A Technical Note on an Apparently Unlisted Integer Sequence from Asymmetric Prime–Composite Neighborhoods

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Abstract

We document an integer sequence defined by a simple asymmetric condition involving prime and composite numbers. For each integer $n \geq 3$, let $a(n)$ be the least positive integer k such that $n - k$ is prime and $n + k$ is composite. We provide initial terms, computational results up to $n = 100000$, summary statistics, graphical observations, and open questions. Searches of the On-Line Encyclopedia of Integer Sequences (OEIS) using initial terms returned no exact match at the time of writing.

1 Introduction

Many integer sequences arise from local prime behavior, such as prime gaps, nearest-prime distances, or counting functions involving primes in intervals. This note considers a simple asymmetric variant: the smallest symmetric offset from n for which the left side is prime and the right side is composite.

Although elementary to define, the resulting sequence displays frequent small values together with occasional larger spikes.

2 Definition

Let $\mathbb{P}$ denote the set of prime numbers. For each integer $n \geq 3$, define

$$a(n) = \min\{k \geq 1 : n - k \in \mathbb{P},\ n + k \notin \mathbb{P},\ n + k > 1\}.$$

Equivalently, $a(n)$ is the least positive integer k such that:

- $n - k$ is prime,
- $n + k$ is composite.

3 Initial Terms

For $n = 3, 4, 5, \ldots$, the sequence begins:

$$1, 2, 3, 3, 2, 1, 6, 5, 4, 9, 2, 1, 10, 5, 4, 7, 2, 1, 4, 3, 4, 1, 2, 7, 8, 5, 6, 19,$$

$$2, 1, 2, 5, 4, 13, 8, 1, 10, 9, 4, 13, 2, 1, 4, 3, 4, 1, 2, 7, 4, 5, 10, 1, 2, 9, 20, \ldots$$

4 Examples

Example 1: $n = 14$

$$14 - 1 = 13 \text{ is prime}, \qquad 14 + 1 = 15 \text{ is composite}.$$

Hence:

$$a(14) = 1.$$

Example 2: $n = 30$

At $k = 19$,

$$30 - 19 = 11 \text{ is prime}, \qquad 30 + 19 = 49 \text{ is composite}.$$

No smaller positive k satisfies both conditions, so:

$$a(30) = 19.$$

5 Computational Results

Direct computation verified that $a(n)$ exists for all tested integers

$$3 \leq n \leq 100000.$$

No exceptions were found in this range.

Summary Statistics

Quantity	Value
Mean of $a(n)$	11.0621
Median of $a(n)$	8
Minimum of $a(n)$	1
Maximum of $a(n)$	134
Standard deviation	10.2241

The empirical mean exceeds the median, indicating a right-skewed distribution with many small values and occasional larger spikes.

6 Graphs

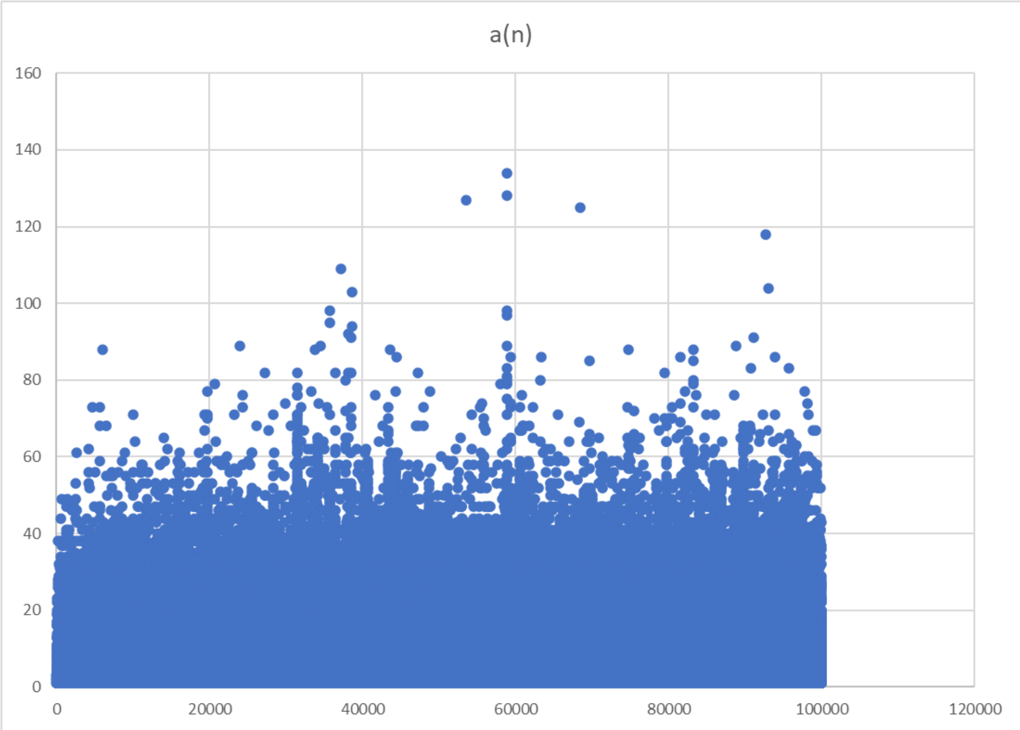

Figure 1: Scatter plot of $a(n)$ for $3 \leq n \leq 100000$.

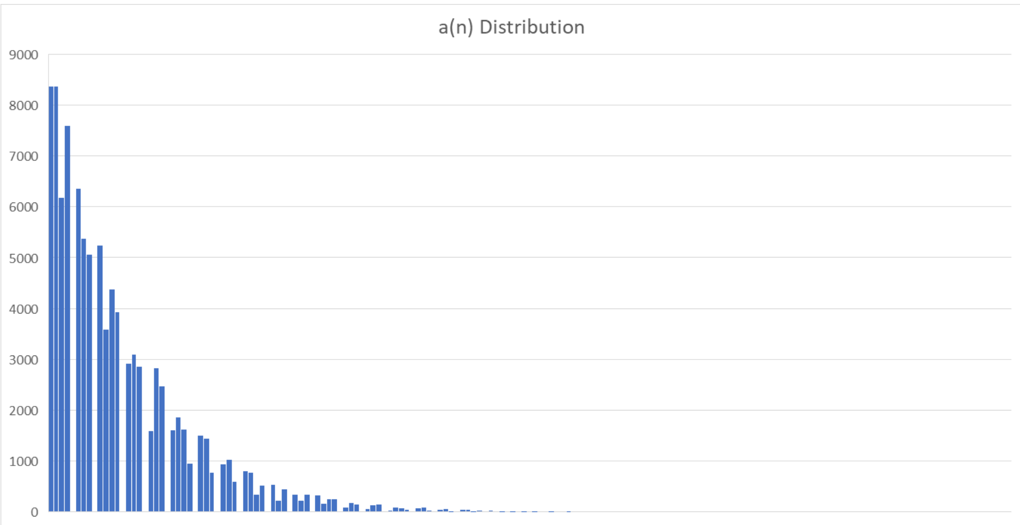

Figure 2: Histogram of computed values of $a(n)$ up to 100000.

7 Observed Behavior

Empirically:

1. Small values occur frequently.

2. Many even integers satisfy $a(n) = 1$, especially when $n-1$ is prime and $n+1$ is composite.

3. Larger values occur irregularly.
4. The maximum observed value up to 100000 is 134.

8 OEIS Search Note

An OEIS search using several initial blocks of terms returned no exact match at the time of writing. This does not exclude equivalent formulations already present under a different description.

9 Algorithm

A direct algorithm is:

```
for n = 3,4,5,...
    for k = 1,2,3,...
        if isPrime(n-k) and isComposite(n+k):
            output k
            break
```

10 Code Availability

Source code, computed data, and manuscript files are available at:

`https://github.com/Dhpla12/prime-composite-asymmetric-sequence`

11 Open Questions

1. Does $a(n)$ exist for every integer $n \geq 3$?
2. Is the sequence unbounded?
3. What is the asymptotic average order of $a(n)$?
4. What proportion of integers satisfy $a(n) = 1$?
5. How are record values related to prime gaps?

12 Conclusion

We documented an apparently unlisted sequence arising from asymmetric prime–composite neighborhoods around integers. The sequence is easy to compute, empirically well-defined for a large tested range, and exhibits nontrivial local variation.

References

[1] G. H. Hardy and E. M. Wright, *An Introduction to the Theory of Numbers*, Oxford University Press.

[2] The On-Line Encyclopedia of Integer Sequences, `https://oeis.org`