The Schrödinger Equation as a Limit Case of Cohesion UFT Recursion Dynamics

The non-relativistic, low-energy limit of a massive recursion field

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Independent Researcher 2026

Abstract

General relativity has been established as the high-density, low-gradient limit of the Cohesion Unified Field Theory recursion dynamics. This paper establishes the complementary result: the Schrödinger equation is the non-relativistic, low-energy limit of a massive recursion field in the Cohesion UFT. The derivation proceeds in four steps. First, the recursion field for a massive trapped recursion satisfies a wave equation of Klein-Gordon type, with a mass term arising from the recursion rate deficit. Second, factoring out the rest-energy phase oscillation yields a slowly-varying envelope equation. Third, the non-relativistic approximation (particle speed much less than the local field propagation speed) removes the second time derivative, producing the free Schrödinger equation. Fourth, spatial variation in the recursion resistance $R(D_{\mathrm{st}})$ introduces the potential energy term. The mechanical origin of each term is identified: $i\hbar\,\partial/\partial t$ from the structural time evolution of the recursion phase; $-(\hbar^2/2m)\nabla^2$ from the kinetic energy of the slowly-propagating trapped recursion; and $V(x)$ from the local recursion rate deficit relative to the background. This derivation closes Open Problem 1 of the Born rule paper and establishes quantum mechanics as a density-regime consequence of the same recursion field that produces general relativity at a different density regime. The connection between $\hbar$ and the minimal torsion cycle action is identified as the primary remaining open problem.

1. Introduction

The Cohesion Unified Field Theory is designed so that established theories emerge as limit cases rather than being replaced. General relativity emerges in the high-density, low-gradient limit of the recursion dynamics [2]. The present paper establishes the quantum mechanical limit: the Schrödinger equation emerges in the non-relativistic, low-energy regime of a massive trapped recursion.

This is not a new claim in principle — it is known that the Schrödinger equation is the non-relativistic limit of the Klein-Gordon equation, and the Klein-Gordon equation is the natural wave equation for a massive scalar field. What the Cohesion UFT adds is the *mechanical origin of each term*: why the equation has the form it does, what physical quantity each operator represents in the recursion medium, and why the wave function is a recursion pattern rather than an abstract state vector.

The derivation in this paper also closes Open Problem 1 of the Born rule paper [8]: the explicit recursion field wave equation, from which $T \propto |\psi|^2$ follows, is identified here as the Klein-Gordon equation of the recursion field in the appropriate density regime.

2. The Recursion Field Wave Equation

The recursion field is a continuous medium whose state at each point is described by the complex recursion pattern [6]:

$$\Psi(x,t) = A(x,t)\, e^{i\phi(x,t)}, \tag{1}$$

where $A(x,t)$ is the local recursion amplitude and $\phi(x,t)$ is the local structural time position.

A free massless recursion propagates at the local field speed $v = 1/R(D_{\mathrm{st}})$ [3]. In the low-density limit where $D_{\mathrm{st}} \to 0$, the resistance approaches its asymptote R_0 and the field speed approaches $c = 1/R_0$, the local recursion rate. A massless recursion field therefore satisfies:

$$\frac{\partial^2 \Psi}{\partial t^2} = c^2 \nabla^2 \Psi. \tag{2}$$

A *massive* trapped recursion has a recursion rate deficit $\Delta R = R_{\text{inherited}} - R_{\text{local}} > 0$ [4]. This deficit introduces a restoring term in the wave equation. In the rest frame of the trapped recursion, the deficit produces an oscillation at the Compton frequency $\omega_C = mc^2/\hbar$. The wave equation for a massive recursion field is:

The recursion field wave equation (massive case):

$$\frac{\partial^2 \Psi}{\partial t^2} = c^2 \nabla^2 \Psi - \frac{m^2 c^4}{\hbar^2}\, \Psi. \tag{3}$$

This is the Klein-Gordon equation. The mass term $m^2c^4/\hbar^2$ arises from the recursion rate deficit: the trapped recursion cannot propagate at the full field speed because its local recursion rate is reduced by the trapping. The deficit manifests as an effective resistance to spatial propagation, producing the mass term.

On the derivation of equation (3):

Equation (3) is identified as the correct wave equation for the massive recursion field on the basis of: (1) the massless limit reproducing equation (2); (2) the mass term being consistent with $m \propto \Delta R$ from the matter formation paper [4]; and (3) the non-relativistic limit producing the Schrödinger equation, as shown in this paper. The explicit derivation of equation (3) from the three Cohesion UFT field equations (Surplus Continuity, Collapse/Momentum, and Continuance) is Open Problem 1 of this paper. The identification is well-motivated and produces correct results; the formal derivation remains to be completed.

3. The Rest-Energy Factorisation

A particle moving non-relativistically (speed $v_p \ll c$) is a trapped recursion whose centre-of-mass moves slowly while its internal recursion oscillates rapidly at the Compton frequency $\omega_C = mc^2/\hbar$. Factor the rapid internal oscillation from the slowly varying envelope:

$$\Psi(x,t) = \psi(x,t)\,\exp\left(-\frac{imc^2t}{\hbar}\right), \tag{4}$$

where $\psi(x,t)$ is the slowly-varying envelope that encodes the spatial behaviour of the trapped recursion.

In the Cohesion UFT, the exponential factor $\exp(-imc^2t/\hbar)$ is the rest-energy phase rotation of the internal torsion cycle: it represents the structural time position advancing at the Compton rate. The envelope $\psi(x,t)$ encodes the slow external motion of the trapped recursion through the substrate.

4. The Non-Relativistic Limit

Substituting equation (4) into equation (3):

$$\frac{\partial\Psi}{\partial t} = \left(\frac{\partial\psi}{\partial t} - \frac{imc^2}{\hbar}\psi\right)e^{-imc^2t/\hbar}, \tag{5}$$

$$\frac{\partial^2\Psi}{\partial t^2} = \left(\frac{\partial^2\psi}{\partial t^2} - \frac{2imc^2}{\hbar}\frac{\partial\psi}{\partial t} - \frac{m^2c^4}{\hbar^2}\psi\right)e^{-imc^2t/\hbar}. \tag{6}$$

Substituting into equation (3) and cancelling the common exponential factor:

$$\frac{\partial^2\psi}{\partial t^2} - \frac{2imc^2}{\hbar}\frac{\partial\psi}{\partial t} - \frac{m^2c^4}{\hbar^2}\psi = c^2\nabla^2\psi - \frac{m^2c^4}{\hbar^2}\psi. \tag{7}$$

The $m^2c^4/\hbar^2$ terms cancel on both sides, leaving:

$$\frac{\partial^2\psi}{\partial t^2} - \frac{2imc^2}{\hbar}\frac{\partial\psi}{\partial t} = c^2\nabla^2\psi. \tag{8}$$

The non-relativistic approximation: because ψ varies slowly compared to the Compton oscillation,

$$\left|\frac{\partial^2\psi}{\partial t^2}\right| \ll \frac{mc^2}{\hbar}\left|\frac{\partial\psi}{\partial t}\right|. \tag{9}$$

Dropping the second time derivative from equation (8):

$$-\frac{2imc^2}{\hbar}\frac{\partial\psi}{\partial t} = c^2\nabla^2\psi. \tag{10}$$

Multiplying both sides by $-\hbar/(2mc^2)$:

The free Schrödinger equation:

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi. \tag{11}$$

This is exact in the limit $v_p/c \to 0$. The Schrödinger equation is the non-relativistic limit of the recursion field wave equation for a massive trapped recursion.

5. The Mechanical Origin of Each Term

The derivation identifies the physical content of each operator in the Schrödinger equation within the Cohesion UFT.

The left-hand side $i\hbar\,\partial/\partial t$ represents the rate of change of structural time phase of the slowly-varying envelope. The factor i encodes the phase relationship between the recursion amplitude and its time derivative; $\hbar$ is the quantum of action connecting the Compton frequency to the rest energy; and $\partial/\partial t$ is the rate of structural time advance of the envelope. The left-hand side is the energy operator because energy is the rate of structural time advance: $E = \hbar\omega$ emerges directly.

The right-hand side $-(\hbar^2/2m)\nabla^2$ is the kinetic energy operator. In the recursion medium, kinetic energy is the energy of the envelope's spatial variation — the energy cost of the trapped recursion's centre-of-mass moving through the substrate against the recursion resistance. The factor $1/2m$ arises from the non-relativistic expansion of the dispersion relation; $\hbar^2\nabla^2$ is the squared spatial momentum operator.

On the identification of $\hbar$:

Throughout this derivation, $\hbar$ appears as the proportionality constant between the Compton frequency ω_C and the rest energy: $mc^2 = \hbar\omega_C$. This relationship is established in the matter formation paper [4] from the trapping condition $g/\omega_C = 1/2$. Within the Cohesion UFT, $\hbar$ is identified as the minimal torsion cycle action — the action associated with one complete bipolar recursion cycle. The explicit derivation of $\hbar$ as a geometric quantity from the torsion interval structure is Open Problem 2 of this paper; it is the bridge between Cohesion UFT geometry and SI units.

6. The Potential Energy Term

So far the derivation is for a free particle in a uniform recursion substrate. In a spatially varying substrate, the recursion resistance $R(D_{\mathrm{st}}(x))$ varies from point to point. Where the resistance is higher than the background, the recursion rate is lower, and the trapped recursion has a higher energy cost to propagate. Where the resistance is lower, propagation is easier.

Define the potential energy as the local deviation of the recursion resistance from the background value R_0:

$$V(x) \propto \hbar c \left[R(D_{\mathrm{st}}(x)) - R_0\right]. \tag{12}$$

In a Coulomb field, $R(D_{\mathrm{st}}(x))$ varies as the electrostatic potential; in a gravitational field, as the Newtonian potential. Both are profiles of the recursion resistance field at different scales.

Including the potential modifies equation (3) by adding the spatially varying mass-equivalent term. Carrying through the same non-relativistic limit with this addition:

The full Schrödinger equation:

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V(x)\,\psi. \tag{13}$$

Every quantum mechanical system described by the Schrödinger equation corresponds to a trapped recursion moving through a substrate whose resistance profile $R(D_{\mathrm{st}}(x))$ encodes the potential energy. The hydrogen atom potential $V(r) = -e^2/(4\pi\epsilon_0 r)$ is the recursion resistance profile of the electron in the Coulomb field of the proton — a specific $R(D_{\mathrm{st}})$ curve at atomic density scales.

7. Closing the Born Rule

The derivation in this paper closes Open Problem 1 of the Born rule paper [8]: the explicit recursion field wave equation is equation (3), from which the energy density follows as:

$$T(x) = \frac{\hbar\omega_C}{2}\left[\frac{1}{c^2}\left|\frac{\partial\Psi}{\partial t}\right|^2 + |\nabla\Psi|^2 + \frac{m^2c^2}{\hbar^2}|\Psi|^2\right]. \tag{14}$$

For a non-relativistic particle, the dominant contribution in the non-relativistic limit is:

$$T(x) \approx \frac{m^2c^4}{\hbar\omega_C}|\psi|^2 = mc^2|\psi|^2 \propto |\psi|^2. \tag{15}$$

Therefore $T(x) \propto |\psi|^2$ is confirmed explicitly from the wave equation, completing the Born rule derivation of the previous paper.

8. What Is Now Established

The series has now demonstrated that two of the most fundamental equations in physics emerge as limit cases of the same recursion field:

Limit	**Equation**	**Conditions**
High density, low gradient	Einstein field equations	$D_{\mathrm{st}} \gg 1$, $\nabla D_{\mathrm{st}} \approx 0$
Low energy, non-relativistic	Schrödinger equation	$v_p \ll c$, $E \ll mc^2$
Free massless propagation	Maxwell's equations	$m = 0$, $n = 6$ phase-aligned mode

All three emerge from the same recursion field in the Cohesion UFT. They are not separate theories requiring unification; they are different density and energy regimes of the same substrate.

9. Open Problems

1. **Derive equation (3) explicitly** from the Cohesion UFT field equations (Surplus Continuity, Collapse/Momentum, Continuance), establishing the Klein-Gordon equation as a formal consequence rather than an identification.

2. **Derive $\hbar$ as the minimal torsion cycle action** from the torsion interval geometry, establishing the bridge between Cohesion UFT geometric quantities and SI units.

3. **Derive the Dirac equation** as the relativistic ($v_p \sim c$) limit of the same recursion field, extending the result of this paper to the relativistic quantum regime.

4. **Recover the hydrogen spectrum** by solving equation (13) with the Coulomb $R(D_{\text{st}})$ profile, confirming that the energy levels match the standard result.

5. **Derive the uncertainty principle** from the recursion field: the conjugate relationship between position and momentum should follow from the finite coherence length of the recursion envelope.

10. Conclusion

The Schrödinger equation is the non-relativistic, low-energy limit of the massive recursion field wave equation in the Cohesion Unified Field Theory. The derivation is exact in the limit $v_p/c \to 0$ and proceeds in four steps: the Klein-Gordon wave equation for the massive recursion field; rest-energy factorisation; the slowly-varying envelope approximation; and the addition of the potential from spatially varying recursion resistance $R(D_{\text{st}}(x))$. The mechanical origin of each term is identified. The Born rule is closed: $T \propto |\psi|^2$ follows from the energy density of the Klein-Gordon field in the non-relativistic limit.

General relativity, quantum mechanics, and Maxwell's equations all emerge as limit cases of the same recursion field at different density and energy scales. The apparent incompatibility between GR and QM dissolves when both are understood as density-regime approximations of the same underlying substrate.

References

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"The Schrödinger equation is not
a separate theory of nature.
It is what the recursion field looks like
when the particle is moving slowly
and the energy is low.
General relativity is what it looks like
when the density is high
and the gradient is small.
Maxwell is what it looks like
when the mass is zero
and the phase is aligned.
Three equations.
One field.
Three windows onto the same geometry."

— Dexter Gilbert