Supplemental Material for

Screened Copy-Time Transport at Cosmic Dawn: A Microscopic Onsager Mechanism for Early Galaxy Compaction and Black-Hole Seed Growth

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April 2026

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1 Fisher projection in matrix form

Let $g_A(Y)$ be the scores of the variables already evolved by the radiation-hydrodynamic solver. Define the Fisher covariance matrix

$$F_{AB} = \langle g_A g_B \rangle, \qquad b_A = \langle s_Q g_A \rangle. \tag{1}$$

Assuming F is invertible on the retained score space, the projected score is

$$s_Q^{\parallel} = g_A (F^{-1})^{AB} b_B, \qquad s_Q^{\perp} = s_Q - s_Q^{\parallel}. \tag{2}$$

The residual Fisher rate is therefore

$$\dot{\mathcal{I}}_Q^{\perp} = \dot{\mathcal{I}}_Q^{\mathrm{raw}} - b_A (F^{-1})^{AB} b_B. \tag{3}$$

Equation (3) is the Schur complement of the Fisher matrix. It is nonnegative. It also gives the actual numerical recipe for post-processing a simulation: build the local score covariance of record variables and subtract the component explained by baseline RHD variables.

2 Radon-Nikodym derivation of the entropy term

Let Y denote residual receiver records after projection. For N_b baryons in a mesoscopic cell, conditional independence over copy units gives the likelihood ratio

$$\frac{\mathrm{d}P(Y \mid N_b, \psi)}{\mathrm{d}P(Y \mid N_b, 0)} = \exp\left(q_Q N_b \psi - \Phi(\psi)\right), \tag{4}$$

where Φ is the normalization cumulant. The part linear in N_b contributes to the coarse-grained entropy; the cell-independent cumulant cancels from baryon-number variations. Passing to the continuum gives

$$\log \frac{\mathrm{d}\Gamma_{\mathrm{cg}}}{\mathrm{d}\Gamma_0} = \int q_Q n_b \psi \, \mathrm{d}^3 x + \text{terms independent of } n_b. \tag{5}$$

Thus

$$\frac{\delta S}{\delta n_b} = -\frac{\mu_B}{T} + q_Q \psi. \tag{6}$$

This is the origin of the copy-time force. No separate mechanical force is inserted.

3 General p-body coefficient

For a p-body channel

$$R = K \int_{V_\ell} n_1 n_2 \cdots n_p \, \mathrm{d}^3 x, \tag{7}$$

write $n_a = \bar{n}_a(1 + \delta_a)$. At fixed cell means,

$$\frac{R}{R_0} = 1 + \sum_{a<b} C_{ab} + \sum_{a<b<c} C_{abc} + \cdots, \tag{8}$$

where $C_{ab} = \langle \delta_a \delta_b \rangle$. If the covariance response to a long mode is $C_{ab} = \chi_{ab,2}\delta_L^2 + O(\delta_L^3)$, then

$$\frac{\partial R^{\perp}}{\partial \delta_L} = 2R_0 \left(\sum_{a<b} \chi_{ab,2} \right) \delta_L + O(\delta_L^2). \tag{9}$$

The Poisson Fisher coefficient is

$$A_2 = 4R_0 \left(\sum_{a<b} \chi_{ab,2} \right)^2. \tag{10}$$

For equal species response $\chi_{ab,2} = \chi_2$, Eq. (10) reduces to

$$A_2 = \left[p(p-1)\chi_2\right]^2 R_0. \tag{11}$$

This derivation is the source of the main-text channel table. One-body rates have no covariance pair and therefore vanish after projection.

4 Resolved clumping estimator

For a smooth field in a cell centered at the origin,

$$\delta_a(\mathbf{x}) = \mathbf{x} \cdot \nabla \log n_a + O(\ell^2 \nabla^2 \log n_a). \tag{12}$$

For a cube of side ℓ, $\langle x_i x_j \rangle = \ell^2 \delta_{ij}/12$, hence

$$C_{ab,\ell} = \langle \delta_a \delta_b \rangle = \frac{\ell^2}{12} \nabla \log n_a \cdot \nabla \log n_b + O(\ell^4 \nabla^4). \tag{13}$$

This estimator is conservative: unresolved turbulent clumping can only be added by an independently calibrated closure. In the absence of such a closure, set the subgrid contribution to zero.

5 Molecular exclusion numbers

With $D = 10^{11}\,\mathrm{m^2\,s^{-1}}$,

$$t_{\mathrm{diff}}(10\,\mathrm{pc}) = \frac{(10 \times 3.086 \times 10^{16}\,\mathrm{m})^2}{10^{11}\,\mathrm{m^2\,s^{-1}}} = 9.52 \times 10^{23}\,\mathrm{s} = 3.02 \times 10^{16}\,\mathrm{yr}. \tag{14}$$

At 100 pc the result is 100 times larger. Molecular copy-time diffusion is therefore excluded as a galaxy-scale compaction channel by dimensional analysis.

6 Hyperbolic stability

Linearizing the Maxwell-Cattaneo current for $\psi = 0$ and constant coefficients gives

$$s\delta n + \mathrm{i}\mathbf{k} \cdot \delta\mathbf{J} = 0, \tag{15}$$

$$\tau_\ell s \delta\mathbf{J} + \delta\mathbf{J} = -D\mathrm{i}\mathbf{k}\delta n. \tag{16}$$

Eliminating $\delta\mathbf{J}$ gives

$$\tau_\ell s^2 + s + Dk^2 = 0. \tag{17}$$

The roots are

$$s_\pm = \frac{-1 \pm \sqrt{1 - 4\tau_\ell D k^2}}{2\tau_\ell}. \tag{18}$$

For all real k, $\operatorname{Re} s_\pm \le 0$. At large k, the signal speed is $\sqrt{D/\tau_\ell}$. This proves the stability and supplies the CFL condition.

7 Finite-volume insertion algorithm

For each hydrodynamic step in a RAMSES/ENZO/AREPO/GIZMO-style solver:

1. Reconstruct primitive variables and RHD group variables on faces.
2. Compute microphysical rate kernels R_r from the same chemistry and radiation tables used by the baseline run.
3. Compute projected Fisher excess using the Schur complement Eq. (3).
4. Compute $\psi = \log(1 + \dot{\mathcal{I}}^\perp/\dot{\mathcal{I}}^\parallel)$ and face-centered gradients.
5. Measure $D_B^{(\ell)}$ from velocity autocorrelation or from the local turbulence closure; apply the speed cap $D_B^{(\ell)}/\tau_\ell \le v_{\max}^2$.

6. Update $\mathbf{J}_{\mathrm{CT}}$ with the hyperbolic relaxation equation.
7. Add CT mass flux at faces and add the corresponding momentum and enthalpy fluxes.
8. Recompute gravity and then apply cooling, chemistry, star formation, feedback and sink accretion.

All source modules after the conservative update are identical in baseline and copy-time runs.

8 Selection likelihood

Given simulation parameters θ and model $\mathcal{M}$, the predicted selected distribution is

$$P(\mathbf{y} \mid S, \mathcal{M}) = \frac{\int P(S \mid \mathbf{y}) P(\mathbf{y} \mid \theta, \mathcal{M}) P(\theta \mid \mathcal{M}) \, \mathrm{d}\theta}{\int P(S \mid \mathbf{y}) P(\mathbf{y} \mid \theta, \mathcal{M}) P(\theta \mid \mathcal{M}) \, \mathrm{d}\theta \, \mathrm{d}\mathbf{y}}. \tag{19}$$

The same survey masks, flux cuts, color cuts, spectroscopic incompleteness, X-ray sensitivity and lensing priors must be used for the baseline and copy-time simulations. This is essential because compact red sources can be overproduced or underproduced by selection alone.

9 Contents of the reproducibility package

The package contains:

- `code/analysis/qict_highstandard_diagnostics.py`: generates all CSV diagnostics and figures;
- `code/simulation/ct_transport_operator.py`: finite-volume reference operator;
- `data/diagnostics`: molecular failure table, effective mobility thresholds, A_2 channel coefficients, observables and receiver window;
- `figures`: six PDF figures used by the manuscript;
- `submission`: cover letter and numerical validation protocol.