

Koide Technical Paper

The Coupling-Modified Koide Relation: A Unified Mass Formula for Leptons and Neutrinos

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Abstract

We present a unified Koide mass relation for both charged leptons and neutrinos using the reciprocal convention $\Sigma m_i / (\Sigma \sqrt{m_i})^2 = Q$. Charged leptons satisfy $Q = 2/3$ exactly (to 5.8×10^{-6} precision with PDG 2024 masses). The physical meaning of $2/3$ is the $SU(3)$ to $SU(2)$ projection ratio: two electroweak dimensions out of three color dimensions. Neutrinos couple through the neutral current Z^0 rather than the charged current W^\pm , modifying the coupling factor by $(1 - \sin^2 \theta_W)$: $Q_\nu = (2/3)(1 - \sin^2 \theta_W) = 0.518006$. Combined with NuFIT 6.0 oscillation constraints, this gives a unique solution $m_1 = 0.4558$ meV, $m_2 = 8.6665$ meV, $m_3 = 50.3409$ meV, $\Sigma m_i = 59.463$ meV. Both relations follow from the same formula with different coupling factors, making the Koide relation a statement about gauge coupling geometry rather than a numerical coincidence.

Keywords: Koide formula · reciprocal convention · charged leptons · neutrinos · SU(3) projection · Weinberg angle · coupling-modified Koide

1. The Koide Formula and Its Conventions

Koide (1983) observed that charged lepton masses satisfy a remarkable relation. The formula has been stated in several equivalent conventions; we use the reciprocal form throughout:

$$Q = \Sigma m_i / (\Sigma \sqrt{m_i})^2 \text{ [reciprocal convention]}$$

Charged leptons (PDG 2024: $m_e=0.511$, $m_\mu=105.658$, $m_\tau=1776.86$ MeV):

$$Q_{\text{leptons}} = 0.66666082 = 2/3 = 0.66666667 \text{ [residual: } 5.85e-06]$$

The standard Koide convention uses $(\Sigma \sqrt{m_i})^2 / \Sigma m_i = 3/2$, which is the reciprocal of our Q multiplied by 3. Both are equivalent; the reciprocal form $Q = 2/3$ is more natural for the physical interpretation.

2. Physical Meaning of 2/3

The factor 2/3 is the SU(3) to SU(2) color projection ratio. In the Standard Model, quarks carry three color charges (SU(3)) while leptons interact via the electroweak SU(2). The Koide relation encodes the projection of three color dimensions onto two electroweak dimensions:

$$Q_{\text{leptons}} = 2/3 = (\text{SU(2) dimensions}) / (\text{SU(3) dimensions})$$

This is the origin of the exact 2/3 — it is a counting ratio, not a free parameter. This interpretation connects the Koide formula to the harmonic framework of Papers I-IV.

3. The Coupling-Modified Koide Relation for Neutrinos

3.1 Physical motivation

Charged leptons couple through the charged current W^\pm boson with the full SU(2) coupling strength (coupling factor $f = 1$). Neutrinos couple only through the neutral current Z^0 boson. The neutral weak coupling differs from the charged current by the Weinberg mixing angle, giving a coupling factor $(1 - \sin^2 \theta_W)$.

THE UNIFIED COUPLING-MODIFIED KOIDE FORMULA

UNIFIED KOIDE FORMULA: $\Sigma m / (\Sigma \sqrt{m})^2 = (2/3) \times f_{\text{coupling}}$ Charged leptons: $f_{\text{coupling}} = 1 \rightarrow Q = 2/3 = 0.6667$ Neutrinos: $f_{\text{coupling}} = (1 - \sin^2 \theta_W) \rightarrow Q_{\nu} = 0.5180$ The Weinberg angle that mixes W^3 and B into physical W^\pm and Z^0 also modifies the Koide mass constraint from the charged to the neutral sector. The same mixing that governs gauge boson masses governs fermion mass ratios.

3.2 The neutrino Koide parameter

$$\sin^2 \theta_W = 1 - (595/675)^2 = 0.222990 \text{ [Paper I, PDG 2025 on-shell value]}$$

$$Q_{\nu} = (2/3)(1 - \sin^2 \theta_W) = (2/3)(1 - 0.222990) = 0.51800640$$

4. Derivation of Neutrino Masses

4.1 Setup

NuFIT 6.0 Normal Ordering (September 2024): $\Delta m^2_{21} = 74.9 \text{ meV}^2$, $\Delta m^2_{31} = 2534.0 \text{ meV}^2$

$$m_2 = \sqrt{m_1^2 + 74.9}, \quad m_3 = \sqrt{m_1^2 + 2534.0} \text{ [all in meV]}$$

$$\text{Koide condition: } f(m_1) = [m_1 + m_2 + m_3] / [\sqrt{m_1} + \sqrt{m_2} + \sqrt{m_3}]^2 = Q_{\nu}$$

$$f(0+) = 0.585611 > Q_{\nu} = 0.518006 \text{ [floor exceeds target – solution exists]}$$

f(m1) is monotonically decreasing with a unique zero near m1 = 0.456 meV.

4.2 Exact solution

m1 = 0.45575481 meV

m2 = sqrt(0.4558^2 + 74.9) = 8.66647059 meV

m3 = sqrt(0.4558^2 + 2534.0) = 50.34091489 meV

Sigma_mi = 59.46314029 meV

Verify: Sigma_m/(Sigma sqrt(m))^2 = 59.463140/114.792289 = 0.5180064015 = Qnu = 0.5180064015 CHECK

Verify: m2^2-m1^2 = 74.9000 meV^2 = 74.9 meV^2 CHECK

Verify: m3^2-m1^2 = 2534.0000 meV^2 = 2534.0 meV^2 CHECK

VERIFIED EXACT SOLUTION

EXACT SOLUTION (all three constraints satisfied to machine precision): m1 = 0.4558 meV m2 = 8.6665 meV m3 = 50.3409 meV Sigma_mi = 59.463 meV Published values in companion Paper III (0.456, 8.667, 50.341, 59.46 meV) are these exact values rounded to 3 decimal places.

5. Comparison of Lepton and Neutrino Relations

Property	Charged Leptons	Neutrinos
Coupling boson	W± (charged current)	Z^0 (neutral current)
Coupling factor f	1 (full strength)	(1 - sin^2 theta_W) = 0.777
Koide Q	2/3 = 0.66667	Qnu = 0.51801
Q precision	5.8×10^-6 (PDG 2024)	Machine precision (derived)
Mass sum	2083.5 MeV	59.463 meV
Ordering	me << mmu << mtau	m1 << m2 << m3 (Normal Ordering)
Mass range	0.511 to 1776.86 MeV	0.456 to 50.341 meV
Lightest/heaviest	0.511/1776.86 = 0.000288	0.456/50.341 = 0.00906

6. The Weinberg Angle as a Koide Rotation

The transition from the charged lepton Koide relation to the neutrino Koide relation is governed entirely by the Weinberg angle:

Q_leptons = 2/3 [W± coupling: theta_W = 0]

$$Q_{\nu} = (2/3)(1 - \sin^2 \theta_W) \text{ [Z}^0 \text{ coupling: actual } \theta_W]$$
$$Q_{\nu}/Q_{\text{leptons}} = (1 - \sin^2 \theta_W) = 0.777010$$

The Koide parameter ratio equals the neutral current coupling factor. As $\sin^2 \theta_W$ runs from 0 to 1, the neutrino Koide parameter runs from 2/3 to 0. At the measured value $\sin^2 \theta_W = 0.22299$, we get $Q_{\nu} = 0.518$. The Weinberg angle rotates the charged-current Koide relation into the neutral-current Koide relation — both are aspects of the same SU(3) to SU(2) projection geometry.

7. Predictions and Tests

Prediction	Value	Test	Timeline
Neutrino mass sum	59.463 meV	Euclid DR1	June 2026
Lightest neutrino mass m1	0.456 meV	Next-gen beta decay (PTOLEMY)	2027+
Normal Ordering confirmed	m1 < m2 < m3	SNO+, Hyper-K	Ongoing
No neutrinoless double-beta decay	0vBB rate < LEGEND-1000	LEGEND-1000	~2030
Qnu changes if sin^2 theta_W changes	Qnu runs with energy	EW precision data	Ongoing

8. Conclusions

The Koide relation in the reciprocal convention $\Sigma m/(\Sigma \sqrt{m})^2 = (2/3) \times f$ unifies charged lepton and neutrino mass relations under a single formula. Charged leptons give $f=1$ via $W\pm$ coupling; neutrinos give $f=(1-\sin^2 \theta_W)$ via Z^0 coupling. The factor 2/3 is the SU(3) to SU(2) projection ratio. The Weinberg angle rotates the Koide relation between the two sectors. The neutrino solution gives $\Sigma m_i = 59.463$ meV, testable by Euclid DR1 in June 2026.

References

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