A modular-charge fifth face of the Unified Equilibrium Law in Quantum Traction Theory: $E_P = (\hbar c/2\pi\ell_P)\, Q_P$ at A7 saturation

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Abstract

Quantum Traction Theory (QTT) [1] reconstructs the basic ontology of physics from a small number of axioms about an absolute tick, a real-valued internal dial, and bundled existence at every world-cell address. One consequence of that reconstruction is the Unified Equilibrium Law (UEL), a single Planck-scale identity asserting that mass, frequency, and the four-density of reality are not independent quantities but distinct *faces* of one primitive capacity unit, the Planck four-cell $4\pi\ell_P^4$. UEL in its established four-face form reads $E_P = m_P c^2 = \hbar\omega_P = \rho_{(4)}(4\pi\ell_P^4)$ with parameter-free dimensional correctors c^2, $\hbar$, and $4\pi\ell_P^4$. We identify a fifth face: information capacity, expressed as the modular charge of an A7-saturated address bundle. The five-face UEL reads

$$E_P = m_P c^2 = \hbar\omega_P = \rho_{(4)}(4\pi\ell_P^4) = \frac{\hbar c}{2\pi\,\ell_P}\,Q_P, \qquad Q_P = 2\pi.$$

The fifth face is forced by the framework's existing axioms (A2 IR identification, A4 internal S^1 dial, A5 visible–hidden factorization, A6 per-address capacity, and A7 modular-bundle saturation $Q_w^{\text{bundle}} = 2\pi$) together with the A7 energy map; no new axiom is introduced. The dimensional corrector $k_Q = \hbar c/(2\pi\ell_P)$ contains no tunable coupling, in the precise sense that the rescaling-invariant content is the product $k_Q\,Q_P = E_P$. Equivalently, $E^{\text{bundle}} = E_P\,S^{\text{bundle}}$ with $S^{\text{bundle}} = 1$ nat at saturation: within QTT, the Planck energy is the energy assigned to one nat of Umegaki relative entropy at one A7-saturated bundle. The same 2π factor that closes the modular bundle in A7 also closes the internal S^1 dial in A4 and drives the spinor half-angle theorem of the parallel QTT structural-identification paper [2]; the two papers use the same real-dial algebra to identify two structural identities. The Bekenstein–Hawking horizon-entropy density $1/(4\ell_P^2)$ integrated over the framework's area unit $A_\Sigma = 8\pi\ell_P^2$ returns 2π, consistent with QTT's calibration of the area unit to one 2π angular modular period. The result does not extend to ordinary low-energy thermodynamic information, where the Landauer scale $k_B T\ln 2$ governs erasure costs.

1 Introduction

Quantum Traction Theory (QTT) [1] is a ground-up reconstruction of the foundations of classical mechanics, quantum mechanics, and general relativity from a short list of axioms: A1 (an absolute tick alongside the laboratory clock), A2 (an endurance budget that produces Newtonian gravity in the IR), A3 (creation that mimics a cosmological constant), A4 (an internal S^1 dial whose quarter-turn is the imaginary unit), A5 (visible–hidden factorization at every world-cell address), A6 (per-address capacity throughputs), and A7 (bundled existence saturating one

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modular circle at every address). The framework recovers Newton's law of gravitation, Maxwell's equations, the Schrödinger equation, charge quantization, and the qualitative structure of quantum measurement from this reconstruction; the imaginary unit i in standard quantum mechanics emerges as the quarter-turn $J^2 = -\mathbf{1}$ on the real two-component dial.

The Unified Equilibrium Law (UEL) is one of the framework's central unification statements. UEL reads

$$E_P \;=\; m_P c^2 \;=\; \hbar\omega_P \;=\; \rho_{(4)}\,(4\pi\ell_P^4), \tag{1}$$

asserting that mass, frequency, and four-density are not independent physical quantities but four equivalent *faces* of one primitive capacity unit — the Planck four-cell $4\pi\ell_P^4$. The dimensional correctors c^2, $\hbar$, and $4\pi\ell_P^4$ that mediate the four faces are parameter-free.

This article identifies a fifth face of UEL: information capacity, expressed as the modular charge of an A7-saturated address bundle. The fifth face is forced by the same axioms that produce the first four. Within QTT's ontology, information is not a thermodynamic add-on to physics, governed by Landauer-scale erasure costs as in the standard treatment [14, 15]; nor is it a horizon-bound construct, governed by Bekenstein–Hawking area-law entropy as in the gravitational treatment [3, 4, 5]. Within QTT, information capacity is one of the equivalent faces of the primitive Planck capacity itself, alongside mass, frequency, and four-density.

The framework inputs to the corollary below are A2 (IR identification $\tilde{\ell} = \ell_P$), A4 (internal S^1 dial), A5 (visible–hidden factorization), A6 (per-address capacity), A7 (saturation of the modular-charge budget at every address, $Q_w^{\text{bundle}} = 2\pi$), and the A7 energy map $E_{\text{bundle}}(w) = (\hbar c/2\pi\ell_P)\,Q_w^{\text{bundle}}$. The mathematical inputs are Tomita–Takesaki theory [8, 9] for modular flow and the relative-entropy formalism of Umegaki–Araki [6, 7].

What this paper contributes. The four-face UEL (1) and the A7 energy map are both already in [1]. This paper isolates and develops the fifth face explicitly. The contributions are:

(1) the explicit five-face form of UEL within QTT, which has not previously been written down;

(2) a Bisognano–Wichmann angular reading [10, 11] of the QTT charge normalization $Q := 2\pi S(\rho\|\omega)$, proposed here as the natural mathematical motivation for the framework's 2π factor;

(3) a unit-discipline that keeps modular charge Q distinct from Umegaki relative entropy S, with the collapsed identity $E^{\text{bundle}} = E_P\, S^{\text{bundle}}$ at $S^{\text{bundle}} = 1$ nat;

(4) the rescaling-invariant statement $k_Q\, Q_P = E_P$, identifying what is and is not absolute in the dimensional corrector;

(5) a three-layer separation of standard mathematics from QTT conventions and QTT postulates (Section 5.2); and

(6) an explicit operational-gap declaration: at present QTT does not provide an operational reconstruction protocol for Q_w from independent observables, and the falsifier is theory-internal until such a protocol exists.

Connection to the parallel structural-identification paper. The same real-dial algebra A4 that closes the internal S^1 at 2π here drives the spinor half-angle theorem of the parallel QTT structural-identification paper [2], in which the QTT two-clock projection constant $I_{\text{clk}} = \cos(\pi/8)$ is identified with the standard T-gate magic-state overlap and the symmetric Clauser–Horne–Shimony–Holt variational angle $\beta = \pi/8$. The two papers use the same real-dial algebra to identify two distinct structural identities — the same 2π closes the dial in both — and form a coordinated programme of structural identifications within the QTT ontology.

Scope statements. First, no claim is made that ordinary low-energy information storage carries Planck-scale energy; everyday bits in computers, detectors, or low-energy cold-atom systems are governed by the Landauer scale [14, 15]. Second, the corollary is formulated in the regulated finite-cell algebra assumed by A5; we make no claim that the naive tensor-product factorization $\mathcal{H} = \bigotimes_w \mathcal{H}_w$ extends to continuum algebraic QFT, where local algebras are typically type III and additional structure (split property, edge modes) is required.

Section 2 fixes notation. Section 3 states UEL in its four-face form. Section 4 reviews the modular-capacity construction with explicit attribution of every 2π that enters, distinguishing standard mathematics from the QTT conventions and the BW reading proposed here. Section 5 states the assumption ledger, the corollary, the rescaling-invariant content, and the three-layer taxonomy. Section 6 carries out the Bekenstein–Hawking calibration check. Section 7 states invalidating conditions and the operational gap. Section 8 closes with the structural unification reading.

2 The two-clock construction

We adopt the kinematic normalization in QTT's two-clock gauge [1]: an absolute clock T and a laboratory clock τ related by a smooth positive lapse $N(x,t)$ in the Einstein gauge. Local observables are referred to spatial slices of τ with future unit normal n^μ, defining the per-address energy

$$E_w(\tau) := \int_{C_w(\tau)} d\Sigma_\mu \, T^{\mu\nu} \, n_\nu, \tag{2}$$

where $C_w(\tau)$ is the finite-cell region around address w on the slice and $T^{\mu\nu}$ is the renormalized stress-tensor expectation value within the framework's regulator (cell construction in [1]). We use $\hbar$, c, and the micro-length $\tilde{\ell}$ as the only dimensional inputs; A2 (the Newtonian limit) fixes $\tilde{\ell} = \ell_P = \sqrt{G\hbar/c^3}$, so that $E_P = \hbar c/\ell_P$, $m_P = \hbar/(c\ell_P)$, and $\omega_P = c/\ell_P$. The symbol i denotes the laboratory imaginary unit; in QTT it is identified with the quarter-turn J on the real two-component dial of A4 [1, Sec. 20.25], $J^2 = -\mathbf{1}$, so the distinction is operational and is not used in the corollary below.

3 Unified Equilibrium Law in its four-face form

UEL in [1] packages four classically distinct routes to the Planck energy as a single identity:

$$E_P \;=\; m_P c^2 \;=\; \hbar\omega_P \;=\; \rho_{(4)}\,(4\pi\ell_P^4). \tag{3}$$

The four-density $\rho_{(4)}$ is the QTT capacity per Planck four-volume, set by A6–A7 in [1]. Reading (3) as a unification statement: each face is converted to energy by a parameter-free dimensional corrector.

Face	Quantity	Corrector
Energy	E_P	—
Mass	m_P	c^2
Frequency	ω_P	$\hbar$
Four-density	$\rho_{(4)}$	$4\pi\ell_P^4$

The fifth face, identified in Section 5, is the modular charge of an A7-saturated bundle. Its corrector has dimensions of energy because the modular charge is dimensionless.

4 Modular charge and the role of 2π (A4–A7 review)

We review the QTT modular-capacity construction at the level needed below; the full construction is in [1]. The aim is to attribute each 2π in the construction to its source, separating standard mathematics from QTT conventions and from QTT postulates.

The first 2π: the S^1 dial circumference (A4). At every address w, A4 attaches an internal S^1 phase circle. The circumference 2π is the angular coordinate on $S^1 \cong \mathbb{R}/2\pi\mathbb{Z}$; the underlying topological invariant is the integer winding of U(1) characters. The 2π that closes the dial in A4 is the same 2π that closes the modular bundle in A7 below, by direct construction in [1].

Visible–hidden factorization (A5). By A5, degrees of freedom decompose over discrete world-cells with a visible–hidden split, $\mathcal{H} = \bigotimes_w \mathcal{H}_w$ and $\mathcal{H}_w = \mathcal{H}_w^{\mathrm{vis}} \otimes \mathcal{H}_w^{\mathrm{hid}}$. This is a regulated finite-cell statement that makes *relative* entropy well-defined at w.

The second 2π: angular modular coordinate (this paper's reading). Fix a faithful local reference state ω_w. Tomita–Takesaki theory [8, 9] associates a canonical modular operator Δ_w and a one-parameter modular flow $\sigma_t^{\omega_w}(A) = \Delta_w^{it} A \Delta_w^{-it}$. In the standard mathematical normalization, ω_w is KMS with respect to σ_t at inverse temperature $\beta = 1$. The framework [1] defines its modular charge using a 2π prefactor, $Q := 2\pi S$, without specifying a unique mathematical motivation for the prefactor. We propose that $Q := 2\pi S$ is naturally read as the angular modular coordinate

$$\theta := 2\pi\, t, \qquad \text{KMS analyticity period } 2\pi \text{ in } \theta, \tag{4}$$

which is the standard setting in physical realizations where the modular flow has a geometric meaning. In the Bisognano–Wichmann theorem [10] the modular flow of the algebra of a Rindler wedge coincides with Lorentz boosts, and the corresponding Unruh temperature carries an explicit $2\pi/a$ factor [11]. The QTT 2π in $Q := 2\pi S$ is therefore not an arbitrary normalization: it matches the angular modular period that appears whenever modular flow has a geometric origin. Combined with the S^1 dial of A4, the same 2π closes both the geometric modular flow and the internal dial.

The anchored modular charge of a state ρ_w supported in $\mathcal{H}_w$ is

$$Q_w(\rho_w \| \omega_w) \;:=\; 2\pi\, S(\rho_w \| \omega_w), \tag{5}$$

where $S(\rho\|\omega) = \operatorname{tr}\rho \ln \rho - \operatorname{tr}\rho \ln \omega$ is the Umegaki relative entropy [6], positive and monotone under completely positive trace-preserving maps [12, 13]. The trace formula in (5) is used at the discrete world-cell level assumed by A5; in continuum algebraic-QFT settings the corresponding object is Araki relative entropy for von Neumann algebras [7].

A6: per-address capacity ceiling.

Axiom 1 (A6, capacity, summary form)**.** *At every address w there exist universal capacity ceilings on energy, power, and action throughputs, fixed by $(\hbar, c, \tilde{\ell})$ and saturable but not exceedable.*

In the modular-charge coordinate, the framework translates A6 into an upper bound $Q_w \le 2\pi$. This bound is QTT-internal; it is not a consequence of standard modular theory, in which $S(\rho\|\omega)$ is generally unbounded. The bound reflects the framework's per-address capacity scales $(\hbar/\tilde{\ell}, \hbar c/\tilde{\ell})$.

A7: bundled-existence saturation.

Axiom 2 (A7, bundled existence, summary form)**.** *There is a universal, saturated budget per address equal to one full angular modular period:*

$$Q_w^{\text{bundle}} \;=\; Q_w^{\text{vis}} + Q_w^{\text{hid}} \;=\; 2\pi, \qquad 0 \le Q_w^{\text{vis}} \le 2\pi. \tag{6}$$

A7 is the load-bearing physical statement: the A6 cap is saturated, so Q_w^{bundle} *equals* 2π at every address rather than merely being bounded by it. The visible–hidden split is determined by local dynamics; the sum is universal and reference-invariant in the sense specified in [1]. To exist at all, an excitation must close the dial.

Remark on the visible–hidden additivity. In general quantum information, relative entropy on a tensor-product split with product reference contains a mutual-information correction, $S(\rho_{AB}\|\omega_A \otimes \omega_B) = S(\rho_A\|\omega_A) + S(\rho_B\|\omega_B) + I(A:B)_\rho$. The framework [1] specifies the visible–hidden decomposition such that the additive form in (6) holds at the bundle level; we use the additive form here and refer the reader to [1] for the underlying decomposition convention.

The A7 energy map. The energy map for a saturated bundle, derived in [1] from $\tilde{\ell} = \ell_P$ via A2, reads

$$E_{\text{bundle}}(w) \;=\; \frac{\hbar c}{2\pi\,\ell_P}\, Q_w^{\text{bundle}}, \tag{7}$$

so that $Q_w^{\text{bundle}} = 2\pi$ recovers the Planck values $E_{\text{bundle}} = E_P$, $M_{\text{bundle}} = m_P$. The linear form of (7) is part of A7 in [1].

Remark on units. We keep Q as the dimensionless modular charge, distinct from S in nats. Because $Q = 2\pi\, S$, a saturated bundle ($Q^{\text{bundle}} = 2\pi$) corresponds to one nat of Umegaki relative entropy ($S^{\text{bundle}} = 1$), *not* to 2π nats. Equivalently, the energy map can be written

$$E^{\text{bundle}} \;=\; \frac{\hbar c}{2\pi \ell_P}\, Q^{\text{bundle}} \;=\; \frac{\hbar c}{\ell_P}\, S^{\text{bundle}} \;=\; E_P\, S^{\text{bundle}}, \tag{8}$$

which makes manifest that $E_P/(2\pi)$ is not an energy per nat but an energy per unit of the angular modular-charge coordinate. Within the A7 energy map, the Planck energy is the energy assigned to one nat of Umegaki relative entropy at one A7-saturated bundle.

5 The fifth face

Assumption ledger. The corollary below uses the framework-internal definitions and inputs:

$$Q := 2\pi S(\rho\|\omega), \quad Q_P = 2\pi, \quad E_w^{\text{bundle}} = k_Q\, Q_w^{\text{bundle}}, \quad k_Q = \frac{\hbar c}{2\pi\,\tilde{\ell}}, \quad \tilde{\ell} = \ell_P. \tag{9}$$

Of these, the first is the QTT angular charge convention; the second is A7 saturation; the third is the A7 energy map; the fourth is dimensional reduction at fixed $\tilde{\ell}$; the fifth is A2's IR identification.

Corollary 1 (Modular-charge fifth face of UEL)**.** *Within QTT, given the ledger* (9)*, the four-face Unified Equilibrium Law* (3) *extends to the five-face identity*

$$\boxed{E_P \;=\; m_P c^2 \;=\; \hbar\omega_P \;=\; \rho_{(4)}\,(4\pi\ell_P^4) \;=\; \frac{\hbar c}{2\pi\,\ell_P}\, Q_P} \tag{10}$$

with $Q_P = 2\pi$ and dimensional corrector

$$k_Q \;\equiv\; \frac{\hbar c}{2\pi\,\ell_P} \;=\; \frac{E_P}{2\pi}. \tag{11}$$

Proof. The first four faces and three equalities are (3). The fifth face and fourth equality follow by evaluating the A7 energy map (7) on $Q_w^{\text{bundle}} = Q_P = 2\pi$:

$$\frac{\hbar c}{2\pi\,\ell_P}\,(2\pi) \;=\; \frac{\hbar c}{\ell_P} \;=\; E_P. \tag{12}$$

□

The fifth face is not a new theorem of quantum information theory and not an independently derived physical scale: it is the explicit unification of the modular-capacity face of the Planck unit with the four faces already recognized in UEL.

5.1 The rescaling-invariant content

A natural question is whether k_Q is absolutely fixed. A rescaling of the dimensionless modular-charge coordinate, $Q' = \alpha\,S$ for any positive α, gives $Q'_P = \alpha$ and $k'_Q = E_P/\alpha$. The product is invariant:

$$k'_Q\,Q'_P \;=\; E_P, \tag{13}$$

so the physically invariant content of Corollary 1 is the product

$$\boxed{k_Q\,Q_P \;=\; E_P} \tag{14}$$

at A7 saturation. The split $k_Q = \hbar c/(2\pi\ell_P)$ and $Q_P = 2\pi$ uses the QTT angular convention $Q := 2\pi S$, with $\alpha = 2\pi$ matching the angular modular period. Adopting any other angular convention rescales k_Q inversely; no additional physical coupling is introduced. This identifies what is and is not absolute in the dimensional corrector.

5.2 Standard mathematics, QTT conventions, QTT postulates

To make the structural content of Corollary 1 unambiguous we classify each ingredient in three layers.

Standard mathematics.

- Existence of modular flow $\sigma_t^{\omega_w}(A) = \Delta_w^{it} A \Delta_w^{-it}$ for faithful normal ω_w, with KMS analyticity at $\beta = 1$ in modular time t [8, 9].
- Positivity and CPTP-monotonicity of relative entropy [6, 7, 12, 13].
- Topological invariance of S^1 winding/character structure.
- Bisognano–Wichmann coincidence of Rindler-wedge modular flow with Lorentz boosts; Unruh's $2\pi/a$ thermal factor [10, 11].

QTT conventions.

- Angular modular coordinate $\theta := 2\pi t$, motivated by Bisognano–Wichmann/Unruh geometric modular flow (this reading proposed here).
- Modular-charge coordinate $Q := 2\pi\,S(\rho\|\omega)$ defined in [1], naturally read as one full angular period per nat under the proposed angular convention.
- Internal dial coordinate on S^1 of circumference 2π (A4).

QTT postulates (imported from [1]).

- A6: per-address capacity ceiling, translating to $Q_w \le 2\pi$ in the angular coordinate.
- A7: saturation $Q_w^{\text{bundle}} = 2\pi$ with the visible–hidden additive decomposition specified in [1].
- A7 energy map: linear $E_w^{\text{bundle}} = k_Q\,Q_w^{\text{bundle}}$.

- A2: IR identification $\tilde{\ell} = \ell_P$, fixing the local energy scale to $\hbar c/\ell_P$.

Dimensional consequence. Given the QTT linear A7 map and A2's IR identification, the coefficient is forced: $k_Q = E_P/Q_P = \hbar c/(2\pi\ell_P)$.

5.3 What is and is not claimed

First, Q is the QTT modular charge defined in [1], not ordinary thermodynamic information; no claim is made that everyday bits in computers, detectors, or cold-atom systems carry $E_P/(2\pi)$ of energy — those are governed by the Landauer scale [14, 15]. Second, the equality is established at A7 saturation; off-saturation behaviour follows (7) but does not extend the four-face UEL itself. Third, no new axiom is introduced; the result is a corollary of A7's energy map at saturation, made explicit and equipped with unit-discipline, the rescaling-invariant statement, and the three-layer classification.

6 Bekenstein–Hawking calibration check

The framework's area unit, fixed by A6–A7 in [1], is

$$A_\Sigma \;=\; 8\pi\,\ell_P^2, \tag{15}$$

written Q_Σ in [1] and renamed here to avoid notational clash with the modular charge Q. The Bekenstein–Hawking entropy of a horizon of area A is [3, 4, 5]

$$S_{\mathrm{BH}}/k_B \;=\; \frac{A}{4\,\ell_P^2}. \tag{16}$$

Evaluated on $A = A_\Sigma$,

$$\frac{A_\Sigma}{4\,\ell_P^2} \;=\; \frac{8\pi\,\ell_P^2}{4\,\ell_P^2} \;=\; 2\pi. \tag{17}$$

Reading. Equation (17) states that the standard dimensionless horizon-entropy functional $A/(4\ell_P^2)$, evaluated on the QTT area unit $A_\Sigma = 8\pi\ell_P^2$, gives the same numerical value as the A7 modular-charge coordinate $Q_P = 2\pi$. Within QTT, this reflects the framework's calibration of the area unit to one full angular modular period: the same 2π that closes the modular bundle in A7 closes the horizon area unit in Bekenstein–Hawking accounting, by direct construction in [1]. Since $Q = 2\pi S$, the saturated Umegaki relative entropy of the bundle is $S^{\mathrm{bundle}} = 1$, not 2π:

$$\boxed{\frac{A_\Sigma}{4\,\ell_P^2} \;=\; Q_P \;=\; 2\pi, \qquad S^{\mathrm{bundle}} \;=\; 1\ \mathrm{nat}.} \tag{18}$$

Status. The agreement is a structural feature of QTT's calibration with semiclassical horizon thermodynamics, not an independent observation. Both sides depend on Planck-scale geometric inputs. The applicability of the leading-order semiclassical density at a Planck-scale area unit is subject to the standard caveat that Planck-scale corrections are precisely where quantum-gravity effects would be expected; we read (17) as a calibration identity, not as a precision prediction.

7 Invalidating conditions and operational gap

The sharp content of Corollary 1 is the dimensionless invariant $k_Q\,Q_P/E_P = 1$ at A7 saturation, with invariance over the choice of angular charge convention as in Section 5.1. The structural-vs-scale separation is summarised in Table 1.

Layer	Statement
Invariant (sharp)	$k_Q\, Q_P = E_P$ at A7 saturation.
Coordinate	$k_Q = \hbar c/(2\pi\ell_P)$ is fixed once the QTT angular convention $Q = 2\pi S$ is fixed. The split between k_Q and Q_P is not absolute; the product is.
Scale	The IR identification $\tilde{\ell} = \ell_P$ is inherited from A2 [1].

Table 1: Three layers of the claim.

Invalidating conditions. The following would each rule out the corollary within the QTT framework:

- A reproducible determination of a per-address modular-budget normalization $Q^\star \neq 2\pi$ in any sector compatible with A6 boundary conditions, in the QTT angular convention (would invalidate A7 saturation).
- An independent measurement of a saturated-bundle energy $E_{\text{bundle}} \neq \hbar c/\ell_P$ at fixed $\tilde{\ell} = \ell_P$ (would invalidate (7)).

Operational gap. At present, QTT does not provide an operational protocol that reconstructs Q_w from independent observables. Without such a protocol the invalidating conditions are theory-internal: they say that if A7's postulates change, the corollary changes. Closing this gap — by either (i) a controlled physical realization of an A6-saturable bundle with independent measurements of E and Q, or (ii) a derivation of A_Σ in [1] that is independent of the angular charge convention so that (17) becomes a non-trivial check — is a precondition for the corollary to acquire empirical falsifiability. Both directions are open problems.

Scope of applicability. The fifth face applies to QTT modular capacity at a Planck address. It does not apply to ordinary low-energy information storage [14, 15], and it does not displace the four-face UEL, which continues to hold in the absence of any modular bookkeeping.

8 Discussion

We have shown that, within QTT's ontological reconstruction, the Unified Equilibrium Law admits a fifth dimensionally consistent face — information capacity, expressed as the modular charge of an A7-saturated address bundle — with no new axiom and no tunable coupling. The fifth face is forced by the framework's existing axioms; the load-bearing physical input is A7 saturation, and the rescaling-invariant content is the product $k_Q\, Q_P = E_P$.

Three structural unification features within QTT. First, the five-face UEL places modular capacity on the same ontological footing as mass, frequency, and four-density: each is converted to energy by a parameter-free corrector once the angular charge convention is fixed. Information is not added on top of physics within QTT — it is one of the equivalent faces of the primitive Planck capacity. Second, at saturation the corollary collapses to the compact identity

$$E_P \;=\; E_P\, S^{\text{bundle}}, \qquad S^{\text{bundle}} \;=\; 1 \text{ nat}, \tag{19}$$

which reads, within the A7 energy map: *within QTT, the Planck energy is the energy assigned to one nat of Umegaki relative entropy at one A7-saturated bundle.* This is a coordinate-free statement and a direct consequence of A2 + A7. Third, the same 2π that closes the modular bundle in A7 closes the internal S^1 dial in A4, calibrates the QTT area unit $A_\Sigma = 8\pi\ell_P^2$ to one angular modular period (Section 6), and drives the spinor half-angle theorem of the parallel structural-identification paper [2] via the same real-dial Pauli algebra. The recurrence of 2π across these constructions is structural, not coincidental: the framework closes its dial once, and several Planck-scale quantities inherit the closure.

Caveats acknowledged honestly. Three caveats sit alongside the unification reading. (i) A7 saturation is the new content; we do not derive A7 in this article. (ii) The angular modular coordinate motivated by Bisognano–Wichmann/Unruh is a reading proposed here; standard Tomita–Takesaki normalization gives $\beta = 1$ in modular time t, and the 2π enters only through the angular reparametrization. (iii) Operational reconstruction of Q_w is open, and the corollary's invalidating conditions are theory-internal until that gap is closed.

One-line summary. Within QTT, at fixed $\tilde{\ell} = \ell_P$, the fifth face of UEL is fixed without tunable parameters: $E_P = \rho_{(4)}(4\pi\ell_P^4) = (\hbar c/2\pi\ell_P)\, Q_P = E_P\, S^{\text{bundle}}$, with $Q_P = 2\pi$ and $S^{\text{bundle}} = 1$ nat at A7 saturation.

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