PROJECTION SPACE

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Abstract. We define the project map $\varphi : \mathbb{C} \to \mathbb{R}, \quad \varphi(a + bi) = a + b$. This map preserve's addition but not multiplication. We examine its properties and discuss its potential relevance to the. Riemann Zeta function were collapse in the complex argument to a single real variable may provide a different perspective on the critical line $Re(s) = 1/2$. Explicit calculations are provided in the appendix.

Contents

1. Introduction

The study of distances in the applying traces back to the ancient Greek geometry with Pythagorean theorem known to Babylon scribes as early as. 19,000 BCE. This was later formalized by Greek mathematicians. Established that for a right triangle reflects A and B, the hypothesis satisfies c^2 =a^2 + b^2. This relationship remained purely geometric for over two millennia. The decisive shift towards. Algebra came in 1637 with René Descartes' "La Géométrie", Which introduce coordinates to the plane. With it, points could be represented as pairs. (x,y) And distance is computed via $\sqrt{x^2 + y^2}$. The cautation framework implicitly through the convex dumpers as points. (a,b) With the formal identification A + bi, Go to full AGA price structure of complex numbers was developed by Bombardier in 1572 and later Euler 1748. a+bi, though the full algebraic structure of complex numbers was developed by Bombelli (1572) and later Euler (1748). But in 19 century the complex analysis had put you into a branch of mathematics that was central and Augustine, Louis Dauchy and Bernard Riemann developed the theory of analytic functions, where rumors 1851 doctoral thesis establishing the geometric foundations and complex variables. Riemann later in 1859 and his memoir on the distribution of prime numbers introduced the Zeta function $\zeta(s)$ for complex numbers denoted by $s = \sigma + it$ Famously conjectured that all non trivial zeros satisfy $\sigma = 1/2$. The focus then was almost entirely on the complex domain. G.H. Hardy Move to

1914 that infinitely many zeros lie on the critical line. However, Hardy and Littlewood in 1921 established conditional results linking the zeros to prime distribution, where Selberg in 1942 charged a position proper proportion lie under line. Conry in 1989 then raised this proportion to approximately 40%. By doing that, however, all of these approaches treated the real and imaginary parts of s as independent coordinates. We apply the real-valued projection φ to complex numbers. The formula $\sqrt{a^2+b^2}$ first appears geometrically as the length of the hypotenuse of a right triangle with perpendicular sides a and b, in ~500 BC. The pythagoras theorem denotes the first variant of it denoted by $c^2 = a^2 + b^2$. René Descartes then in 1637 introduced the coordinates (x,y) on a plane, where the distance from $(0, 0)$ to (x, y) denoted by $\sqrt{a^2+b^2}$. This is the same Pythagorean formula, now in a numerical coordinate system. With it Descartes effectively turned geometry into algebra by denoting the algebraic characters a and b as part of the coordinate. In 1859 Riemann [1] conjectured all non-trivial zeros lie on $Re(s) = 1/2$. He based this on a zeta function denoted by zeta function $\zeta(s)$. This implied that all non-trivial zeros have a real part 1/2. Hardy **hardy1914?**, in 1914, then proved that infinitely many are on that the critical line $Re(s) = 1/2$. This is the first unconditional result showing the line is not empty but does not prove the case of possible zeros off it. However, given the cardinality of the problem, utilizing 1D reduction of a complex number can reduce the complexity of the problem by reinterpreting the problem as one of the sum of the complex number values and not an isolated problem on the real part of the complex numbers. We denote the standard methodology as $\sqrt{(a^2+b^2)}$. This treats a and b as independent squared lengths. Instead, we use in this paper the sum of a and b and denote: $\sqrt{((a+b)^2)}$ as form. The dimensionality reduction also leads to loss of information as we denote in the preliminaries. The none-invertible projection relies only on addition as the other properties of standard multiplication, injectivness and more are not required.

2. **Preliminaries**

2.1. **Arithmetic Operation P.** We define $P((a+b)\times i) = (a^2+b^2)\times(-1)$ as new arithmetic operation within this formal system.

Under this complex number i is -1. Because -1 times a sum of a real numbers does result in the same total value we can collapse P to φ

2.2. **Projection Complex Number.** We denote projection complex number as

$$s = (a+b)i$$

Where we take the sum of $a+b$ and then multiply it with i.

2.3. **Underlying Set.** Let

$$S := C$$

as set of ordered pairs:

$$x = a + bi \mod (a, b), a, b \in R$$

Without change to the elements themselves, only to interpretation rules.

2.4. **Projective map.** Let projection be

$$\varphi : C \to \mathbb{R}, \quad \varphi(a+bi) = a+b$$

This is a lossy real-valued compression of S.

2.5. **Addition in S.** Standard complex addition is retained:

$$(a+bi) \oplus (c+di) = (a+c) + (b+d)i$$

Projection interaction:

$$\varphi(z_1 \oplus z_2) = \varphi(z_1) + \varphi(z_2)$$

So additive structure is preserved under projection.

2.6. **Multiplication in S.** Standard multiplication is retained:

$$(a+bi) \otimes (c+di) = (ac-bd) + (ad+bc)i$$

However, the projection of S is not multiplicative as we denote as:

$$\varphi(z_1 \otimes z_2) \neq \varphi(z_1)\varphi(z_2)$$

So multiplication is not preserved under projection.

2.7. **Cohomology.** Let $\underline{\mathbb{R}}_S$ be the constant sheaf on S. Then we denote the map

$$\varphi : S \to \mathbb{R}$$

Which induces

$$\varphi^* : H^0(\mathbb{R}, \underline{\mathbb{R}}) \to H^0(S, \underline{\mathbb{R}})$$

Remark. φ^* is an isomorphism because φ preserves addition and both spaces are connected. As both spaces are contractible, there is no higher cohomology.

2.8. **Allowed operations.** The allowed operations are: addition on a, b, squaring real numbers, square root of nonnegative reals. The forbidden operation are complex multiplication unless rewritten in real form.

3. **Results**

3.1. **Theorem.** The isomorphism $C \cong \mathbb{R}^2$ holds.

Proof. Let

$$\mathbb{C} \cong \mathbb{R}^2$$

With natural identification map

$$(a + bi) \longleftrightarrow (a, b)$$

Where the map is one to one, onto and preserves addition a scalar multiplication, thus it is a vector space isomorphism over R.

3.2. **Observation.** The results using arithmetic P are the same as in normal arithmetic but under constraint arguments as showcased in the Appendix, such are two values with the same magnitude but different signs, result into the total value under Arithmetic P to become 0 for all equal sized real numbers. This is different hen normal arithmetic as in it the value would result into a root none 0 but in the system P it results into root fo P. This indicates that there is a collapse possibility during the calculations, before the the square root operation.

3.3. **Conjecture.** Several mathematicians have attempted to modify the Riemanns at a function power terminates argument structure. However, I not have to answer in a manner purpose in this paper. Perverts in 1882 introduced a generalized zeta function

$$\zeta(s, a) = \sum_{n=0}^{\infty} \frac{1}{(n + a)^s}$$

Which changed the assignment from N to N + A but left the complex numbers $s = \sigma + it$ completely untouched. Lerch in 1887 went further with the large Zeta function denoted by

$\Phi(z, s, a) = \sum_{n=0}^{\infty} \frac{z^n}{(n+a)^s}$

Which added a complex phase factor z^n. That modified the entire algebraic structure of the sum. But the argument s remained a standard complex number with 2 independent dimensions. Three months old analytic continuation extended the Zeta function from half plane with rear part greater than one to all as not equal to 1 via contour integration, which was a modification of the domain of S but not of the form of S itself which stayed as $\sigma + it$. Harry and Littlewood in the 1920s developed approximation function equation which was studying this other function for S near the critical line by approximating the infinite sum of definite expressions. The method of evaluation. Was changed but again preserved a two dimensional complex input. Then touring a 1943 Perform computational verification of zeros which was using Turing Gram method working entirely with the standard presentation $\sigma + it$. However, more recently, spectral interpretations. Function, which is following Hilbert and Polya's famous suggestion, attempts to find a Hermitian operator whose eigenvalue correspond to the non trivial zeros. This however changes the framework entirely, but still respects a complex nature of zeros as points in the plane. In all these cases, though, the input to the Zeta function remains fundamentally two dimensional, with a real part and an imaginary part treated as independent quantities. The projection space introduced in this paper collapses the complex argument $\sigma+it$ Into a single real number $\sigma+t$ Via the projection map $\varphi(a+bi) = a+b$, Thereby reducing the problem from two dimensions to one dimension while preserving additive structure. For the conjecture we note the zeta function:

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}, \quad \mathrm{Re}(s) > 1$$

Where the explicit domain is:

$$s = \sigma + it, \quad \sigma > 1, \ t \in \mathbb{R}$$

But his conjecture denotes that all 0s might lie on $\mathrm{Re}(s) > 1/2$. With this our space S and projected complex numbers we can break down the input of the Zeta function by not requiring the imaginary parts i and collapse the complex number from $2D$ to 1D$ this which means to change from $a+bitoa+b$ as in the computational intermediate step it becomes -1 and thus trivial as it gets multiplied by the sum of $a+b$. Then we get the simplified Zeta function domain of where the domain is

$$s = (\sigma + t), \quad \sigma > 1, \ t \in \mathbb{R}$$

This reduces the complexity of the zeta function and shows that the result lives in the projection. It also gives us a new angle via the Real Line instead of the complex plane which might reveal symmetries.

Proof Sketch. If s equals the sum of the computed value of a complex number and the projection complex number is equal to a complex number then if projection complex number a and complex number b, $a = b$ for s in the zeta function.

4. **Commutativity Diagram**

The Maple structures the simpler ones while preserving essential operations of modern arithmetics. This idea was systematically developed during the 19th century through the work of Everest Galloway and Augustine Louise Couche. And later it was given a Structucual formualtion by Emmy Noether. In the context of group theory, such reductions are formalized via homomorphism, which are maps between algebraic structures that preserve the relevant operations. Such mappings enable the study of given structure through its image in a simpler or more tractable setting. This facilitates the analysis of otherwise intractable problems.

Concurrently, development of complex analysis introduced canonical examples of such structure preserving maps. Complex numbers admit natural projections onto the real numbers. The real part which assigns to each complex number its real component constitutes the homophsim, with respect to addition, as it satisfies the identity

$$\mathfrak{R}(z_1 + z_2) = \mathfrak{R}(z_1) + \mathfrak{R}(z_2)$$

This property establishes it as morphism of additive groups and thus provides a concrete instance of high how high dimensional algebraic objects can be systematically reduced while preserving the linear structure. This historically has led to projections as playing out an important role not only in pure mathematics but also in applied disciplines were complex. Valued formulations are routinely interpreted via their real components.

80 Per manifestation of these ideas appears An analytic number 0. Were specific focus on the work of Bernard Riemann on the Zeta function. The associated functional equation exhibits a symmetry relating values of S and 1 -, S, thereby inducing a reflection about the critical line defined by value Re(s) =1/2. The conjecture that all non trivial zeros lie on this line is the idea behind the Riemman Conjecture. This means that. Particularly in pure math as well. It shows. That we're part is as important as in other fields. Such as the structure invariant that comes from the Ripper. The symmetry has motivated approaches that attempt to project complex analytic phenomena onto real variable frameworks with the aim of reducing analytical complexity while presumably essential features of the problem. Those structural principles were further obstructed, however, in the mid 20th century to the development of category theory by Samuel Eilenberg in Saunders Mclean. Mathematical objects are studied in terms of their relationships within this framework. And are encoded as morphisms. In the preservation of structure is expressed to commutative diagrams. Those diagrams formalized the requirement that different compositions of morphisms yield identical results, thereby ensuring coherence across transformations. The notion, however, of reducing complexity via structure preserving maps is thus subsumed into a general categorical language.

These ideas. May be interpreted as an early form of dimensional reduction, which is the process of mapping a higher dimensional structure onto a lower dimensional one in a manner that preserves specified operations. However, such rejections are inherently limited, such as the algebraic properties of the mapping. For instance, while a rear part map preserves addition, does not preserve multiplication, and hence cannot surface for more morphism or fields. Such limitations reflect fundamental constraints that have been understood since the formalization of algebraic structures, however. In this paper we just want to introduce a commutative diagram with C being a complex number in the R as a real number, which follows from the definitions defined prior. We denote this commutative diagram as

$$
\begin{tikzcd}
C \arrow[r, "\text{standard op}"] \arrow[d, "\phi"'] & C \arrow[d, "\phi"] \\
R \arrow[r, "\text{simpler op}"] & R
\end{tikzcd}
$$

As addition holds, we can work entirely in R and still get the right answer in C as the commutativity assures this for addition. This creates a reduced complexity that is not only beneficial for the conjectured Re(s)=1/2 but also possibly other topics.

[1] [2] [3] [4] [5] [6] [7] [8] [9] [10] [11] [12] [13] [14] [15] [16] [17]

5. References

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6. Appendix: Explicit calculations

For these examples use the definition of the “Real-valued projection” operation. The following operations are only allowed arithmetic ruleset in S:

6.1. **Equality.** We denote the complex number

$$s = (2 + 2) \times (-1)^1/2$$

We then compute

$$s = (2^2 + 2^2) \times (-1)$$

Then

$$(4 + 4) \times (-1) = 8$$

Then we take the square root of 8 and denote and omit its sign

$$\sqrt{8}$$

6.2. **Unequal.** We denote the complex number

$$s = (3 + 2) \times (-1)^1/2$$

We then compute

$$s = (3^2 + 2^2) \times (-1)$$

Then

$$(9 + 4) \times (-1) = -13$$

Then we take the square root of -13 and denote and omit its sign

$$\sqrt{13}$$

6.3. **One zero.** We denote the complex number

$$s = (0 + 2) \times (-1)^1/2$$

We then compute

$$s = (0^0 + 2^2) \times (-1)$$

Then

$$(0 + 4) \times (-1) = -4$$

Then we take the square root of -4 and denote and omit its sign

$$\sqrt{4}$$

6.4. **Cancellation.** We denote the complex number

$$s = (-2 + 2) \times (-1)^1/2$$

We then compute

$$s = (-2^2 + 2^2) \times (-1)$$

Then

$$(-4 + 4) \times (-1) = 0$$

Then we take the square root of 0 and denote and omit its sign

$$\sqrt{0}$$

Remark. The space works but as this example shows, it loses information including for cases such as $a + b = 0$.

6.5. **Both negative.** We denote the complex number

$$s = (-2 + -2) \times (-1)^1/2$$

We then compute

$$s = (-2^2 + -2^2) \times (-1)$$

Then

$$(-4 + -4) \times (-1) = 8$$

Then we take the square root of 8 and denote and omit its sign

$$\sqrt{8}$$

6.6. **None-Zero Mixed signs.** We denote the complex number

$$s = (-2 + 5) \times (-1)^1/2$$

We then compute

$$s = (-2^2 + 5^2) \times (-1)$$

Then

$$(-4 + 10) \times (-1) = -6$$

Then we take the square root of -6 and denote and omit its sign

$$\sqrt{8}$$