONAL/THEOREM. Validated.

12B.7 No-Go 21.AG (Uniform Rate Suppression)

Exact PRyMordial BBN kills the entire uniform-s family. At the D/H-matching s = 0.892, Y_p = +5.73σ. Uniform scalar dressing does not mimic ΔN_eff: it produces the wrong abundance pattern because Γ suppression and H enhancement are kinetically non-isomorphic. Status: COMPUTATIONAL NO-GO. Validated.

12B.8 Additional Results (Conformal and Interaction Routes)

Conditional 21.AA (scoped): Conformal metric-leg coupling not derived; if assumed, the induced dressing is conformal. Conditional 21.AC (scoped): Massive BBN can respond to conformal dressing in principle, but this does not prove N_eff = 3.044 + F_abs. Stiff a^{-8} dilution route: correct θ* insulation (residual 10^{-28} at recombination) but catastrophic BBN time profile (ΔN_eff,eff = 3840 at 10 MeV, correct only at 1 MeV, gone by 0.1 MeV). Local expansion variance: ρ_rad/ρ_total = 1 throughout BBN, so the tracking factor collapses to a constant √(1+F_abs) giving ΔN_eff,eff = 2.75, far too large. Chiral g_A modification: wrong D/H direction. Phase-space Jacobians: catastrophic or flat. G_F shift: equivalent to uniform scaling. Geometric chemical potential: ±20σ failures.

12B.9 Complete Kill List

Nineteen routes tested and killed: (1) Assembly gap ΔN_eff — decoupled. (2) V(α) radiation — scope boundary. (3) Bogoliubov dispersion — homogeneous conformal probe kills scale-factor-sourced dispersion (21.Ba); full 3+1 mode-matching remains open (21.Bb). (4) H_0 Jacobian — no transfer theorem. (5) Baryon freeze-in — sign REVERSED under expanding convention but route DEAD (see §8.5). (6) Metric tensor-product — metric is singlet (21.M1). (7) Wigner-Eckart — allows j≥1 (21.P). (8) Weinberg-Witten — curved spacetime (21.R). (9) Optical filtration (I−E_opt) — operator in fixed-point algebra (21.Z). (10) Conformal θ* insulation — breaks acoustic scale by 1.56% (21.AB1). (11) Conformal a^{-4} sourcing — gives a^{-8} (21.AD). (12) Broad admissibility uniqueness — counterexamples (21.AF). (13) Uniform local rate suppression — Y_p blows up (21.AG). (14) Stiff a^{-8} dilution — correct θ* insulation, catastrophic BBN time profile. (15) Local expansion variance — tracking factor collapses, ΔN_eff too large. (16) Chiral g_A modification — wrong D/H direction. (17) Phase-space Jacobians — catastrophic or flat. (18) Effective G_F shift —

equivalent to uniform scaling. (19) Geometric chemical potential — lepton asymmetry, ±20σ failures.

12C. The Surviving Specification

The nineteen no-gos (eighteen P_resp routes plus freeze-in) constrain what the response map MUST look like:

Must enter through H(z): Every weak-sector modification fails. Only genuine energy density in the expansion rate produces the correct D/H + Y_p correlation.

Must be approximately constant during BBN: The a^{-8} route fails because it spikes at 3840 ΔN_eff at 10 MeV and collapses through the BBN window.

Must decouple before recombination: The θ* insulation no-go (21.AB1) forbids any permanent addition to C_bare.

Must be non-separable and channel-dependent: The uniform rate collapse theorem (21.AG) kills all separable scalar dressings.

Must come from the puncture geometry: The number F_abs = 0.36879 is derived from the locally nontracial partition function with zero parameters.

One surviving Paper 22 target:

Non-separable radiation response: A channel-dependent, energy-selective operator on A_weak ⊗ B(H_A)^{SU(2)} that maps local nontracial puncture data into an effective BBN-era source which naturally decouples before recombination.

13. Connection to Open Foundation Problems

13.1 The Reduced / Unreduced Boundary

Part I sharpens the scope wall: scalar acoustic modes close on the reduced stack; vector/tensor require the noncentral SU(2) structure. Part II confirms: the reduced stack is exhausted for radiation; the 0.3622 weight lives in exactly the noncentral sector the reduction erases.

13.2 The Reduced → Full Algebra Extension

Paper 20 identified three premises needed for ΔN_eff: P_fullgrav, P_puncham, P_radmap. Paper 21 substantially advances this program: local nontraciality is proved (21.O), the puncture fixed-point algebra is characterized (21.Y), and the strongest numerical candidate F_abs = 0.36879 is derived from the full-spectrum bounded Casimir response (0.066% from target). The earlier j=1+2 partition fraction 0.3622 (1.7% from target) is now understood as a precursor shadow of the same family. The remaining target is the response map P_resp: a non-separable, channel-dependent, energy-selective operator that delivers F_abs to BBN observables without entering C_bare as permanent radiation. Eighteen P_resp routes have been killed; freeze-in reversed sign under the expanding convention but is independently dead (ω_b,geom already below target). These nineteen results precisely constrain what P_resp must look like.

13.3 The Assembly Gap

The assembly gap (H_0 = 61.06, age = 15.07 Gyr) [SUPERSEDED: bare branch retired by Paper 29; root cause was a normalization error] is confirmed decoupled from ΔN_eff and from the H_0 observable-class Jacobian. Paper 29 resolves the assembly gap through the conditional Schur branch (H_0 = 68.91).

13.4 Conformal Transparency

Theorem 21.Ba proves that source-free Maxwell on the exact 3+1 FRW/OS interior is conformally invariant at the homogeneous level, retroactively validating Paper 18's Bogoliubov Spectrum Theorem assumption that scale-factor-sourced mode squeezing vanishes. The full 3+1 mode-matching kernel (including dynamic backscattering) remains open (21.Bb).

14. Open Problems

Active unresolved problems.

1. R4 normalization (Paper 17): load-bearing for GTTP. [Pending verification]

Open Problems Closed in This Paper

1. AC1 acoustic ruler premise (Paper 20 Open #2): Closed by AC1 Reduction Theorem (P21.A5). J_θ = x^{−1/2}√(1+γ^2) now theorem-grade.

Appendix A: The Complete Derived Foundation

Master mathematical reference for the IO framework (Papers 1–21). Carried forward from Paper 13 Appendix A, updated with Paper 21 results. Theorem-grade entries only. All results classified by the status system of §1.4.

A.0 Master Mathematical Reference

All numerical values, organized by derivation chain. Updated through Paper 21.

Formula / Quantity	Value	Description	Source
M_U	4.50×10^{53} kg	Measured input	Paper 1

γ_{BI}	0.2375	Barbero-Immirzi (LQG)	External
R_U	4.40×10^{26} m	Observable universe radius	Paper 1
CMP: $\langle K\rangle = \ln \Delta$	—	DERIVED	Paper 10
BDP: $f_b = 2\gamma/x$	—	DERIVED	Paper 12
GTTP: $\sigma = K_{gauge}$	—	DERIVED	Paper 13
$r_s = 2GM_U/c^2$	6.685×10^{26} m	Schwarzschild radius	Paper 1
$x = r_s/R_U$	1.51899	Compactness ratio	Paper 1
$\tau_{total} = \pi r_s/(2c)$	111.0 Gyr	Total expansion half-cycle	Paper 1
$\Delta = x^4(1+\gamma^2)$	5.6240	Spatial decoupling	Papers 8/9
$\sqrt{\Delta}$	2.3715	Matter projection	Paper 9
$\langle K\rangle = \ln \Delta$	1.72705	Total modular energy	Paper 10
$K_{geom} = 4 \ln x$	1.67199	Geometric modular (96.8%)	Paper 10
$K_{gauge} = \ln(1+\gamma^2)$	0.05487	Gauge modular (3.2%)	Paper 10
$N_{horizon} = 2/\sqrt{e}$	1.2131	Horizon Kruskal factor	Paper 2
$\gamma = \sqrt{r_s/l_P}$	6.43×10^{30}	Amplification factor	Paper 2
T_{IO}	2.6635 K	Interior Hawking temp	Paper 1
$\lVert K\rVert = \lVert\Gamma\rVert = 1/r_s$	—	Equal spectral norms	Paper 9
$K \perp \Gamma$	—	Orthogonality	Paper 9
A_{tan} eigenvalues	$(-\gamma\pm i)/r_s$	Two gauge channels	Paper 9
$f_b = 2\gamma/x$	0.31271	BDP baryon fraction	Paper 12

$F = \langle K\rangle x/(8\gamma)$	1.3807	Rational correction	Paper 12
$f_b \times F = \langle K\rangle/4$	0.43176	Exact identity	Paper 12
$\omega_{b,eff}$	0.02910	Acoustic baryon loading	Paper 12
α-ladder	$\alpha = 1$	1-form, unique D/H match	Paper 12
$P_k = \exp(-\Delta/2)$ [Schur]	0.0601	Paper 11	
$P_k = \Delta^{(-1/x^2)}$ [i.i.d.]	0.4731	[ACTIVE — THEOREM]	Paper 10
$\Omega_{k,obs}$	−0.046	Projected curvature	Papers 10/11
H_0,obs (P10 map)	67.58 km/s/Mpc [ACTIVE — THEOREM]	Paper 10 projection	Paper 10
H_0,obs (Schur)	68.91 km/s/Mpc	Schur-based [CONDITIONAL]	Paper 11
θ^*	0.5962°	−0.5σ	Paper 12
ℓ_1	220.0	exact	Paper 12
R (shift ratio)	1.7489	−0.3σ	Paper 12
$D/H \times 10^{-5}$	2.51	-0.55 sigma	Paper 12
Y_p	0.2478	+0.70 sigma	Paper 12
BAO χ^2	19.8	< ΛCDM (22.0)	Paper 12
$T_{obs} = T_{IO} \times x^{K_{gauge}}$	2.7253 K	GTTP temperature	Paper 13
$x^{K_{gauge}}$	1.02320	Gauge thermal factor	Paper 13
$V(\alpha) = -2 \ln(\cos \alpha)$	—	Kähler potential on CP^1	Paper 13
$\gamma_{required}$ (FIRAS)	0.23789	0.16% from canonical	Paper 13
$n_{transfer} = 1$	51.7σ margin	Transfer ladder	Paper 13

$a = \dim(S^2)/2$	1	Normalization	Paper 13
$C_2(A)/C_2(\Gamma)$	$1+\gamma^2$	Casimir gauge invariance	Paper 14
$A\_S^T A\_S$ eigenvalue	$(1+\gamma^2)/r\_s^2$	Degenerate ($J^2=-I$)	Paper 14
$N(\theta^*=0.5962°)$	5.634	0.18% from Δ	Paper 14
$\exp(K\_gauge/2)$	1.02782	$\neq$ x^K_gauge (1.02320)	Paper 14
$\Lambda \times r\_s^2$	$9\pi^2/[4(1+\gamma^2)]$	Universal (x-indep)	Paper 14
$f\_b \times x = 2\gamma$	0.475	Universal (x-indep)	Paper 14
σ = K_gauge	—	DERIVED (Theorem 17.1; R4 premise)	Paper 17

A.1 Detailed Derivation Catalog

Complete step-by-step derivation chain. Steps 1–41 from Papers 1–13 (carried forward from Paper 13 Appendix A). Steps 42–56 are Paper 14 new results. Steps 57–65 are Paper 15 new results. Steps 66–76 are Paper 16 new results. Steps 77–84 are Paper 17 new results. Steps 85–96 are Paper 18 new results. Steps 97–105 are Paper 19 new results. Steps 106–118 are Paper 20 new results. Steps 119–136 are Paper 21 new results. Each step includes formulas, physical context, verification counts, and status label.

A.1.1 The Geometric Chain (Papers 1–3)

Step 1: Schwarzschild radius (Paper 1 §2.1). $r\_s = 2GM\_U/c^2 = 6.685 \times 10^{26}$ m. The observable universe sits within a factor 1.5 of its own Schwarzschild radius. STATUS: DERIVED (standard GR, exact)

Step 2: OS interior metric (Paper 1 §2.4). $ds^2 = -d\tau^2 + a^2(\eta)[d\chi^2 + \sin^2\chi\, d\Omega^2]$, $a(\eta) = (r\_s/2)(1 - \cos\eta)$. Spatial curvature k = +1 (closed). $g\_\tau\tau = -1$ everywhere: all gravitational effects are encoded in the scale factor, not the lapse. STATUS: DERIVED (Oppenheimer-Snyder 1939, exact)

Step 3: Total expansion half-cycle (Paper 1 §2.2). $\tau\_total = \pi r\_s/(2c) = 111.0$ Gyr. Current age as fraction: 13.8/111.0 = 12.4% (early, gentle regime). STATUS: DERIVED

Step 4: Current epoch — two constraints (Paper 1, Paper 2 §8). Temporal: $\eta\_t = 1.371$ (from $\tau(\eta\_t) = 13.8$ Gyr). Spatial: $\eta\_s = \arccos(1 - 2R\_U/r\_s) = 1.893$ rad (from $R(\eta\_s)$ =

R_U). These are DIFFERENT η values. The OS cycloid has one degree of freedom. This over-determination is the Space-Time Decoupling (§A.1.2). STATUS: DERIVED

Step 5: Compactness ratio (Paper 1 §2.1). $x = r_s/R_U = 1.51899$. STATUS: DERIVED

A.1.2 The Space-Time Decoupling (Paper 2 §8)

Step 6: Decoupling ratio (Paper 2 §8.3). $\Delta_{cycloid} = \tau(\eta_s)/t_{obs} = (\eta_s - \sin \eta_s)/(\eta_t - \sin \eta_t) = (1.893 - 0.947)/(1.371 - 0.980) = 0.946/0.391 = 2.418$. Does NOT match $\Delta_{geometric} = x^4(1+\gamma^2) = 5.624$. The temporal link is withdrawn under the expanding convention. STATUS: DERIVED

Step 7: Observer's time dilation. $dt/d\tau|_{R=R_U} = 1/\Delta = 0.1778$. The cosmological clock runs at 17.78% of the interior proper time. STATUS: DERIVED

Step 8: Kruskal lapse (Paper 2 §6.1). $\lim_{\eta \to \pi^-}[\tilde{\tau}/T_K] = 2/\sqrt{e} = 2e^{(-1/2)} \approx 1.2131$. Proof: Taylor expansion of OS cycloid in Kruskal coordinates. SymPy residual = 0. Zero free parameters. STATUS: DERIVED (exact)

A.1.3 The Holographic Amplification Chain (Papers 1–2)

Step 10: Carlip-Virasoro derivation (Paper 2 §4). Central charges of horizon Virasoro algebra (Carlip 1998, 1999): $c_{3D} = 6r_s^2/l_P^2$ (full 2-sphere, all angular modes), $c_{1D} = 6r_s/l_P$ (radial sector, s-wave reduction). Interior dimensional reduction: r is timelike inside horizon; observer counts modes along time axis. $T_{IO}/T_H = \sqrt{(c_{3D}/c_{1D})} = \sqrt{(r_s/l_P)} = \gamma = 6.431 \times 10^{30}$. SymPy residual = 0. 35/35 verification checks. STATUS: DERIVED

Step 11: Interior temperature (Paper 1 §4.1). Full position-dependent formula: $T(r) = \hbar c\gamma/(4\pi r\, k_B)$. At R_U: $T_{IO} = \gamma \times T_{Hawking} \times x = 2.6635$ K. Geometric mean identity: $T_{IO}^2 = T_{Hawking} \times T_{Planck}$ (cross-model confirmed, Haug-Tatum 2024). $T(z) = T_{IO} \times (1+z)$, identical to ΛCDM. STATUS: DERIVED

Step 12: Greybody factor Γ = 1 (Paper 1 §5.3). Interior Regge-Wheeler potential $V_l(r) = f(r)[l(l+1)/r^2 + (1-s^2)r_s/r^3]$ where $f(r) = 1 - r_s/r$. For $r < r_s$: $f(r) < 0$, so $V_l < 0$ for all r, all spins $s \leq 1$. No barrier — all modes propagate freely. Interior radiation is exact blackbody: $\Gamma(\omega) = 1$. Explains COBE/FIRAS perfection (< 50 ppm). STATUS: DERIVED (theorem)

Step 13: Cosmological invariant (Paper 3 §4). $T_{CMB} \times R_U = \hbar c\gamma/(4\pi k_B) = 1.172 \times 10^{27}$ K·m. Algebraic identity holding at every epoch — verified across five spot-checks. STATUS: DERIVED

Step 14: Continuity Theorem (Paper 3 §5.3). For all $\tau_{obs} \in (0, \tau_{total})$, the framework returns complete self-consistent observables varying smoothly. Proof:

composition of three analytic maps. The observer coordinate is NOT fine-tuned. STATUS: DERIVED (theorem)

Step 15: No-Go for semiclassical QFT (Paper 2 §3). Semiclassical QFT on stationary Schwarzschild CANNOT produce T_IO. Five candidate mechanisms examined and excluded (Bogoliubov coefficient analysis, 36/36 verification checks). The 30-order gap requires quantum gravitational physics. STATUS: DERIVED (theorem)

A.1.4 Dark Energy (Papers 1, 8)

Step 16: Torsion cosmological constant (Paper 1 §6). From Einstein-Cartan-Holst action with fermion torsion coupling: $L_{int} = -(3\kappa/16)[\gamma^2/(1+\gamma^2)](\bar{\psi}\gamma_5\gamma_a\psi)^2$. Holographic spin density: $s = 3\hbar/(8l_P^2\ \gamma\ r_s)$. Resulting $\Lambda = 9\pi^2/[4r_s^2(1+\gamma^2)]$. With position correction: $\rho_{\Lambda,eff} = \rho \times x^2 = 6.05 \times 10^{-27}$ kg/m³ (observed: 5.96, error 1.5%). STATUS: DERIVED

Step 17: γ_BI-free derivation (Paper 2 §7.1). $\Lambda_{IO} = 3(H_0^2/c^2 + 1/R_U^2 - r_s/R_U^3)$. Contains NO γ_BI. Matches Λ_obs to 2.4%. STATUS: DERIVED

Step 18: Self-consistency at horizon (Paper 1 §8.2). σ_junction = σ_torsion EXACTLY (algebraic identity for any Λ). Energy parameter $e^2 = 1 + \Lambda_{eff}\, r_s^2/3 = 8.28$. STATUS: DERIVED (exact)

A.1.5 The Spectral Norm Chain (Paper 9)

Step 19: Ashtekar-Barbero connection (Paper 9). $A = \Gamma + \gamma_{BI} K$ in PG slicing. $K \perp \Gamma$ (Pythagorean theorem: K and Γ occupy perpendicular subspaces, verified by explicit SVD). $||A||/||\Gamma|| = \sqrt{(1+\gamma^2)}$ in PG. Paper 14 confirms: frame-dependent in 3D, but S^2-restricted quantities give $(1+\gamma^2)$ universally regardless of slicing. STATUS: DERIVED (Paper 14 upgrades to intrinsic formulation)

Step 20: Matter projection (Paper 9). $\sqrt{\Delta} = x^2 \times \sqrt{(1+\gamma^2)} = x^2 \times ||A||/||\Gamma|| = 2.3715$. Applies to ALL energy terms (matter and bare Λ) because both are sourced by interior physics projected through the horizon's gauge connection. STATUS: DERIVED

Step 21: Spatial decoupling (Papers 2, 8, 9). $\Delta = x^4(1+\gamma^2) = (\sqrt{\Delta})^2 = 5.624$. Single route: spectral norm squared (Paper 9). The former K_Λ correction route (Paper 8) is retired (Paper 9 v1.5 ansatz removal). The cycloid integral route ($\Delta_{cycloid} = 2.418$) does not match under expanding convention. STATUS: DERIVED (multiply confirmed)

A.1.6 The Modular Energy Chain (Paper 10)

Step 22: Total modular energy (Paper 10, CMP). $\langle K\rangle = \ln \Delta = \ln[x^4(1+\gamma^2)] = 4 \ln x + \ln(1+\gamma^2) = K_{geom} + K_{gauge} = 1.7271$. The Conformal Modular Principle: identification of horizon's total modular energy with ln Δ. STATUS: DERIVED (CMP)

Step 23: K_gauge derivation (Paper 10 §10.1). On S^2 horizon: A_tan = $(-\gamma I + J)/r\_s$ where $J^2 = -I$ (SO(2) generator). Covariant Laplacian: $-\Delta\_A|\_tan = (1+\gamma^2)/r\_s^2 \times I_2$. Gaussian determinant ratio: K_gauge = $-\ln(Z\_A/Z\_\Gamma) = \ln(1+\gamma^2) = 0.0549$. STATUS: DERIVED (algebraic identity)

Step 24: K_geom derivation (Paper 10 §10.2). K_geom = $\ln[a(\pi)^4/a(\eta\_s)^4] = 4 \ln(r\_s/R\_U) = 4 \ln x = 1.6725$. From conformal 4-volume measure $\sqrt{(-g)} = a^4 \times \sqrt{(-\tilde{g})}$. Power 4 = 3 (spatial S^3) + 1 (conformal lapse: $d\tau = a\, d\eta$). Boltzmann counting: $\Omega(a) \propto a^4$, modular energy = entropy difference $\Delta S\_B = 4 \ln x$. STATUS: DERIVED via elimination (three supporting arguments)

Step 25: No-Go Theorem (Paper 10 §10.3). Nine independent QFT approaches computed; all yield zero or wrong values for K_geom. Deepest reason: conformally coupled scalar $\chi = a\varphi$ on OS interior satisfies free wave equation on static Einstein universe $R \times S^3$. Conformal vacuum $|0\rangle$ is time-independent in conformal time. Relative entropy = 0. K_geom is a gravitational measure effect, not a matter-field quantity. STATUS: DERIVED (theorem). CRITICAL: conformal fields do NOT acquire K_geom.

A.1.7 The Curvature Suppression (Papers 10–11)

Step 26: P_k = $\Delta^{(-1/x^2)} = 0.4731$ from i.i.d. modular Hamiltonian (Paper 10 v2.0 §5.3). Cumulant expansion in thermodynamic limit. STATUS: THEOREM.

Step 27: Schur complement formula f_eff = $N/(2 \ln \Delta)$, P_k = $\exp(-N/2)$. STATUS: DERIVED (theorem).

A.1.8 The Projection Map (Paper 10)

Step 28: f = $R\_U^2/r\_s^2 = 1/x^2 = 0.4334$ (observer's accessible fraction) — DERIVED

Step 29: H_0,geom = 58.4 km/s/Mpc; $\Omega\_m = 0.197$, $\Omega\_k = -0.130$, $\Omega\_\Lambda = 0.933$ — DERIVED (standard Friedmann on OS interior)

Step 30: Projection map: $N = \Omega\_m\sqrt{\Delta} + \Omega\_k\, P\_k + \Omega\_\Lambda\sqrt{\Delta}$; H_0,obs = H_0,geom $\times \sqrt{N}$ — DERIVED (Paper 10 §9)

A.1.9 The Baryon Dictionary (Paper 12)

Step 31: BDP: f_b = $2\gamma/x = 0.3127$. Baryons = gauge-coupled dust — DERIVED (BDP)

Step 32: α-ladder: discrete geometric exponents. D/H selects $\alpha = 1$ (1-form). — DERIVED (uniqueness by elimination)

Step 33: Master identity: f_b × F = $\langle K\rangle/4$ (exact) — DERIVED (algebraic identity)

Step 34: Complete BBN scorecard (v1.5 corrected): D/H at -0.55 sigma, Y_p at +0.70 sigma, chi^2 = 0.80 - DERIVED. (v1.4 reported D/H at -1.2 sigma, Y_p at +0.6 sigma using YPBBN output and pre-Paper-22-v1.1 amplitudes.)

A.1.10 The Gauge Thermal Transfer (Paper 13)

Step 35: GTTP: d(ln ω)/dλ = K_gauge in conformal coordinate λ = ln(r/R_U). T_obs = T_IO × x^K_gauge = 2.7253 K — DERIVED (Theorem 17.1)

Step 36: Temperature Transfer Theorem: five structural pieces forcing T_obs = T_IO × x^K_gauge:

Piece 1: Transfer ladder n = 1 (51.7σ over nearest competitor) — DERIVED (uniqueness by elimination)

Piece 2: KMS rigidity (only uniform rescaling preserves Planck form) — THEOREM

Piece 3: Segment additivity (Cauchy equation forces Φ(λ) = σλ) — THEOREM

Piece 4: Source selector (σ = K_gauge, unique survivor of conformal no-go) — DERIVED (Theorem 17.1)

Piece 5: Exact identity ln(T_obs/T_IO) = K_gauge × ln(x) — DERIVED (algebraic identity)

Step 37: V(α) = −2 ln(cos α), α = arctan(γ_BI). Fubini-Study Kähler potential on CP^1 — DERIVED

Step 38: V-hierarchy: exp(V) = $1+\gamma^2$ (Paper 9), V′′ = $2(1+\gamma^2)$ (Paper 10), V′ = 2γ (Paper 12), V = K_gauge (Paper 13) — DERIVED

Step 39: γ_BI from FIRAS inversion: γ_required = 0.23789 (0.16% from canonical). EPRL γ = 0.274 excluded at 33σ — DERIVED

Step 40: Normalization a = $\dim(S^2)/2 = 1$ from S^2 topology × Gaussian determinant exponent — DERIVED

Step 41: Eight dead mechanisms excluded (modular flow, additive constant, modified dispersion, bulk norm, wave equation, surface gravity, mode normalization, path holonomy) — DERIVED (systematic elimination)

A.1.11 Paper 14 New Results: Kill Shot B (Frame Covariance)

Step 42: 3D spectral norm ||A||/||Γ|| = $\sqrt{(1+\gamma^2/\alpha^2)}$ is FRAME-DEPENDENT (varies with slicing parameter α). PG (α = 1) is special, not universal — THEOREM

Step 43: S^2-restricted quantities give $(1+\gamma^2)$ regardless of slicing. K_tan = −I/r_s is the null second fundamental form K^(n)_AB = −(1/r_s)q_AB — THEOREM

Step 44: A_S = (−γI + J)/r_s admits horizon-intrinsic formulation via null geometry + γ — THEOREM

Step 45: Five-step slicing-invariance proof: q_AB intrinsic → Γ_S foliation-free → K^(n) 4D-covariant → A_S foliation-free → K_gauge foliation-free — THEOREM

Step 46: SU(2) gauge invariance via Casimir: $C_2(A)/C_2(\Gamma) = 1+\gamma^2$ for all three su(2) directions — THEOREM

Step 47: Uniqueness theorem: $A\_S^T A\_S = (1+\gamma^2)/r\_s^2 \times I_2$ from $J^2 = -I$. Any dimensionless, scale-independent, SU(2)-gauge-invariant scalar from pulled-back horizon connection data is $f(1+\gamma^2)$ — THEOREM

Step 48: Three covariant routes to $\sqrt{\Delta}$ without 3D PG norm: Gaussian determinant, curvature ratio, Laplacian eigenvalue — all give 2.37150358 — THEOREM

Step 49: Fifteen competing mechanisms systematically eliminated (Lambda_CS, torsion propagator, geometric mean of Lambdas, pair creation with ±K_gauge/2, and eleven others) — DERIVED (systematic elimination)

Step 50: Interior/exterior asymmetry: observable corrections are interior projection effects. No-hair theorem blocks exterior access to R_U,int. Cross-cancellation of x_ext and x_int blocked — DERIVED

Step 51: σ = K_gauge identification: analogous to β = 2π/κ in Hawking temperature in the limited sense that both equate a geometric c-number with a thermal observable, but requires the A-B gauge sector beyond standard BW — DERIVED (Theorem 17.1)

A.1.12 Paper 14 New Results: Kill Shot A (N_eff = Δ Closure)

Step 52: N = exp(⟨K⟩) is a normalization identity (Z = Tr(ρ) = 1), not a derivation — DERIVED (normalization identity)

Step 53: Self-consistency fixed point dead. Age loop trivially flat (Δ_output varies 0.31% across N = 1–50) — DERIVED (no-go)

Step 54: Schur inversion N = P_k^(−1/f) = Δ is a tautology on the i.i.d. formula (which gives Δ by construction), not a derivation of the Schur mode count — DERIVED (no-go)

A.1.13 Paper 14 New Results: Kill Shot C (H_0 Composite)

Step 55: H_0 depends on two projections: H_0^2 = [matter]$\sqrt{\Delta}$ + [curvature]P_k. No independent H_0 projection exists — DERIVED (reduction)

Step 56: V(α) cannot assign H_0 a single form degree (mixes matter and curvature projections) — DERIVED

A.1.14 Paper 15 New Results: Quaternionic Gauge Sector

Step 57: Quaternionic Norm Theorem. $Q(a+v) = a^2+||v||^2$ is the unique positive normalized multiplicative SU(2)-invariant quadratic observable on $\mathbb{H}$. Proof: degree-2 homogeneity, conjugation invariance, multiplicativity, normalization, and positivity force the reduced norm. SymPy residual = 0. Monte Carlo: max error 5.7×10^{-14}. STATUS: THEOREM.

Step 58: Calibration No-Go. Independent channel rescaling preserves KMS, detailed balance, surface gravity, isotropy, modular-Hessian, but changes λ freely. STATUS: DERIVED (no-go theorem).

Step 59: Within the quaternionic SU(2)-carrier, degree-2 homogeneity, multiplicativity, conjugation invariance, normalization, and positivity force the reduced norm, hence $\lambda = 1$. STATUS: THEOREM.

Step 60: Tangential slice. $Q(q_\gamma) = 1+\gamma^2$. Exact for all $\gamma \in \mathbb{R}$. STATUS: DERIVED.

Step 61: Half-density weight. Background-free pairing: w+w=1, w=1/2. STATUS: DERIVED.

Step 62: CMP transgression gives $t(\eta) = x(\eta)^2$ (scalar coefficient of the half-density $T(\eta)$). Within the transgression class analyzed here, CMP selects x^2 whereas ordinary metric transgression gives x. STATUS: DERIVED.

Step 63: Boundary No-Go. Local Schwarzschild horizon scalars are x-independent. x requires R_U from bulk. STATUS: DERIVED (no-go theorem).

Step 64: $K_{geom}(\eta) = 4 \ln x(\eta)$ holds identically along the OS cycloid. Exact algebraic identity. 1001-point scan + 5000-point Monte Carlo: max error 1.55×10^{-76}. STATUS: DERIVED.

A.1.15 Paper 16 New Results: The Gauge Projection Degree

Step 65: Tangential Transfer Homogeneity Theorem. $\tau(T_\gamma) = (1+\gamma^2)^{(p/2)}$. $\tau = \sqrt{(1+\gamma^2)}$ iff p = 1 (for all γ). STATUS: THEOREM.

Step 66: Form-degree-one response. $\delta_{cA}F = c\delta_A F$. $F_{mix,\gamma} = \sqrt{(1+\gamma^2)}\, F$. STATUS: THEOREM.

Step 67: No universal 1-form selector. Explicit counterexample families. STATUS: DERIVED (no-go).

Step 68: Carrier representation. $\rho_u(q_\gamma) = T_\gamma$. Eliminates shared-sector bridge axiom. STATUS: DERIVED.

Step 69: Slice-family collapse. On C_J: $(a^2+b^2)^{(p/2)}$. Global uniqueness fails; slice uniqueness holds. STATUS: THEOREM.

Step 70: Axiom elimination. $s^2 = Q$ algebraic. $F_1 = \sqrt{Q}\, F$, $F_2 = Q\, F$. STATUS: DERIVED.

Step 71: Two-fiber transfer degree. M_K degree 1. E_K degree 2. M_K not determined by $K^\dagger K$ alone. STATUS: THEOREM.

Step 72: Marginal trapping locality. $||A\_tan(r)||/||\Gamma\_tan(r)|| = \sqrt{(1+\gamma^2 r\_s/r)}$. Equipartition iff $r = r\_s$. STATUS: THEOREM.

Step 73: Universal transport no-go. Breaks GTTP at 20.15σ, BDP at $\alpha\_eff = 0.934$, acoustic at 5.17%. STATUS: DERIVED (no-go).

Step 74: Timelike locality. r timelike inside horizon. Standard AQFT locality within the local bilinear class. STATUS: DERIVED.

Step 75: Composite uniqueness. Four options all distinct. $\sqrt{(1+\gamma^2)}$ unique within the local bilinear class. STATUS: THEOREM (P1–P6).

A.1.16 Paper 17 New Results: The Modular Projection Theorem

Step 76: Shared Hilbert space construction. $H\_IO = \Gamma\_s(L^2(\mathbb{R}, d\nu) \otimes H\_g)$. STATUS: DERIVED.

Step 77: Product flow no-go. $\Delta\_prod = \Delta\_ph \otimes \Delta\_g$ leaves photon observables blind to K_gauge. STATUS: DERIVED (no-go).

Step 78: A-vacuum GNS construction. $\omega\_A(X) = \int \varphi\_\kappa((id \otimes \tau\_\kappa)(X(\kappa)))\, d\mu\_A(\kappa)$. Max KMS residual 4.58×10^{-16}. STATUS: DERIVED.

Step 79: Reduction theorem (Theorem 17.R). Compact SU(2) averaging + irreducibility forces $M\_th \otimes Z\_g$. STATUS: THEOREM.

Step 80: Fiberwise KMS inheritance (Theorem 17.K). Built fiberwise from KMS states. STATUS: THEOREM.

Step 81: IO rigidity package R1–R4. R1–R3 inherited (Papers 13–14). R4: one-e-fold normalization (Paper 17 normalization premise — not derived from first principles). Forces $G\_IO = D \otimes \hat{K}\_g$. STATUS: DERIVED (G1–G6; R4 is an explicit premise).

Step 82: Modular Projection Theorem (Theorem 17.1). $d(\ln \omega)/d\lambda = K\_gauge$ and $T\_obs = T\_IO \times x^{K\_gauge}$. STATUS: DERIVED (G1–G6).

Step 83: Foundation closure. All high-priority upstream gaps closed within the reduced Schwarzschild tangential thermal sector. STATUS: DERIVED.

A.1.17 Paper 18: CMP Modular Realization

Step 84: Gravitational history algebra. STATUS: DERIVED.

Step 85: Relative modular operator. $K\_geom = 4 \ln x$. STATUS: THEOREM.

Step 86: CMP Modular Realization Theorem (18.C). $\langle K \rangle = \ln \Delta$. STATUS: THEOREM.

A.1.18 Paper 18: BDP Modular Derivation

Step 87: BDP observable type: 1-form from open covariant transport. STATUS: DERIVED.

Step 88: Derivative selection rule. $V' = 2\gamma$. STATUS: DERIVED.

Step 89: 1-form scaling: $1/x$. STATUS: DERIVED.

Step 90: BDP Theorem (18.B). $f_b = 2\gamma/x$, $\alpha = 1$ structural. STATUS: THEOREM.

A.1.19 Paper 18: V(α) Uniqueness

Step 91: Reduced gauge center one-dimensional. STATUS: DERIVED.

Step 92: $V(\alpha)$ Uniqueness (18.V). Picard-Lindelöf. STATUS: THEOREM.

A.1.20 Paper 18: Acoustic Entropy-Rank

Step 93: Reduced acoustic algebra. STATUS: DERIVED.

Step 94: CMP-derived covariance. Measure = Δ. STATUS: DERIVED.

Step 95: Entropy-Rank Theorem (18.N). Rank = Δ. Math valid; N_{eff} physical ID withdrawn. STATUS: THEOREM (math only).

A.1.21 Paper 18: Modular Bogoliubov Spectrum

Step 96: Bogoliubov Spectrum Theorem (18.β). $T_{obs} = T_{IO} \times x^{K_{gauge}}$. STATUS: THEOREM.

A.1.22 Paper 19: Baryon Scalarization and IO Friedmann Equation

Step 97: P(k) crisis diagnosis. STATUS: DERIVED.

Step 98: Baryon Scalarization Theorem. $\alpha = 3/2$, $\omega_b = 0.01705$. BOSS $\chi^2 = 73.06$. STATUS: THEOREM (S1–S6).

Step 99: Universal projector no-go. STATUS: DERIVED.

Step 100: Geometric clock factor. $H^2_{obs,geom} = x^2 H^2_{bare}$. STATUS: DERIVED.

Step 101: Primitive redshift certification. STATUS: THEOREM.

Step 102: RT upgrade chain. STATUS: DERIVED.

Step 103: IO Friedmann Equation (homogeneous scope). STATUS: DERIVED.

Step 104: Level-mixing no-go. STATUS: DERIVED.

Step 105: Global assembly dissolution. STATUS: DERIVED.

A.1.23 Paper 20: Acoustic Projection and BBN Radiation Algebra

Step 106: Bare readout H_0_obs = Δ^{1/4} H_0_bare. STATUS: DERIVED.

Step 107: OS identity Ω_m/|Ω_k| = x. STATUS: DERIVED.

Step 108: Ω preservation under readout. STATUS: DERIVED.

Step 109: Acoustic History Reduction Theorem (20.1). J_r = x^{1/2}. STATUS: THEOREM.

Step 110: D_A classification. Weight x, gauge-neutral. STATUS: DERIVED.

Step 111: J_θ,geom = x^{−1/2}. STATUS: DERIVED.

Step 112: Degree-2 no-go. STATUS: DERIVED.

Step 113: Acoustic Phase-Calibration Theorem (20.2). J_θ = x^{−1/2}√(1+γ^2). θ* = 0.599°. STATUS: DERIVED.

Step 114: Twelve eliminated paths. STATUS: DERIVED.

Step 115: BBN multi-channel incompatibility. STATUS: DERIVED.

Step 116: BBN measurement immunity. STATUS: DERIVED.

Step 117: IO kinetic correction δ_IO = 0.044. STATUS: DERIVED.

Step 118: BBN radiation algebra (20.RAD1). STATUS: THEOREM.

A.1.24 Paper 21: AC1 Derivation and Radiation Response

Step 119: Optical Necessity (P21.A1). STATUS: DERIVED.

Step 120: Primitive Angular Stage (P21.A2). STATUS: DERIVED.

Step 121: No Second A-Action (P21.A3). STATUS: DERIVED.

Step 122: Collection Preservation (P21.B2). STATUS: THEOREM.

Step 123: Spherical-Harmonic Preservation (P21.B3). STATUS: THEOREM.

Step 124: Scalar-Sector No-New-Slot (P21.B4). STATUS: THEOREM.

Step 125: A4 Bridge Theorem (P21.B5). STATUS: THEOREM.

Step 126: Primitive-Stage Inheritance (P21.B6). STATUS: THEOREM.

Step 127: AC1 Reduction Theorem (P21.A5). J_θ = x^{−1/2}√(1+γ^2). STATUS: THEOREM.

Step 128: Sky-Slot Inheritance (P21.A4). STATUS: THEOREM.

Step 129: Assembly gap decoupling (21.C). STATUS: DERIVED.

Step 130: H_0 Jacobian no-go (21.A). STATUS: DERIVED.

Step 131: Conformal transparency (21.Ba). STATUS: THEOREM.

Step 132: $\Delta N_{eff} = 0$ (21.Bb). STATUS: DERIVED.

Step 133: Baryon freeze-in dead (21.E). STATUS: THEOREM.

Step 134: Tensor coupling classification (21.F). STATUS: THEOREM.

Step 135: Temperature-Independence (21.G). STATUS: THEOREM.

Step 136: SU(2) Geometric Weight (21.I). $Z_{punc} = 1.4221$. STATUS: THEOREM.

A.3 Key Identities (Updated from Paper 13)

All identities from Papers 1–13 carry forward. New Paper 14 identities:

[P14] $A_S^T A_S = (1+\gamma^2)/r_s^2 \times I_2$ (uniqueness, from $J^2 = -I$)

[P14] $C_2(A)/C_2(\Gamma) = 1+\gamma^2$ (Casimir gauge invariance)

[P14] $K^{(n)}_{AB} = -(1/r_s)q_{AB}$ (null second fundamental form, foliation-free)

[P14] $Q_{cov} = |z|^2$ where $z = -\gamma + i$, giving $|z|^2 = 1+\gamma^2$

[P14] $\Lambda_{torsion} \times r_s^2 = 9\pi^2/[4(1+\gamma^2)]$ (universal, x-independent)

[P14] $f_b \times x = 2\gamma$ (universal, x-independent)

[P15] $Q(a+v) = a^2+||v||^2$ (quaternionic reduced norm, unique)

[P15] $Q(q_\gamma) = 1+\gamma^2$ (tangential slice)

[P15] $w = 1/2$ (background-free pairing)

[P15] $t(\eta) = x(\eta)^2$ (CMP transgression, not metric)

[P16] $\tau(T_\gamma) = (1+\gamma^2)^{(p/2)}$ for all normalized O(2)-invariant degree-p functionals

[P16] $||A_{tan}(r)||/||\Gamma_{tan}(r)|| = \sqrt{(1+\gamma^2 r_s/r)}$ (position-dependent, horizon-local at $r = r_s$)

[P16] $A_{tan} = L_\gamma \Gamma_{tan}$ with $L_\gamma = I + \gamma J$, $|L_\gamma| = \sqrt{(1+\gamma^2)}I$ (frame translation)

[P17] $G_{IO} = D \otimes \hat{K}_g$ (unique coupled generator under R1–R4)

[P17] $d(\ln \omega)/d\lambda = K_{gauge}$ (modular flow consequence)

[P18] $K_{CMP} = K_{geom} + K_{gauge}$ (Theorem 18.C)

[P18] $f_b = 2\gamma/x$, $\alpha = 1$ structural (Theorem 18.B)

[P18] $V(\alpha)$ uniqueness (Theorem 18.V)

[P19] $\omega_b(O) = \omega_b(\alpha=1)\, x^{1-\alpha_O}$

[P19] $\alpha = 3/2$ for clustering (THEOREM)

[P20] $J_r = x^{1/2}$, $J_\theta = x^{-1/2}\sqrt{(1+\gamma^2)}$

[P20] $\Omega_m/|\Omega_k| = x$

[P21] AC1 promoted (Theorem P21.A5)

[P21] $\Delta N_{eff} = 0$ (Theorem 21.Bb)

[P21] $Z_{punc} = 1.4221$, $f_E = 0.3622$ (Theorem 21.I)

A.4 Numerical Values (Paper 14 Additions)

All values from Paper 13 carry forward. New Paper 14 values:

$C_2(A)/C_2(\Gamma) = 1+\gamma^2 = 1.05641$ (Casimir ratio)

$||A\_S^T A\_S|| = (1+\gamma^2)/r\_s^2$ (degenerate eigenvalue, uniqueness)

$N(\theta^* = 0.5962°) = 5.634$ (acoustic scale selector for N_eff)

$N\_eff/\Delta = 5.634/5.624 = 1.0018$ (0.18% match)

$P\_k(\text{Schur}, N=\Delta) = \exp(-\Delta/2) = \exp(-2.812) = 0.060$

$\Delta\_output(\text{age loop}) = 1.095–1.099$ (trivially flat, 0.31% variation)

$H_0(\text{Schur, CONDITIONAL}) = 68.91$ km/s/Mpc

$\exp(K\_gauge/2) = \sqrt{(1+\gamma^2)} = 1.02782$ ($\neq$ x^K_gauge = 1.02320)

$Q(q\_\gamma) = 1+\gamma^2 = 1.05640625$

$t(\eta\_obs) = x^2 = 2.3073$ (scalar coefficient of half-density T)

coeff[$O(\eta\_obs)$] = 5.6241

Step 64 identity check Monte Carlo max error = 1.55×10^{-76}

$s\_h = \sqrt{(1+\gamma^2)} = 1.027816$ (horizon spectral factor)

GTTP violation under universal transport: 20.15σ

T_obs = 2.7253 K (GTTP, Theorem 17.1)

K_gauge position in FIRAS band: −0.325σ from best fit

A.5 Status Summary (Updated through Paper 21)

CMP: ⟨K⟩ = ln Δ — THEOREM (Theorem 18.C, C1–C5)

BDP: f_b = 2γ/x, α = 1 — THEOREM (Theorem 18.B, B1–B5; α = 1 structural from 1-form type)

GTTP: σ = K_gauge — DERIVED (Theorem 17.1; R4 normalization premise, Open Problem 1)

V(α) = −2 ln(cos α) — THEOREM (Theorem 18.V, V1–V3; Picard-Lindelöf uniqueness)

Acoustic entropy-rank = Δ — THEOREM (Theorem 18.N; mathematical result valid, physical N_eff identification WITHDRAWN)

Bogoliubov spectrum — THEOREM (Theorem 18.β; T_obs = T_IO × x^{K_gauge})

K_gauge horizon-intrinsic — THEOREM (Paper 14)

$(1+\gamma^2)$ uniqueness on S^2 — THEOREM (Paper 14)

Gauge projection degree p = 1 — THEOREM (Paper 16, P1–P6)

A.6 Self-Consistent Transport No-Go Recalculation (Paper 16)

This appendix provides the explicit self-consistent recalculation underlying the universal transport no-go (§10.4). Within the Paper 10/13 projection implementation, replacing $\sqrt{\Delta}$ by $\sqrt{\Delta}$_new propagates catastrophically through the IO Friedmann equation, yielding T_obs = 2.73759 K (20.15σ from FIRAS).

A.6.1 The IO Friedmann Equation

The IO Friedmann equation (Paper 10, §9) is:

$$H^2(z) = [H^2\_m(z) + H^2\_\Lambda\text{,bare}] \times \sqrt{\Delta} + H^2\_k(z) \times \Delta\text{^}(-1/x^2)$$

where the H^2_i(z) are the geometric (pre-projection) Hubble contributions computed from the OS interior, $\sqrt{\Delta} = x^2\sqrt{(1+\gamma^2)} = 2.37150$ is the matter projection, and Δ^$(-1/x^2)$ = P_k is the curvature-sector projection. At z = 0, this yields H_0,obs = 67.58 km/s/Mpc (Paper 10).

Important: the projection factors $\sqrt{\Delta}$ and P_k multiply the individual H^2 components, not the density parameters Ω_i directly. The resulting H_0,obs depends nonlinearly on $\sqrt{\Delta}$ through the full Friedmann integral, not through a simple algebraic rescaling of H_0,geom.

Under universal transport, the gauge projection doubles: $\sqrt{(1+\gamma^2)} \rightarrow (1+\gamma^2)$. Therefore:

Published: $\sqrt{\Delta} = x^2 \times \sqrt{(1+\gamma^2)} = 2.30734 \times 1.02782 = 2.37150$

Modified: $\sqrt{\Delta}\_\text{new} = x^2 \times (1+\gamma^2) = 2.30734 \times 1.05641 = 2.43747$

Ratio: $\sqrt{\Delta}\_\text{new} / \sqrt{\Delta} = 1.02782$ (2.78% increase)

$\Delta\_\text{new} = (2.43747)^2 = 5.9413$

A.6.2 Perturbative H_0 Estimate

Since the energy sector dominates the IO Friedmann equation at z = 0 (the curvature term is suppressed by P_k ≈ 0.06), H^2_obs scales approximately as $\sqrt{\Delta}$ to leading order:

$H^2\_0\text{,new} / H^2\_0\text{,published} \approx \sqrt{\Delta}\_\text{new} / \sqrt{\Delta} = 1.02782$

$H\_0\text{,new} \approx H\_0\text{,published} \times \sqrt{(1.02782)} = 67.58 \times 1.01382 = 68.51$ km/s/Mpc

This linear estimate is illustrative. The exact value requires self-consistent iteration because H_0 determines R_U (through the Friedmann age integral), which in turn determines x = r_s/R_U, which feeds back into $\sqrt{\Delta}\_\text{new} = x^2(1+\gamma^2)$.

A.6.3 Self-Consistent Iteration

The self-consistent iteration proceeds as follows:

Step 1. Start with H_0,new = 68.51 km/s/Mpc (perturbative estimate).

Step 2. Compute the new age $t\_\text{obs} = \int\_0\text{^}\infty \, dz/[(1+z)H(z)]$ using the modified Friedmann equation. Require t_obs = 13.8 Gyr.

Step 3. The age constraint determines R_U,new, hence $x\_new = r\_s/R\_U,new$.

Step 4. Recompute $\sqrt{\Delta}\_new = x\_new^2(1+\gamma^2)$ and iterate from Step 2 until convergence.

Convergence (Supplement S1: paper16_transport_consistency_audit_checks.py [21], Newton scheme):

Iteration 0: $H\_0 = 68.51$, $x = 1.531$, $\sqrt{\Delta}\_new = 2.479$, $t\_age = 13.45$ Gyr

Iteration 1: $H\_0 = 69.12$, $x = 1.540$, $\sqrt{\Delta}\_new = 2.508$, $t\_age = 13.67$ Gyr

Iteration 2: $H\_0 = 69.38$, $x = 1.544$, $\sqrt{\Delta}\_new = 2.521$, $t\_age = 13.76$ Gyr

Iteration 3: $H\_0 = 69.44$, $x = 1.545$, $\sqrt{\Delta}\_new = 2.524$, $t\_age = 13.79$ Gyr

Iteration 4: $H\_0 = 69.45$, $x = 1.5454$, $\sqrt{\Delta}\_new = 2.525$, $t\_age = 13.80$ Gyr (converged)

Final converged values:

$H\_0,converged = 69.45$ km/s/Mpc

$R\_U,converged = 4.326 \times 10^{26}$ m

$x\_converged = r\_s / R\_U,converged = 6.685 \times 10^{26} / 4.326 \times 10^{26} = 1.5454$

A.6.4 Propagation to T_obs

The GTTP formula (Paper 13) is $T\_obs = T\_IO \times x\text{\^{}}K\_gauge$, where $K\_gauge = \ln(1+\gamma^2) = 0.05487$. $T\_IO = \hbar c\gamma/(4\pi R\_U\ k\_B)$ scales as $1/R\_U$:

$T\_IO,new = T\_IO,published \times (R\_U,published / R\_U,converged) = 2.6635 \times (4.400 / 4.326) = 2.6635 \times 1.01711 = 2.7091$ K

$x\_new\text{\^{}}K\_gauge = (1.5454)\text{\^{}}0.05487 = \exp(0.05487 \times \ln 1.5454) = \exp(0.05487 \times 0.4352) = \exp(0.02388) = 1.02417$

$T\_obs,analytic = 2.7091 \times 1.02417 = 2.7746$ K

The full self-consistent iteration (Supplement S1 [21]), which also accounts for the modified baryon sector feedback on the Friedmann integral, gives:

$T\_obs,converged = 2.73759$ K

Both the analytic estimate (2.7746 K) and the full iteration (2.73759 K) are catastrophically high relative to FIRAS.

A.6.5 GTTP Significance

$T\_obs,modified - T\_obs,FIRAS = 2.73759 - 2.7255 = 0.01209$ K

$\text{Deviation} = 0.01209 / 0.0006 = 20.15\sigma$

A.6.6 BDP α-Ladder Derivation

The BDP (Paper 12) defines the baryon fraction via $f_b = 2\gamma/x^\alpha$, equivalently $f_b \times x^\alpha = 2\gamma$. Define the baryon product $p_b := f_b \times x$. At the published values, $p_b = 2\gamma = 0.475$ and $\alpha = 1$ exactly. Under universal transport, the modified baryon density shifts $p_{b,new} = 0.48822$ (from the self-consistent Friedmann iteration, §A.6.3). The effective α is determined by $p_{b,new} \times x_{new}^{(\alpha-1)} = 2\gamma$, giving:

$$\alpha_{eff} = 1 + \ln(2\gamma / p_{b,new}) / \ln(x_{new})$$

Numerical evaluation with the converged values $p_{b,new} = 0.48822$, $x_{new} = 1.5454$:

$$\alpha_{eff} = 1 + \ln(0.475 / 0.48822) / \ln(1.5454) = 1 + \ln(0.97291) / 0.43524 = 1 + (-0.02745) / 0.43524 = 1 - 0.06308 = 0.93692$$

This analytic one-step estimate gives $\alpha_{eff} \approx 0.937$. The full self-consistent iteration (Supplement S1: paper16_transport_consistency_audit_checks.py) additionally iterates the BDP master formula $\omega_{b,eff} = (\ln \Delta/4) \times \omega_{m,geom}$ with the modified Δ, accounting for baryon-sector feedback on the Friedmann integral. This converges to:

$$\alpha_{eff,converged} = 0.93437$$

The difference between the one-step estimate (0.93692) and the converged value (0.93437) reflects the baryon feedback correction. Both values are far from the integer 1, confirming the loss of 1-form geometric structure.

The loss of the integer $\alpha = 1$ breaks the 1-form geometric interpretation required by the BDP.

A.6.7 Acoustic Closure

$$N(\theta^*) / \Delta_{new} = 5.634 / 5.9413 = 0.9483 \quad (5.17\% \text{ deviation})$$

Compared to $N(\theta^*)/\Delta = 5.634/5.624 = 1.0018$ (0.18%) in the published framework. The deviation degrades by a factor of 29.

The universal transport hypothesis is rejected at 20.15σ by GTTP, with corroborating failures in BDP ($\alpha_{eff} = 0.934$) and acoustic closure (5.17%).

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Acknowledgments and AI Disclosure

This paper was developed through a multi-AI collaborative pipeline. Claude (Anthropic, Opus 4.6) served as pipeline orchestrator, managing prompt sequencing across Part I (two adversarial rounds) and Part II (nineteen investigation routes), maintaining the derived stack, designing the multi-round validation strategy, and identifying the a^{-8} dilution route as a potential resolution of the θ*-insulation / BBN-constant tension. Codex/ChatGPT (OpenAI) performed all theorem construction and numerical computation: Part I Rounds 1–2 (A1–A5, B0–B6), the SU(2) partition function (Z_punc = 1.4221), the P_multlift decomposition, the puncture fixed-point algebra (21.Y), the optical-kill no-go (21.Z), the absolute local Casimir load (F_abs = 0.36879), the λ=4 conditional uniqueness (21.X), all exact PRyMordial BBN computations including the uniform-s kill, stiff-dilution test, and non-separable weak-sector tests. Wolfram/ChatGPT (OpenAI, 5.4 Pro Extended) served as primary adversarial validator, independently reviewing all thirty results and performing the holistic publication review. Key contributions: identified the A4 gap (Part I Round 1), proposed the hybrid response functional architecture, diagnosed the reference-state error that led to the optical filtration paradigm revision, proposed the Casimir-based response and relative-entropy routes, confirmed the puncture fixed-point algebra kills the optical-kill mechanism, and recommended the admissibility classification program. Gemini (Google DeepMind, 2.5 Pro) provided foundational physical insights: the “lens vs light” framing (convergent with Claude, Part I); the SU(2) geometric freeze-out concept that initiated the partition function investigation; the measurement-dependence paradigm; the identification that the reduced algebra is the physical low-temperature phase of the boundary; the Metric Tensor-Product selection rule attempt (21.M, partially survived); the local decoherence

principle (became Theorem 21.O); the Weinberg-Witten / bulk propagation attempt; the V(α) composite dressing interpretation (f_b + K_gauge); the λ=4 spacetime-dimension identification; the conformal probe projection; and the three non-separable weak-sector routes (chiral g_A, phase-space Jacobian, G_F shift). David Fife (PI) directed all investigations, enforced pipeline discipline, maintained investigative continuity across nineteen successive routes, spotted the j=2 convergence between Codex's Clebsch-Gordan classification and Gemini's mechanism, identified the a^{-8} dilution connection between two no-gos, and made all final physics decisions.