The Present Hubble Horizon as a Radiative Decoupling Boundary

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Abstract

Hawking (1975) established that the Schwarzschild horizon has a thermal Planckian spectrum. In the ideal blackbody luminosity limit, this gives the reduced spherical normalization

$$X \equiv \frac{L}{R^2T^4}\frac{\hbar^3c^2}{k_B^4} = \frac{\pi^3}{15},$$

the Stefan–Boltzmann normalization reduced by the area of the unit sphere.

In spatially flat FLRW cosmology, the Hubble radius $R_H = c/H$ coincides with the apparent horizon, and the standard definition of the vacuum fraction gives the dimensionless identity

$$\Lambda R_H^2 = 3\Omega_\Lambda.$$

The proposal made here is not a modification of the Einstein or Friedmann equations, and not a determination of Λ alone. It is a boundary value for this existing dimensionless horizon combination:

$$\Lambda_0 R_{H,0}^2 = \frac{\pi^3}{15}.$$

The numerical coefficient is supplied by the reduced spherical blackbody normalization, while the cosmological slot in which it is placed is the standard combination ΛR_H^2.

It follows immediately that

$$\Omega_{\Lambda,0} = \frac{\pi^3}{45} \simeq 0.689, \qquad \Omega_{m,0} = 1 - \frac{\pi^3}{45} \simeq 0.311.$$

Standard flat-FLRW kinematics then give

$$z_{\rm acc} \simeq 0.643, \qquad q_0 \simeq -0.534, \qquad H_0 t_0 \simeq 0.954,$$

corresponding to $t_0 \simeq 13.85$ Gyr for $H_0 = 67.4\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$.

The product $\Lambda R_H^2(z)$ follows the standard monotonic trajectory

$$0 \longrightarrow 1 \longrightarrow \frac{\pi^3}{15} \longrightarrow 3,$$

corresponding respectively to the matter-dominated past, the acceleration threshold, the present boundary value, and the asymptotic de Sitter limit. In this framework, the Schwarzschild horizon supplies the established thermal radiative anchor, while the Hubble horizon supplies the cosmological boundary on which the same reduced spherical normalization is tested.

1 Introduction

The cosmological constant Λ enters the Einstein field equations as a universal coefficient, but its numerical value is not fixed by the local equations of motion. In the spatially flat Friedmann–Lemaître–Robertson–Walker (FLRW) background with pressureless matter and a cosmological term, this freedom appears as the present vacuum fraction $\Omega_{\Lambda,0}$, normally inferred from data rather than derived from a boundary condition.

The relevant quantity in the present work is not Λ alone. It is the dimensionless product ΛR_H^2. This product already appears in standard flat FLRW cosmology. With

$$R_H = \frac{c}{H}, \tag{1}$$

the standard definition of Ω_Λ gives

$$\Lambda R_H^2 = 3\Omega_\Lambda. \tag{2}$$

The proposal below fixes the present value of this existing dimensionless combination. It does not introduce a new dynamical equation.

In the spatially flat case considered here, the apparent horizon coincides with the Hubble radius. This is the boundary used throughout the paper. The boundary condition is imposed only at the present epoch:

$$\Lambda_0 R_{H,0}^2 = \frac{\pi^3}{15}. \tag{3}$$

This condition selects a definite present-epoch member of the standard flat matter-plus-Λ family.

A separate line of development concerns the thermodynamics of horizons. Bekenstein [1] and Hawking [2] established that black hole horizons carry entropy and temperature, and that the Schwarzschild horizon has a thermal spectrum. Gibbons and Hawking [3] extended horizon thermodynamics to cosmological horizons. Jacobson [4], Padmanabhan [5], and Verlinde [6] developed related thermodynamic readings of gravitational dynamics.

The present work uses the Schwarzschild–Hawking horizon only as the canonical established case of an ideal thermal radiative horizon. In the ideal blackbody luminosity limit, its Planckian emission satisfies the reduced spherical normalization

$$X = \frac{\pi^3}{15}. \tag{4}$$

The question asked here is whether the same reduced spherical radiative normalization closes the present Hubble horizon through the dimensionless FLRW combination ΛR_H^2.

The argument is organized as follows. Section 2 defines the reduced spherical radiative normalization. Section 3 records the Schwarzschild–Hawking ideal blackbody anchor. Section 4 states the standard dimensionless FLRW combination. Section 5 formulates the cosmological closure condition. Section 6 derives the late-time consequences and compares them with observations. Section 7 describes the trajectory of ΛR_H^2 across cosmic history. Section 8 records the associated boundary-count interpretation. Section 9 states the falsifiability conditions.

2 The Reduced Spherical Radiative Normalization

Consider an ideal spherical blackbody radiative boundary of radius R, temperature T, and bolometric luminosity L. Define

$$X \equiv \frac{L}{R^2 T^4} \frac{\hbar^3 c^2}{k_B^4}. \tag{5}$$

For a blackbody surface,

$$L = 4\pi R^2 \sigma T^4, \qquad \sigma = \frac{\pi^2 k_B^4}{60\hbar^3 c^2}. \tag{6}$$

Substitution gives

$$X = 4\pi\frac{\pi^2}{60} = \frac{\pi^3}{15}. \tag{7}$$

The factor 4π is the area of the unit sphere. The factor $\pi^2/60$ is the Stefan–Boltzmann coefficient written in fundamental constants. Equation (7) is therefore the reduced spherical blackbody normalization.

This value is independent of the specific radius, temperature, and luminosity of any particular ideal emitter. It characterizes an ideal spherical radiative boundary.

3 The Schwarzschild–Hawking Anchor

For a Schwarzschild black hole of mass M, the horizon radius is

$$R_s = \frac{2GM}{c^2}, \tag{8}$$

and Hawking's result gives the temperature

$$T_H = \frac{\hbar c^3}{8\pi GMk_B} = \frac{\hbar c}{4\pi k_B R_s}. \tag{9}$$

The corresponding ideal blackbody luminosity obtained by applying the Stefan–Boltzmann law to the Schwarzschild radius and Hawking temperature is

$$L_H = 4\pi R_s^2\sigma T_H^4 = \frac{\hbar c^6}{15360\pi G^2M^2}. \tag{10}$$

This is the ideal spherical blackbody luminosity, before greybody factors, spin-dependent transmission coefficients, and species-dependent emission channels.

Using Eq. (5),

$$X_{\mathrm{BH}} = \frac{L_H}{R_s^2T_H^4}\frac{\hbar^3c^2}{k_B^4} = \frac{\pi^3}{15}. \tag{11}$$

The Schwarzschild–Hawking horizon therefore realizes the reduced spherical radiative normalization in the ideal blackbody limit. This section does not derive the coefficient from black-hole physics. It records the established case in which a thermal horizon functions, in this ideal limit, as a Planckian spherical emitter.

4 The Standard Dimensionless Horizon Combination

In spatially flat FLRW cosmology,

$$R_H = \frac{c}{H}. \tag{12}$$

The vacuum fraction is

$$\Omega_\Lambda = \frac{\Lambda c^2}{3H^2}. \tag{13}$$

Therefore,

$$\Lambda R_H^2 = \Lambda\frac{c^2}{H^2} = 3\Omega_\Lambda. \tag{14}$$

Equation (14) is standard. It is not a new relation proposed in this paper.

The physical proposal is instead that the present value of this standard dimensionless combination is fixed by the reduced spherical radiative normalization:

$$\Lambda_0 R_{H,0}^2 = \frac{\pi^3}{15}. \tag{15}$$

Thus the framework does not assign primary physical significance to Λ alone, nor to R_H alone. The relevant object is the dimensionless product ΛR_H^2.

5 Cosmological Closure

Combining Eq. (14) with Eq. (15) gives

$$3\Omega_{\Lambda,0} = \frac{\pi^3}{15}, \tag{16}$$

and therefore

$$\Omega_{\Lambda,0} = \frac{\pi^3}{45} \simeq 0.6890. \tag{17}$$

For a spatially flat background,

$$\Omega_{m,0} = 1 - \Omega_{\Lambda,0} = 1 - \frac{\pi^3}{45} \simeq 0.3110. \tag{18}$$

The present matter–vacuum partition is therefore fixed analytically from one boundary value.

Equation (15) is not derived from the Hawking luminosity. The Schwarzschild–Hawking calculation identifies $\pi^3/15$ as the reduced normalization of an ideal spherical radiative horizon. The cosmological proposal is that the same reduced normalization supplies the present boundary value of ΛR_H^2.

The proposal is not that the Hubble horizon is a black-hole horizon. The claim is narrower: an established reduced spherical radiative normalization is applied to the standard dimensionless FLRW horizon combination and then tested against the observed late-time density partition.

6 Derived Late-Time Quantities

The following consequences use only the flat matter-plus-Λ background and the closure-fixed values in Eqs. (17) and (18).

6.1 Acceleration transition

For matter plus a cosmological term, the acceleration threshold is

$$\ddot{a} = 0. \tag{19}$$

Equivalently,

$$\rho_m = 2\rho_\Lambda. \tag{20}$$

Therefore,

$$\Omega_{m,0}(1 + z_{\rm acc})^3 = 2\Omega_{\Lambda,0}. \tag{21}$$

Substitution gives

$$z_{\rm acc} = \left(\frac{2\pi^3/45}{1 - \pi^3/45}\right)^{1/3} - 1 \simeq 0.643. \tag{22}$$

6.2 Present deceleration parameter

For the same background,

$$q_0 = \frac{1}{2}\Omega_{m,0} - \Omega_{\Lambda,0}. \tag{23}$$

Using Eqs. (17) and (18),

$$q_0 = \frac{1}{2} - \frac{\pi^3}{30} \simeq -0.534. \tag{24}$$

6.3 Cosmic age

For a flat matter-plus-Λ universe,

$$H_0 t_0 = \frac{2}{3\sqrt{\Omega_{\Lambda,0}}} \sinh^{-1} \sqrt{\frac{\Omega_{\Lambda,0}}{\Omega_{m,0}}}. \tag{25}$$

Substitution gives

$$H_0 t_0 \simeq 0.954. \tag{26}$$

For

$$H_0 = 67.4\,\mathrm{km\,s^{-1}\,Mpc^{-1}}, \tag{27}$$

this corresponds to

$$t_0 \simeq 13.85\,\mathrm{Gyr}. \tag{28}$$

6.4 Observational comparison

Quantity	Closure value	Observational value	Reference
$\Omega_{\Lambda,0}$	0.6890	0.6847 ± 0.0073	Planck 2018 [7]
$\Omega_{m,0}$	0.3110	0.3153 ± 0.0073	Planck 2018 [7]
z_{acc}	0.643	$0.61 \lesssim z_t \lesssim 0.79$	[8]
q_0	-0.534	$\simeq -0.527$	derived from [7]
t_0	13.85 Gyr	13.797 ± 0.023 Gyr	Planck 2018 [7]

Table 1: Late-time cosmological quantities fixed by the closure condition $\Lambda_0 R_{H,0}^2 = \pi^3/15$, compared with standard observational determinations.

No entry in the table is fitted independently. The listed quantities follow from the single boundary value in Eq. (15), the assumed flat matter-plus-Λ background, and the observational value of H_0 only where a dimensional age is quoted.

7 The $\Lambda R_H^2(z)$ Trajectory

With constant Λ_0 and

$$R_H(z) = \frac{c}{H(z)}, \tag{29}$$

the product $\Lambda_0 R_H^2(z)$ evolves monotonically in the flat matter-plus-vacuum background.

Epoch	$\Lambda_0 R_H^2(z)$	Comment
Matter-dominated past, $z \to \infty$	0	negligible vacuum fraction
Acceleration onset, $z = z_{\mathrm{acc}}$	1	threshold value
Present epoch, $z = 0$	$\pi^3/15 \simeq 2.067$	boundary closure
Asymptotic future, $z \to -1$	3	pure de Sitter limit

Table 2: Distinguished values of the standard dimensionless combination ΛR_H^2 in the flat matter-plus-Λ background under the proposed present boundary condition.

The value $\Lambda R_H^2 = 1$ at acceleration onset follows from the acceleration condition and is independent of the numerical value $\pi^3/15$. The value $\Lambda R_H^2 \to 3$ is the pure de Sitter limit. The closure condition fixes the present position of the trajectory:

$$0 \longrightarrow 1 \longrightarrow \frac{\pi^3}{15} \longrightarrow 3. \tag{30}$$

Thus 0, 1, and 3 are standard values associated with the flat matter-plus-Λ background, while $\pi^3/15$ is the proposed present boundary value.

8 Boundary Counts

The horizon area count in Planck units is

$$N = \frac{A_H}{\ell_P^2} = \frac{4\pi R_H^2}{\ell_P^2}. \tag{31}$$

For the present Hubble horizon, this gives the familiar order

$$N_0 \sim 10^{122}. \tag{32}$$

This section is not used in deriving the late-time cosmological results. It records a structural consequence of reading the same boundary relation in terms of horizon area counts.

At a Planck curvature limit,

$$\Lambda_{\max} = \ell_P^{-2}. \tag{33}$$

Applying the same reduced boundary relation,

$$\Lambda R^2 = \frac{\pi^3}{15}, \tag{34}$$

gives

$$R_{\min} = \sqrt{\frac{\pi^3}{15}}\,\ell_P \simeq 1.4377\,\ell_P. \tag{35}$$

The corresponding minimum boundary count is

$$N_{\min} = \frac{4\pi R_{\min}^2}{\ell_P^2} = \frac{4\pi^4}{15} \simeq 25.98. \tag{36}$$

This does not constitute a complete theory of the early universe. It records the finite boundary count implied if the same closure relation is formally extended to the Planck curvature limit.

9 Falsifiability

The closure condition makes specific quantitative predictions within the assumed flat matter-plus-Λ background.

First, a decisive determination of

$$\Omega_{\Lambda,0} \neq \frac{\pi^3}{45} = 0.6890 \tag{37}$$

would falsify the closure condition in that background.

Second, under spatial flatness, a corresponding determination of

$$\Omega_{m,0} \neq 1 - \frac{\pi^3}{45} = 0.3110 \tag{38}$$

would falsify the same condition.

Third, improved kinematic determinations of the acceleration transition provide an additional test, since the closure fixes

$$z_{\text{acc}} \simeq 0.643. \tag{39}$$

Fourth, independent constraints excluding a constant-Λ background at high significance would challenge the assumptions under which the closure is formulated.

Upcoming and ongoing surveys such as DESI, Euclid, LSST, and future supernova compilations can test the closure by improving the precision of $\Omega_{\Lambda,0}$, $\Omega_{m,0}$, and z_{acc}.

10 Discussion

The proposal is structurally minimal. The Einstein field equations are unchanged. The Friedmann equations are unchanged. The added element is a single present-epoch boundary condition on the standard dimensionless combination ΛR_H^2.

The coefficient $\pi^3/15$ is not introduced as an arbitrary number. It is the reduced spherical blackbody normalization. The cosmological location of the coefficient is also not arbitrary: ΛR_H^2 is the standard dimensionless product equal to $3\Omega_\Lambda$ in flat FLRW cosmology. The proposal therefore connects a radiative normalization with an existing horizon combination.

The framework does not claim that the present Hubble horizon is a Schwarzschild horizon. It does not claim that the ideal Hawking blackbody formula is a complete description of physical black hole evaporation. It does not provide a microscopic derivation of Λ. The claim is narrower: the reduced spherical radiative normalization realized by ideal thermal radiative horizons supplies the present boundary value of the cosmological horizon combination.

In this reading, black holes provide the established theoretical anchor, ordinary spherical blackbody radiation provides the geometric normalization, and cosmology provides the empirical test. The observed present value of the vacuum fraction is read as the projection

$$\Omega_{\Lambda,0} = \frac{1}{3}\frac{\pi^3}{15} = \frac{\pi^3}{45} \tag{40}$$

of a present horizon boundary condition.

The boundary-count interpretation gives a secondary reading of the large hierarchy 10^{122}. In this formulation, the hierarchy is associated with the present horizon area count

$$N_0 = \frac{4\pi R_{H,0}^2}{\ell_P^2}, \tag{41}$$

rather than only with a bulk vacuum-energy mismatch. This interpretation is not required for the late-time cosmological predictions.

11 Conclusion

The reduced spherical radiative normalization

$$\frac{\pi^3}{15} \tag{42}$$

appears for an ideal spherical blackbody and is realized by the Schwarzschild–Hawking horizon in the ideal blackbody luminosity limit. In flat FLRW cosmology, the standard dimensionless horizon combination is

$$\Lambda R_H^2 = 3\Omega_\Lambda. \tag{43}$$

The proposal made here is that the present Hubble horizon satisfies the boundary condition

$$\Lambda_0 R_{H,0}^2 = \frac{\pi^3}{15}. \tag{44}$$

This single condition fixes

$$\Omega_{\Lambda,0} = \frac{\pi^3}{45} \simeq 0.689, \qquad \Omega_{m,0} \simeq 0.311, \tag{45}$$

and gives

$$z_{\text{acc}} \simeq 0.643, \qquad q_0 \simeq -0.534, \qquad H_0 t_0 \simeq 0.954. \tag{46}$$

At the Planck curvature limit, the same relation gives the finite formal boundary count

$$N_{\min} = \frac{4\pi^4}{15} \simeq 25.98. \tag{47}$$

The result is a radiative boundary closure for the present Hubble horizon. It introduces no new field, modifies no local gravitational equation, and is falsifiable through improved determinations of late-time cosmological parameters.

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