Geometric Wave Engineering: Pseudo-Surfaces of Variable Negative Gaussian Curvature as a Geometric Basis for Programmable Wave Control

Vladimir Khaustov
Independent Researcher
ORCID: 0009-0007-3657-2309
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Abstract

This paper presents Geometric Wave Engineering (GWE) as a proposed design discipline in which the macroscopic geometry of a wave domain is treated as an independent control variable alongside material, boundary conditions, and excitation. The core geometric objects are pseudo-surfaces of variable negative Gaussian curvature generated from hyperbolic, parabolic, and elliptic profiles. The most developed construction is the higher-order pseudo-hyperboloid built by a recursive interval rule and a Merge operation that forms a common internal volume without artificial connecting segments. Pseudo-paraboloids and pseudo-ellipsoids generalize the same constructive logic to other generators. In addition to the geometric and computational construction, numerous stochastic ray-tracing studies have already been carried out for all three pseudo-surface families. Without entering into detailed numerical data here, these studies qualitatively support many of the criteria C1–C8 and indicate remarkable possibilities for controlling waves of different physical nature and across broad frequency ranges within the proposed GWE framework. The present preprint focuses on definitions, construction rules, representative figures, functional zones, and a compact verification roadmap.

Keywords

Geometric Wave Engineering; pseudo-surface; pseudo-hyperboloid; pseudo-paraboloid; pseudo-ellipsoid; variable negative Gaussian curvature; Merge operation; programmable wave control; wave confinement; localized ring states.

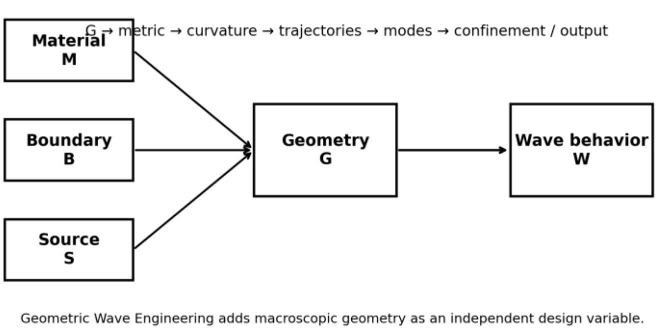

Figure 1. Geometric Wave Engineering as an extension of the classical material–boundary–source triad.

1. Introduction

Wave control is a central task in optics, acoustics, radio engineering, microwave systems, elastodynamics, and quantum-wave mechanics. Classical engineering usually operates with three main design variables: material properties, boundary conditions, and excitation. The present work argues that macroscopic geometry itself can act as a fourth independent design variable.

In Geometric Wave Engineering, geometry is not treated as a passive container. Instead, it is regarded as a programmable factor capable of influencing trajectories, modal organization, localization zones, leakage channels, and controlled output. In symbolic form, the observable wave behavior may be written as $W = F(M, B, S, G)$, where G denotes geometry. The proposed program places special emphasis on the chain G → metric → curvature → trajectories → modes → confinement / output.

The aim of this paper is not to claim a complete universal proof for every wave process at once. Its goal is more precise: to introduce a reproducible class of pseudo-surfaces and to show that they constitute a coherent geometric platform for systematic wave-engineering research.

2. Pseudo-surfaces of variable negative Gaussian curvature

A pseudo-surface is considered here as a surface of revolution generated by a specially designed conic profile. Its regular working region is the subset where the surface is smooth and where the Gaussian curvature K is well defined. A pseudo-surface of variable negative Gaussian curvature is one for which there exists an open non-empty regular subset on which $K < 0$ and K is non-constant.

The geometric family studied in this paper contains three canonical branches: pseudo-hyperboloids, pseudo-paraboloids, and pseudo-ellipsoids. They are united not by a single local formula, but by a common constructive philosophy: an analytically defined generator, recursive or rowed extension to higher order, and a Merge operation for the internal volume.

3. Three canonical generator families

Pseudo-hyperboloids are generated from open hyperbolic branches. In the vertical second-order case the generator distance is $\rho(|x|) = R - b\sqrt{(|x|/a)^2 - 1}$, with $a \le |x| \le L$, where $L = a\sqrt{1 + (R/b)^2}$.

Pseudo-paraboloids are generated from two mirrored parabolic branches. In the vertical second-order case the generator distance is $d(|x|) = R - 2\sqrt{f|x|}$. In Figure 3 the parabolic generator is shown for $h = 0$.

Pseudo-ellipsoids are generated from two elliptic quarter-segments. For the vertical type shown here, the representative parameters are $a = 1$, $K = 1.5$, $h1 = 0$, and $h = 0$, with $R = b + h$ and $b = aK$. Figure 3 shows both the generators and the real second-order vertical sections produced from the computational scripts.

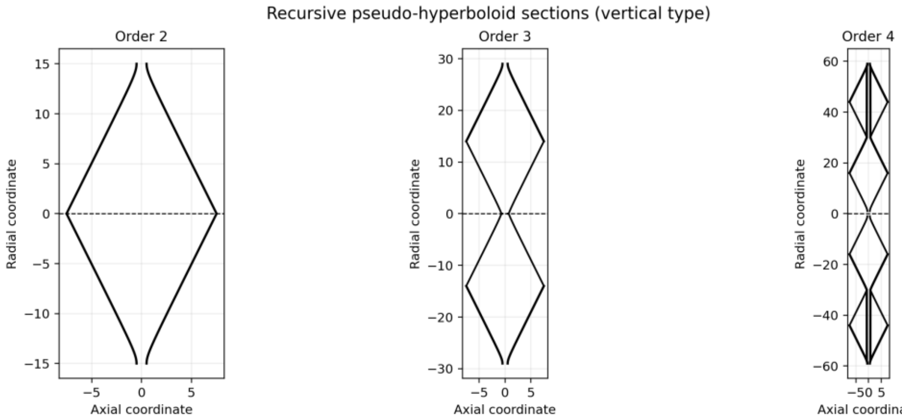

Figure 2. Vertical pseudo-hyperboloid sections for orders 2–4, illustrating recursive growth and Merge-based common-volume formation.

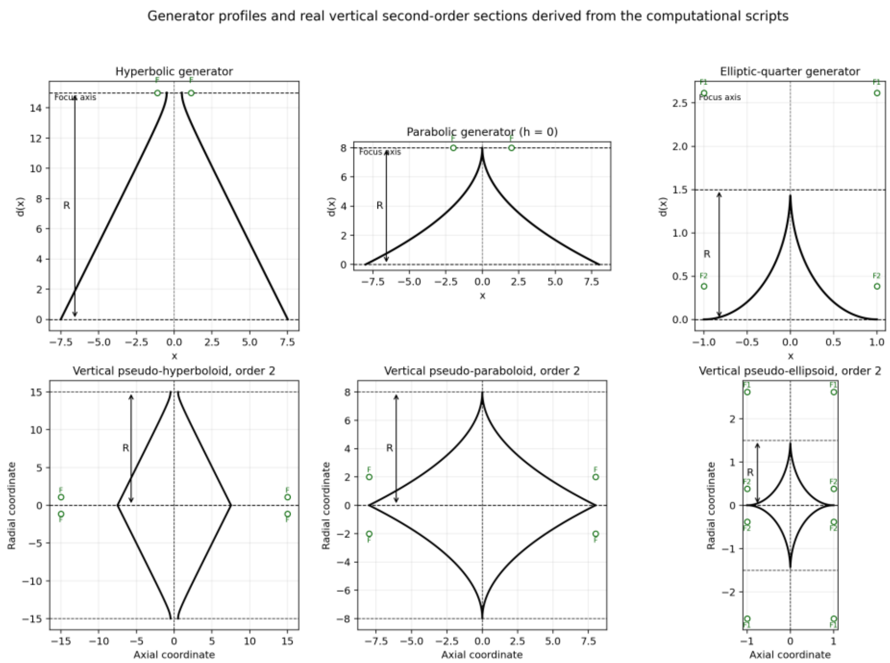

Figure 3. Top row: the three canonical generators with service lines, foci, and the parameter R. Bottom row: the real vertical second-order 2D sections produced from the computational scripts for the three pseudo-surface families.

4. Recursive construction and the Merge operation

For higher-order pseudo-hyperboloids, the constructive rule may be written as $B_2 = \{d\}$ and $B(k+1) = \{R_k + r, R_k - r : r \in B_k\}$. The formal number of branches is $N_n = 2^{(n-2)}$. However, a physical internal volume is not merely a list of branches. At each axial coordinate, one must work with intervals and merge intersecting or touching intervals to obtain the common internal volume.

This Merge operation is fundamental. It does not delete the generating geometry as such; it removes only duplicated overlap in the final common volume. Separated components must remain separated, and the construction must not introduce artificial straight connecting segments. The same constructive philosophy is used in the parabolic and elliptic families.

5. Computational evidence and current verification status

The geometric definitions are supported by explicit Python scripts for rowed pseudo-hyperboloids, pseudo-paraboloids, and pseudo-ellipsoids. These scripts generate base profiles, second-order sections, higher-order sections, 3D surfaces of revolution, row configurations, and Merge-based common internal volumes.

Beyond pure geometry, extensive stochastic ray-tracing experiments have already been performed by the author for all three pseudo-surface families. A detailed numerical exposition is beyond the scope of the present paper, but the accumulated results qualitatively support many criteria from the verification ladder C1–C8. In particular, they indicate stable localization zones, controllable circulation, structured output behavior, and non-trivial wave-management possibilities associated with these geometries.

Accordingly, the present article positions GWE not as a finished closed theory, but as a rapidly developing scientific direction grounded in explicit geometry, computational construction, and an already active verification program.

6. Functional zones and engineering interpretation

Pseudo-surfaces naturally contain focal loci, ring zones, equatorial regions, necks, polar regions, output windows, overlaps, and Merge transitions. These are geometric zones first; their wave function must be verified by computation or experiment. Nevertheless, they provide a practical language for discussing how geometry may organize localization, circulation, delay, controlled release, and coupling between internal regions.

The central engineering hypothesis is that a distributed and variable negative curvature, together with recursive or rowed topology, can create wave-management opportunities not easily accessible in more conventional simple cavities.

Table 2. Functional zones

Zone	Meaning	Wave-engineering interpretation
Focal locus	Generator-associated focus region	Candidate localization or trajectory-concentration zone
Ring / equator	Closed annular region	Circulation, ring states, controlled output
Neck / transition	Topologically critical passage	Delay, coupling, leakage control
Merge transition	Union of overlapping intervals	Common-volume formation without duplicate walls

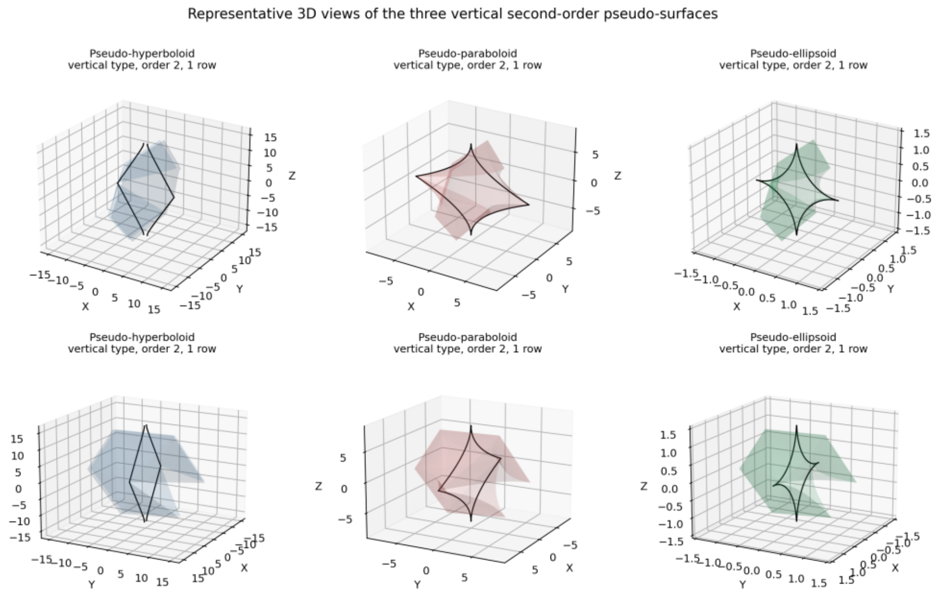

Figure 4. Representative 3D views of the three vertical second-order pseudo-surfaces. Two view angles are shown for each family.

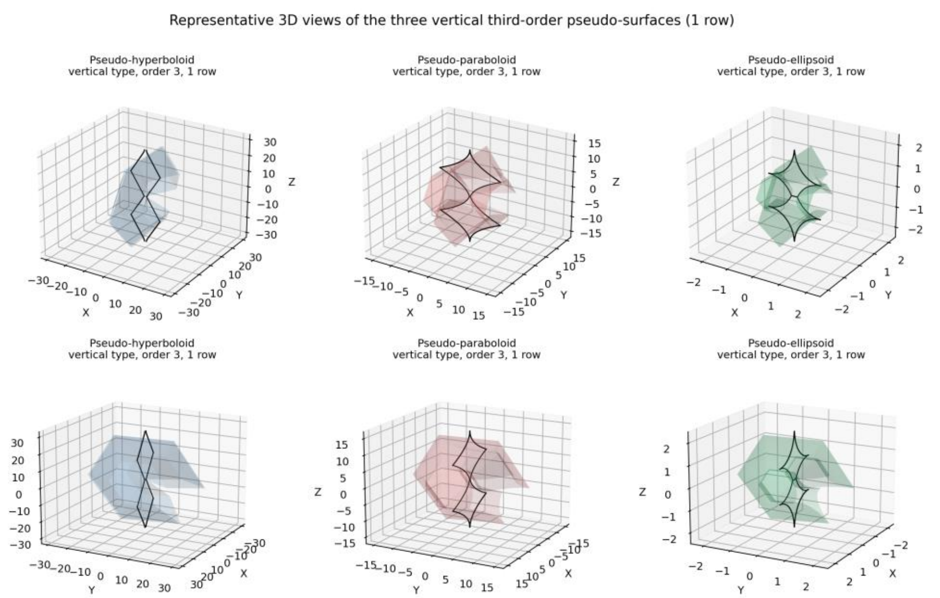

Figure 5. Representative 3D views of the three vertical third-order pseudo-surfaces (1 row). Two view angles are shown for each family.

7. Verification roadmap C1–C8

The proposed verification ladder consists of eight criteria: C1 (geometry and reproducibility), C2 (localization or attractor zones), C3 (spectral windows), C4 (confinement-output balance), C5 (directional output), C6 (scale behavior), C7 (cross-physics transfer), and C8 (robustness).

At the current stage, C1 is established directly by the analytical definitions and scripts. A significant part of C2–C8 has already received qualitative support from stochastic ray-tracing studies across all three pseudo-surface families. Dedicated publications should present these results in full numerical and experimental detail.

8. Applications and outlook

Potential application areas include open resonators, wave concentrators, mode selectors, delay structures, structured radiators, acoustic cavities, and wave-guiding domains for multi-zone control. The immediate practical value of the pseudo-surface framework lies in its parameterized geometry: a, b, R, f, K, h1, offsets, row count, and row overlap can all be used as design variables.

The most realistic next step is a coordinated program of ray, Helmholtz, Maxwell, and acoustic simulations, followed by prototyping and comparison with classical cavities of similar scale.

9. Conclusion

Geometric Wave Engineering is proposed here as a new scientific and engineering direction in which geometry becomes a primary programmable factor for wave control. Pseudo-hyperboloids, pseudo-paraboloids, and pseudo-ellipsoids form the first coherent family of pseudo-surfaces supporting this program.

The combination of explicit generators, higher-order recursive construction, Merge-based common-volume formation, and growing stochastic ray-tracing evidence suggests that these geometries deserve systematic worldwide attention. The present paper is intended as a concise, public-facing foundation for that program.

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