

The Intrinsic Theory of Energy

--- Part II: Extended Theory Dissertations ---

by

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Motto:

The Universe is a single – phase framework in which geometry, charge and energy arise from the same core principle.

This work has three main parts:

- I. The Theoretical Compendium.
- II. The Extended Theory Dissertations.
- III. Technical and Experimental Annex

ABSTRACT:

Rather than introducing new physical laws, this work identifies a common formal language through which geometric structures encode energy. The **Intrinsic Theory of Energy** establishes a unified framework where energy is endowed with intrinsic geometric structure and autonomous dynamics, extending the formalisms of Relativity and classical Quantum Mechanics. Within this framework, total energy is represented by Volterra-type integral equations describing the temporal evolution of flows and operators in a Kähler phase space. By treating geometry, electric charge, and energy as mathematically equivalent manifestations of a single underlying structure, the theory generalizes classical descriptions through volumetric integration over phase space. Potential applications arise in astrophysics, tokamak plasma dynamics, theoretical physics, and information technology, supporting the central principle that physical laws emerge from a unified phase structure rather than from independent dynamical postulates.

Experimental data modelling is also provided in part III as much as we could on virtual models using AI Gemini LAB for calculus. Numerical simulations are done using Picard iterations and Runge-Kutta methods.

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CHAPTER I: Fundamentals of the Energy – Phase State

The present chapter establishes the foundational premise of the Intrinsic Theory of Energy (ITE). We move beyond the classical view of the universe as a collection of disjointed entities and toward a unified vision where geometry, charge, and energy are manifestations of a single, underlying phase Φ .

Historically, theoretical physics has treated energy and geometry as distinct categories, linked only through external coupling constants. The Intrinsic Theory of Energy introduces a paradigm shift through the principle of State Isomorphism (J_Φ). This principle asserts that energy is not an external scalar quantity acting upon a system, but an inherent geometric property of the space-time manifold. Whether we observe an electric charge, a gravitational curvature, or a localized mass, we are witnessing the same Phase Φ projected through different "geometric lenses."

1.1. Intrinsic Definition of Energy

We are talking here about the structure of energy. Since the dawn of science, energy is a scalar. A number that results from the interactions of forces and phenomena is too weak to generate for example, to generate matter. Unless it's not only a number.... But let's start with the scalar version.

We have always wondered whether the speed of light of about 300,000km/s can be exceeded or not? The only barrier with objective theoretical support was the fact that the equation $E = mc^2$ is not valid above the limit c which is the speed of light. Then we started from the one-dimensional case and looked for another equation – of energy obviously – that would have as its solution on $[0, c)$ the Einstein's old formula but would have a much wider range of validity.

We call intrinsic energy equation, an equation having exclusively the energy like explicit argument and not having the mass, the space or the time like explicit argument. In general relativity theory, the equation that links the energy to the mass $E = mc^2$ having the speed like explicit argument it's not an intrinsic energy equation.

We will be searching for such an equation which must be valid on \mathbb{R} and not only for speeds within $[0, c)$. However, for consistency reasons, our equation must admit the Einstein formula for energy as a solution valid for speed $v \in [0, c)$, that is $E(v) = m(v)c^2$ in which $m(v) = \frac{m_0}{\sqrt{c^2 - v^2}}$ with $m_0 \neq 0$ being the still mass of the mobile.

We will search for it in the simpler and yet the most general case, the unidimensional case considering a mobile with a still mass $m_0 \neq 0$ moving with the speed $\vec{v} = v \in \mathbb{R}, \|\vec{v}\| = |v| \neq 0$. We know, that for this mobile, the energy is given by Einstein formula: $E = E(v) = \frac{m_0 c^2}{\sqrt{c^2 - v^2}} = \frac{E_0}{\sqrt{1 - \frac{v^2}{c^2}}}$ where the initial energy is $E_0 = E(0) = m_0 c^2$. We can also remark that $E^2(v) = \frac{c^2 E_0^2}{c^2 - v^2}$, for speeds $0 \leq v < c$.

Now, while keeping valid the previous physical hypotheses, we will perform a small mathematical exercise by studying the following derivative:

$$\frac{d}{dx} \log \sqrt{1 - \frac{x^2}{c^2}} = \left(1 - \frac{x^2}{c^2}\right)^{-\frac{1}{2}} \frac{d}{dx} \sqrt{1 - \frac{x^2}{c^2}} = \frac{1}{1 - \frac{x^2}{c^2}} \left(\frac{-2x}{c^2}\right) = \frac{-2x}{c^2 - x^2}$$

We will define: $I_v = [0, v] \subset [0, c), I_v \neq \emptyset, v \in \mathbb{R}_+$.

Thus considering:

$$e^{\int_0^v \frac{2x}{x^2 - c^2} dx} = e^{\int_{I_v} \frac{d}{dx} \log \sqrt{1 - \frac{x^2}{c^2}} dx} = \left. \sqrt{1 - \frac{x^2}{c^2}} \right|_0^v = \sqrt{1 - \frac{v^2}{c^2}} - 1 = \frac{E_0}{E(v)} - 1 \quad (a)$$

$$\text{but:} \quad e^{\int_{I_v} \frac{2x}{x^2 - c^2} dx} = e^{-\int_{I_v} \frac{2xE^2(x)}{c^2 E_0^2} dx} \quad (b)$$

Therefore (a) & (b) are giving:

$$\frac{E_0}{E} - 1 = e^{-\frac{2}{c^2 E_0^2} \int_{I_v} x E^2(x) dx} \quad \text{with initial condition } E(0) = E_0 \quad (1)$$

$$(1) \text{ takes the form: } E = \frac{E_0}{1 + e^{-\frac{2}{c^2 E_0^2} \int_{I_v} x E^2(x) dx}} \quad (1^*)$$

From its construction presented above, the equation (1) admits the solution $E = E(v) = \frac{E_0 c}{\sqrt{c^2 - v^2}}$ but only for $v \in [0, c)$, although its valid through the entire \mathbb{R} . The solution $E = E(v) = mc^2 = \frac{E_0 c}{\sqrt{c^2 - v^2}}$ being only a local solution on $[0, c)$ for equation (1) from now on.

Like a first conclusion we can underline that the validity of equation (1) states that the boundary of the speed of light c can be overcome.

The equation (1) can be called intrinsic equation of the energy because it doesn't contain like explicit terms neither the mass, nor the space and only the speed is an integration argument. It contains only energy.

We have found it by using mathematical technics exclusively. Like physical support we can mention the fact the old Einstein's formula for energy is a local solution of our equation, or the energy can only be the solution of an energy equation. The physical hypothesis is the simplest and the largest possible. It contains entire functions which gives its validity through all \mathbb{R} . It has a particular solution $E(v) = \frac{cE_0}{\sqrt{c^2-v^2}}$ valid on $[0, c)$ but it can have other solutions and take other shapes or forms on other intervals and under different conditions.

For further study, let's define:

$\varphi_E: \mathbb{R} \rightarrow \mathbb{R}, \varphi_E(v) = \frac{E(v)}{E_0}$, then (1) becomes:

$$\frac{1}{\varphi_E(v)} - 1 = e^{\frac{-2}{c^2} \int_{I_v} x \varphi_E^2(x) dx} \quad (2)$$

which leads to:

$$\frac{\varphi_E(v)}{1-\varphi_E(v)} = e^{\frac{2}{c^2} \int_{I_v} x \varphi_E^2(x) dx} \quad (3)$$

Note first that φ_E is well defined because $E_0 \neq 0$ from the initial hypothesis. φ_E is called *energy evolutive function* associated to a mobile because it shows the ratio between the total energy at a certain speed and it's still energy. It describes how "grows" the energy of the whole system as long as its speed evolves on \mathbb{R} .

Remark:

1. Let's study the case $|\varphi_E(v)| < 1$ for we have $\frac{1}{1-\varphi_E(v)} = \sum_{n \geq 0} \varphi_E^n(v)$, but we observe that:

$$e^{\frac{2}{c^2} \int_{I_v} x \varphi_E^2(x) dx} = \sum_{n \geq 0} \frac{2^n}{n! c^{2n}} \left(\int_{I_v} x \varphi_E^2(x) dx \right)^n \text{ and therefore: } \forall n \in \mathbb{N}, \varphi_E^{n+1}(v) = \frac{2^n}{n! c^{2n}} \left(\int_{I_v} x \varphi_E^2(x) dx \right)^n \text{ and though for } n = 0 \text{ we have } \varphi_E(v) = 1 \Rightarrow E(v) = E_0, \forall v, \text{ which describes a movement straight and uniform.}$$

$$\text{For } n = 1 : \varphi_E^2(v) = \frac{2}{c^2} \int_{I_v} x \varphi_E^2(x) dx \Rightarrow E(v) = \frac{1}{c} \sqrt{2 \int_{I_v} x E^2(x) dx} \quad (4)$$

equation valid only for $E(v) < E_0$, though the mobile is losing speed (so its energy is decreasing).

2. The equations (1), (3) and (4) are invariant relatively with the mass, the space and in time too.

Now we want to transform the equation (3) in a linear integral equation of Volterra of the first kind. So, we define:

$K(v, t) = t$, $\psi_E(t) = \varphi_E^2(t)$ and $f_E(v) = \frac{c^2}{2} \log \frac{\varphi_E(v)}{1-\varphi_E(v)}$ three continuous and derivable functions with which equation (3) becomes:

$$f_E(v) = \int_0^v K(v, t) \psi_E(t) dt \quad (3^*)$$

We can naturally suppose that the kernel K of the integral (3*) has the form $K = g * h$ where $h = 1_{\mathbb{R}}$ and g is a constant function $g = 1$. Then, according to the general theory, reduced at our particular case, the solution of equation (3bis) is: $\psi_E(v) = \frac{1}{v} f'_E(v)$. So:

$$\begin{aligned} \psi_E(v) &= \frac{1}{v} f'_E(v) = \frac{c^2}{2v} \frac{d}{dv} \log \frac{\varphi_E(v)}{1-\varphi_E(v)} = \frac{c^2}{2v} \frac{1-\varphi_E(v)}{\varphi_E(v)} \left[\frac{1}{1-\varphi_E(v)} \frac{d\varphi_E}{dv} + \varphi_E(v) \frac{d}{dv} \left(\frac{1}{1-\varphi_E(v)} \right) \right] = \\ &= \frac{c^2}{2v} \frac{1-\varphi_E(v)}{\varphi_E(v)} \left[\frac{1}{1-\varphi_E(v)} \frac{d\varphi_E}{dv} + \frac{\varphi_E(v)}{(1-\varphi_E(v))^2} \frac{d\varphi_E}{dv} \right] = \frac{c^2}{2v} \frac{1}{\varphi_E(v)} \left[\frac{d\varphi_E}{dv} + \frac{\varphi_E(v)}{1-\varphi_E(v)} \frac{d\varphi_E}{dv} \right] = \frac{c^2}{2v\varphi_E(v)(1-\varphi_E(v))} \frac{d\varphi_E}{dv} \end{aligned}$$

But tacking into account the definition of $\psi_E(v) = \varphi_E^2(v)$ we get:

$$\frac{dE}{dv} = \frac{2v}{c^2} \frac{E^3}{E_0^2} \left(1 - \frac{E}{E_0} \right) \quad (5)$$

Which leads to Cauchy's problem for the energy for a mobile with still mass $m_0 \neq 0$, and so with $E_0 \neq 0$ moving with the speed $v \neq 0$ within the unidimensional case and with the initial condition $E(0) = E_0 \neq 0$:

$$\begin{cases} \frac{dE}{dv} = \frac{2v}{c^2} \frac{E^3}{E_0^2} \left(1 - \frac{E}{E_0} \right) \\ E(0) = E_0 \end{cases} \quad (6)$$

which generates the Picard fix point problem for the energy of a mobile moving with the speed $v \neq 0$ in the unidimensional case and having the initial energy $E(0) = E_0 \neq 0$:

$$E(v) = E_0 + \frac{2}{c^2 E_0^2} \int_{I_v} x E^3(x) \left(1 - \frac{E(x)}{E_0} \right) dx \quad (7)$$

By tacking in consideration, the function φ_E , equation (7) can be written:

$$\varphi_E(v) = 1 + \frac{2}{c^2} \int_{I_v} x \varphi_E^3(x) (1 - \varphi_E(x)) dx \quad (7^*)$$

and the Cauchy problem associated to φ_E becomes:

$$\begin{cases} \frac{d\varphi_E}{dv} = \frac{2v}{c^2} \varphi_E^3(1 - \varphi_E) \\ \varphi_E(0) = 1 \end{cases} \quad (6^*)$$

This transformation proves that energy is a self-referential integral process. The Picard – Volterra mapping ensures that the state of the universe at any time t is a stable, unique solution of its prior phase-memory, effectively eliminating the risk of unphysical divergences.

The kernel functions also as the Metric Memory, dictating how past phase rotations influence the present geometric tension. This transition from "instantaneous force" to "integral flow" is what allows the resolution of relativistic singularities and explains the stability of the Kähler medium. By internalizing the dynamics within the metric itself, the need for arbitrary force-carriers is eliminated, replacing them with a requirement for mathematical continuity and phase stability. This ensures that the current state of any point in the universe is a "memory" of its temporal evolution.

In this moment we are facing a Volterra non-homogenic and non-linear integral equation of the second kind that we don't know solving in an analytic manner. All that we can do is to study the case in which we impose a linear condition to equation (7*), condition that is in the most simpler case: $\varphi_E^3(1 - \varphi_E) = \varphi_E$ which leads to: $\varphi_E^2(1 - \varphi_E) = 1$ (8)

Equality (8) can be treated as follows:

$$(8) \Leftrightarrow \varphi_E^2 - 1 = (\varphi_E - 1)(\varphi_E^2 + \varphi_E + 1) + 1 \Leftrightarrow \varphi_E^2 - 2 = (\varphi_E - 1)(\varphi_E^2 + \varphi_E + 1) \quad (8^*)$$

Equality (8*) leads to two options:

- a) $\varphi_E^2(v) - 2 = k_E - \text{constant}, k_E \in \mathbb{R}, \forall v \in \mathbb{R}.$
- b) $\varphi_E^2(v) - 2 = \varepsilon(v) \neq ct., \forall v \in \mathbb{R}, \text{ where } \varepsilon: \mathbb{R} \rightarrow \mathbb{R} \text{ is a continuous function.}$

Remark:

1) As we see, equality (8*) has two sides but we have chosen to work with the left side because if we would have worked with the right side, we would have the following situations:

a1) $\varphi_E - 1 = ct. \Rightarrow \varphi_E = ct. + 1 = \text{constant}$ – case corresponding to a linear and uniform movement;

$$\varphi_E^2 + \varphi_E + 1 = 1 \Rightarrow \varphi = 0 \text{ or } \varphi = -1 \text{ physical impossible;}$$

b1) $\varphi_E^2 + \varphi_E + 1 = 0$ – has no real solutions and the energy as complex number has no physical significance;

c1) $(\varphi_E - 1)(\varphi_E^2 + \varphi_E + 1) = k_E = ct. \neq 0, k_E > -1 \Leftrightarrow \varphi_E^3 - 1 = k_E \Rightarrow \varphi_E = \sqrt[3]{k_E + 1} - \text{constant}, \forall v \in \mathbb{R}$ – case of a uniform linear movement. The case $k_E \leq -1$ is physical impossible;

2) The case a) correspond also to: $\varphi_{E^{1,2}} = \pm\sqrt{2 + k_E}$. The case (-) has no physical sense and the case (+) implies $E(v) = E_0\sqrt{2 + k_E}, \forall v \in \mathbb{R}$ – so uniform linear movement.

Taking into account the previous remark, we will work on the case b) in which $\varphi_E^2(v) - 2 = \varepsilon(v) \neq ct., \forall v \in \mathbb{R}$, with $\varepsilon: \mathbb{R} \rightarrow \mathbb{R}$, so:

$\varphi_E^2 - 2 - \varepsilon = 0 \Rightarrow \varphi_{E^{1,2}} = \pm\sqrt{2 + \varepsilon}$. Since the case (-), has no physical sense, we introduce φ_{E^1} in (7*) and we get: $\sqrt{2 + \varepsilon(v)} = 1 + \frac{2}{c^2} \int_0^v x \sqrt{2 + \varepsilon(x)} dx$. In order to simplify, we define: $\alpha_E(v) = \sqrt{2 + \varepsilon(v)} \Rightarrow \varepsilon(v) = \alpha_E^2(v) - 2$ and we get:

$$\alpha_E(v) = 1 + \frac{2}{c^2} \int_0^v x \alpha_E(x) dx, \forall v \in \mathbb{R} \quad (9)$$

Equation (9) is an Volterra linear integral equation of the second kind and we will search its solution using the classical method of the « resolvent ».

Using the iterative kernels, the resolvent of the equation (9) becomes:

$$R\left(v, x, \frac{2}{c^2}\right) = \sum_{n \geq 1} \frac{2^{n-1}}{c^{2(n-1)}} \frac{(v^2 - x^2)^{n-1}}{(2n-2)!} = e^{\frac{2}{c^2}(v^2 - x^2)} \text{ and though the solution is:}$$

$$\alpha_E(v) = 1 + \frac{2}{c^2} \int_0^v x e^{\frac{2x^2}{c^2}} dx = \frac{1}{2} e^{\frac{2v^2}{c^2}} + \frac{1}{2} \text{ and then we have:}$$

$$\varepsilon(v) = \alpha_E^2(v) - 2, \varphi_{E^1}(v) = \sqrt{2 + \varepsilon(v)} \Rightarrow E(v) = \frac{E_0}{2} \left(1 + e^{\frac{2v^2}{c^2}}\right), \forall v \in \mathbb{R} \quad (10)$$

As we can see, the solution (10) respects the initial condition $E(0) = E_0 \neq 0$.

Physical Interpretation of the Linearized Form

A remarkable property of the new energy equation in its linearized form:

$E = \frac{E_0}{2}(1 + e^{2\beta^2})$, where we note $\beta = \frac{v}{c}$, is its behavior at low velocities $v \ll c$.

By applying the Taylor series expansion for the exponential term ($e^x \approx 1 + x \dots$):

$$E \approx \frac{E_0}{2}(1 + 1 + 2\beta^2) = E_0 + E_0\beta^2$$

Substituting $\beta = \frac{v}{c}$ and $E_0 = m_0c^2$, we obtain: $E \approx m_0c^2 + m_0v^2$

This result is profoundly significant as it demonstrates that the new Intrinsic Energy Equation naturally incorporates the classical rest energy (m_0c^2) and a kinetic term (m_0v^2). While the classical Newtonian kinetic energy is expressed as $\frac{1}{2}mv^2$, our Volterra – derived model introduces a scaling factor that anticipates the non-linear transition required to bypass the light barrier.

This confirms that the proposed theory does not contradict classical mechanics in the subluminal limit, but rather extends and unifies it within a global, non-singular framework.

Practical explanation: linearization stands for "Approximate Regime for Laminar Flows". What does it actually mean? It means that we ignore turbulence and high-order particle interactions. It is similar to going from supersonic aerodynamics (where everything is nonlinear and shocks occur) to subsonic aerodynamics (where the air is "friendly"). Where does it lead? It leads to a "Steady-State Acceleration" type solution. In linear particle accelerators, if we used this linearization condition, we would obtain a highly predictable particle flow, where the energy increases exponentially but in a controlled manner (see equation (10)), without generating massive parasitic gamma radiation that usually occurs in violent collisions.

Conclusion: Linearization is useful for Propulsion Engineering, where we want a continuous and stable flow, not an explosive one. We will approach this matter again later on.

1.2. Beyond Wave-Particle Duality

While standard Quantum Mechanics relies on wave-particle duality, ITE proposes Phase Singularity Resolution. What is conventionally perceived as a "particle" is, in fact, a localized node of high phase density – a region where the geometry of space undergoes a critical rotation. This rotation frequency determines the observed mass and charge, effectively translating the "dialect" of geometry into the "dialect" of observable energy.

Presently we will pursue a vector construction meant to put in evidence the energetic space associated to the movement of a mobile with initial energy nonzero. Our construction is such that we can recall the results obtained previously in the unidimensional case.

Let: $\vec{v} \in \mathbb{R}^n, \vec{v} = (v_1, \dots, v_n)$ where $v_i = \frac{dx_i}{dt}$ and $e_i = \left(\underbrace{0, \dots, 1, \dots, 0}_{i\text{ème position}} \right), i = \overline{1, n}$ $\{e_i\}_{i=\overline{1, n}}$ being the canonical basis of \mathbb{R}^n .

We define: $u_i: \mathbb{R}^n \rightarrow \mathbb{R}^n, u_i(v) = \left(\underbrace{0, \dots, v_i, \dots, 0}_{i\text{ème position}} \right) = v_i e_i, i = \overline{1, n}$

Then: $(E \circ u_i)(v) = E(0, \dots, v_i, \dots, 0), i = \overline{1, n}$.

We define: $E_i: \mathbb{R} \rightarrow \mathbb{R}, E_i(v_i) = E(0, \dots, v_i, \dots, 0) = (E \circ u_i)(v), i = \overline{1, n}$

and we form the vector: $\vec{E} = (E_1, \dots, E_n) \in \mathbb{R}^n$

which is the energy vector associated to the movement of a mobile at the speed $\vec{v} \in \mathbb{R}^n$ and having initial energy $E_0 \neq 0$. So, we observe that $u_i, i = \overline{1, n}$ leaves the space \mathbb{R}^n invariant because $\sum_{i=1}^n u_i(v) = v, \forall v \in \mathbb{R}^n$, and so $u_i(v)$ is the restriction of vector v on the axis i .

We note $E_i = E_i(v_i), i = \overline{1, n}$ the projection of $E: \mathbb{R}^n \rightarrow \mathbb{R}$ on i axis, $i = \overline{1, n}$.

Taking into account the equations from the previous chapter regarding the unidimensional case, we can write, for example, the gradient of the vector field \vec{E} of the intrinsic energy-density manifold, with help from the above construction:

$$\begin{aligned} \nabla E &= \left(\frac{\partial E_1}{\partial v_1}, \dots, \frac{\partial E_n}{\partial v_n} \right) \in \mathbb{R}^n; \frac{\partial E_i}{\partial v_i} = \frac{2v_i E_i^3}{c^2 E_0^2} \left(1 - \frac{E_i}{E_0} \right), i = \overline{1, n} \\ \nabla E &= \frac{2}{c^2 E_0^2} \left(v_1 E_1^3 \left(1 - \frac{E_1}{E_0} \right), \dots, v_n E_n^3 \left(1 - \frac{E_n}{E_0} \right) \right), \text{ we note: } \tilde{E} = \left(1 - \frac{E_i}{E_0} \right)_{i=\overline{1, n}}, \text{ and we get:} \\ \nabla E &= \frac{2}{c^2 E_0^2} (\vec{v} \cdot \vec{E}^3) \cdot \tilde{E} \end{aligned} \quad (11)$$

as vector, where $\forall \vec{x}, \vec{y} \in \mathbb{R}^n, \vec{x} \cdot \vec{y} = (x_1 y_1, \dots, x_n y_n)$.

Pursuing the construction of the vector energy field of the intrinsic energy-density manifold, associated to the movement of our mobile: $\vec{E} = (E_1, \dots, E_n)$, $E_i = E_i(v)$, $v \in \mathbb{R}^n$ that we just constructed, we recall from the general theory of Hermitian vector spaces that:

Then, we recall from the general theory of Hermitian vector spaces that:

$$\forall v, \forall \vec{E} \in \mathbb{R}^n, \exists \phi_v \text{ such as } \phi_v(E) = \langle v | \hat{H} | \Psi_E \rangle = \langle v | E \rangle \text{ where } \Psi_E \text{ is the "state" having the energy } E \text{ at the speed } v. \quad (12)$$

where ϕ_v is a linear operator from the dual space associated. $\phi_v(E)$ represents the projection of the energy state Ψ_E onto the velocity basis via the Hamiltonian operator \hat{H} , acting as a phase evolution operator. $\phi_v(E)$ is not a static object, but a dynamic, evolving one. If the classical Hamiltonian \hat{H} represents the energy of a system in a fixed state, $\phi_v(E)$ is a coupling operator (an evolution operator in Hermitian space) that "links" energy to velocity. It describes the intrinsic interaction of the mobile with the environment / space during movement. It shows how the energy state Ψ_E is "projected" onto the velocity base via the Hamiltonian operator \hat{H} . It is a phase evolution operator.

$$\text{So, we get:} \quad E_i = \frac{\partial}{\partial v^i} \phi_v(E), i = \overline{1, n} \quad (13)$$

$$\text{and:} \quad \frac{\partial E_i}{\partial v^i} = \frac{\partial^2}{\partial v^{i^2}} (\phi_v(E)), \forall i = \overline{1, n} \quad (14)$$

$$\text{which leads for the divergence operator to: } \text{div} E = \Delta \phi_v(E) \quad (15)$$

that is basically the equation of state of the vacuum in our model. We observe that the divergence of the total energy dictates the phase curvature.

On the relations (13) and (15) we can make the following remarks:

- i. as we observe, ϕ_v is the potential of the energy field $\vec{E} = \vec{E}(v)$ of the intrinsic energy-density manifold, described in this chapter, and that for each v belonging to the trajectory of the movement of our mobile. This is valid $\forall n \in \mathbb{N}$ and for any type of movement of a mobile. We recall that the field \vec{E} concerns the total energy of the mobile. So, we emphasize that ϕ_v is different from the potential energy of the mobile. The field \vec{E} is generated at any time during the movement of the mobile.
- ii. if $\text{div} E = ct$. that implies that the field \vec{E} must also be constant or whirling. To simplify the study, we can assume $ct = 0$ and we get the situation $\text{div} E < 0$ corresponding to a convergent energy field that implies a slow-down motion and the situation $\text{div} E > 0$ corresponding to a divergent energy field that implies an accelerated movement.

with respect to E_i and keeping the energy relative variation like argument of the solution.

We get: $E_i = \frac{cE_0}{2v_i} \left(\frac{1}{2} - 2\tilde{E}_i \right)^{-\frac{1}{2}}, \forall i = \overline{1, n}$ which is an expression of the parts of the potential of the energy field associated to the movement of the mobile having $E_0 \neq 0$, expression in function of its relative variation on each axis i .

If we develop equation (21) we get:

$$\frac{1}{2} + 3 \frac{v_i^2 E_i^2}{c^2 E_0^2} - 4 \frac{v_i^2 E_i^3}{c^2 E_0^2} = 0 \text{ and we note } \frac{v_i^2}{c^2 E_0^2} = a_i(v, E_0), \text{ where } a_i: \mathbb{R}^{n+1} \rightarrow \mathbb{R} \text{ thus:}$$

$$E_i^3 - \frac{3}{4} E_i^2 - \frac{1}{8a_i} = 0 \Leftrightarrow (E_i - 1)(E_i^2 + E_i + 1) = \frac{3}{4} E_i^2 - \frac{8a_i - 1}{8a_i} \quad (22)$$

This whole approach means that we have found an isomorphism between the space of differential operators and the linear algebraic space. It shows that energy is not chaos, but a structure that can be calculated with watchmaker precision. If the Laplacian is zero, it means we have perfect equilibrium, a potential state that is just "waiting" for speed to become dynamic.

Remark:

The processing of the left member of (22) is like the one from the previous remark.

1. Again, we have two cases:

$$\text{a. } \frac{3}{4} E_i^2 - \frac{8a_i - 1}{8a_i} = k_{E_i} - \text{constant}, k_{E_i} > 0 \Rightarrow E_i = \frac{2}{\sqrt{3}} \sqrt{\widetilde{k}_{E_i} - \frac{c^2 E_0^2}{8v_i^2}}, \text{ where}$$

$\widetilde{k}_{E_i} = k_{E_i} + 1$. We select only the option (+) for negative energy has no physical meaning. We also observe that physical meaning is valid only if: $k_{E_i} > \frac{c^2 E_0^2}{8v_i^2} - 1$.

$$\text{b. } \frac{3}{4} E_i^2 - \frac{8a_i - 1}{8a_i} = \beta_i(v) \neq \text{constant}, \text{ where } \beta_i: \mathbb{R}^n \rightarrow \mathbb{R} \text{ is the energy field potential modeling function. Then we have:}$$

$$E_i(v) = \frac{2}{\sqrt{3}} \sqrt{\widetilde{\beta}_i(v) - \frac{c^2 E_0^2}{8v_i^2}}, \text{ où } \widetilde{\beta}_i(v) = \beta_i(v) + 1, \forall i = \overline{1, n} \quad (23)$$

2. Under the assumption of linearization of equation (7*) we can now find the form of energy field potential modeling function on each axis, β_i by using the equation (10).

We get: $\beta_i = \frac{3}{4} E_0^2 e^{\frac{4v_i^2}{c^2}} - 1 + \frac{c^2 E_0^2}{8v_i^2} = \frac{E_0^2}{4} \left(\frac{3}{2} e^{\frac{4v_i^2}{c^2}} + \frac{c^2}{2v_i^2} \right) - 1$, from where:

$$\widetilde{\beta}_i(v) = \frac{E_0^2}{4} \left(\frac{3}{2} e^{\frac{4v_i^2}{c^2}} + \frac{c^2}{2v_i^2} \right) \quad (24)$$

What actually is the Phase Φ ?

Let's imagine an infinite ocean of possibilities.

Phase is not a particle, but the state of vibration and coordination of this ocean as a whole, down to the smallest structural unit, wherever it may be.

1. At the Micro level (Quantified):

Phase Φ is the fundamental "pulse." It's that heartbeat that turns nothing into something. When we say that the information unit Ψ_{3B} is the smallest unit, that's right: Ψ it's just the "package" of geometry and energy, but Φ it's what makes it exist in a certain state. At the micro level, the phase Φ is what says, "Now you are electric charge," or "Now you are curvature." It's the "source code" of reality.

2. At the Macro level:

When these small units (Ψ_{3B}) come together, their phases overlap (interference). If the phases are in harmony, we have stable matter, from random objects to planets, stars.

If the phases are in chaos, we have noise or void. The macro series are, in fact, the harmonics of that fundamental vibration of Φ .

When the smallest unit re-enters "Minkowski", it does not fall apart because Phase Φ is a topological invariant. Even if the surrounding space flattens (Minkowski), the phase "knot" that constitutes it remains tied. It is like a knot on a string: you can stretch the string as much as you want (flat geometry), but the knot (matter) stays there because it is a property of the way the string (phase) is twisted.

Definition: Phase Φ is the measure of the geometro-dynamical correlation between space, charge and energy. It is not a simple property of a wave but is the state of synchronization of the void that allows the emergence of structure.

At this stage, the MIU (Minimal Information Unit or we can say Minimal Phase Accumulation Unit) does not have a defined topology (it is not a sphere, it is not a point, it is not a torus, etc...). It is a density / probability distribution of phase over the entire domain Ω_{I_v} . The concrete form of these intervening factors will be deduced later in the chapters dedicated to them, where the phase 'crystallizes' into observable geometry.

If in the standard model the phase is a local scalar in a Hilbert space. In our model, the Phase Φ is not a simple local scalar in a Hilbert space, but a bi-directional global connection operator. It dictates the fundamental mechanism by which the curvature of space is translated into electric flux and, conversely, how the dynamics of charge and energy sculpt the local geometry. This transformation symmetry ensures that no component of the MIU is isolated, with the phase acting

as the common denominator in which information flows freely between the metric and electromagnetic states.

Thus, the minimum unit of accumulation is:

$$\delta\Phi = \int_{\partial\Omega_{I_v}} \langle \Psi | \phi_v | \Psi \rangle d\sigma_\Omega$$

where:

$\Psi = \langle G | \Phi | Q, E \rangle$ and G is geometry, Q and E are the electrical load and the energy

Ω_{I_v} is the volume between v_X and v_Y , $\forall X$ and Y on the trajectory of the mobile

Ω_{I_v} it is also called jump volume as we will see further in the Intrinsic Theory of Energy

Invariance: Regardless of local variations of the environment, the integral on Ω_{I_v} remains constant, ensuring the stability of matter creation upon reentry into the flat metric.

As we will see throughout the paper, in this model of the Intrinsic Theory of Energy (ITE), we have eliminated the separation between "object" and "law".

- **In classical physics:** We had a particle (the object) and a force (the law).
- **In ITE:** We only have **Phase Φ** .
 - When it "spins" in some way, we call it **Geometry**.
 - When it "flows" in some way, we call it **a Charge**.
 - When it "vibrates" with a certain intensity, we call it **Energy**.

Because in Ω_{I_v} , the three components are not added arithmetically, but are **interfered harmonically**.

- The geometry (G_{jk}) gives the fundamental frequency.
- The load (Q_j) gives the amplitude.
- Energy provides resonance.

When all three are in phase, the MIU is indestructible. Any attempt to "break" it in the "Minkowski" space would require infinite energy to undo this phase knot. That is why it is the fundamental unit of matter/phase accumulation, that is why it has no specific form but composes any particle. It is an informational-material unit.

Furthermore, the tensor representation of equation (3) confirms that this energy flow is intrinsically coupled to the Kähler metric. It becomes the equation of an energy flow on a Cauchy hypersurface (Σ):

$$E(v) = E_0 + \frac{2}{c^2 E_0^2} \int_{\Sigma} x^{\mu} E^3 \left(1 - \frac{E}{E_0}\right) d\Sigma_{\mu} \quad (25)$$

It demonstrates that what we once perceived as static "mass" is, in reality a stable fixed point within a dynamic iteration of space-time geometry.

Here our old integral on I_v becomes a flow integral that depends on the path metric. As we have written before, if $\text{div} E = 0$ we have perfect equilibrium (a potential state). In tensor language that means that the Cauchy Engine is conservative i. e.: $\nabla^{\mu} \nabla_{\mu} \phi = 0$ – the Laplace equation in tensor form.

The stability of the Phase Φ is not merely a mathematical conjecture; it is an intrinsic property of information that persists even when the physical medium (sand, memory, or vacuum) is subject to extreme entropy.

1.3 Charge as Metric Tension

The appearance of charge is the physical result of the Phase Operator ϕ_v forcing the Kähler metric into a state of local accumulation. In this framework, we eliminate the classical separation between the "object" and the "law".

- Geometric Isomorphism: Charge, mass, and energy are "isomorphic dialects" of the same underlying phase-state Φ .
- The Nature of the Knot: A particle is fundamentally a Phase Knot where the rotation frequency is high enough to prevent the metric from dissolving back into the flat vacuum.
- Flux Definition: Mathematically, charge (q) is defined as the flux of the phase gradient across the boundary of the Minimal Information Unit (MIU):

$$q = \oint_{\partial \Omega_{MIU}} \nabla \Phi \cdot dS \quad (26)$$

This implies that charge, mass, and geometry are "isomorphic dialects" of the same phase-state. A particle is simply a Phase Knot where the rotation frequency prevents metric dissolution.

- Unified Dialect: While geometry (G_{jk}) provides the fundamental frequency of the system, the charge (Q_j) provides the amplitude, and energy provides the resonance. When these three are in phase, the resulting MIU becomes a stable, indestructible unit of matter.

1.4 The Phase Jump and the Volume of Displacement (Ω_{I_v})

Traditional physics describes motion as a continuous trajectory through space. ITE replaces this with the Phase Jump Mechanism.

- **Non-Local Resonance:** The transport of an MIU from point A to point B is a reconfiguration of the global phase Φ . When the phase at A is "rotated out" (annihilation), it must instantaneously "rotate in" at B (creation) to satisfy the Global Conservation of Phase.
- **The Responsibility Constraint:** Because the universe maintains a constant phase balance, any creation "out of nothing" without a corresponding source-sink resonance triggers a massive Metric Back-Reaction. This is the fundamental physical law that prevents "free energy" or uncompensated events — the universe "charges" a price for every change in its geometric memory. This will be explained and enhanced furthermore as the work goes on.

1.4.1 The Conservative Nature of Phase Evolution

In the ITE, transport is not a displacement of matter through a passive vacuum, but a continuous re-mapping of the phase Φ . The fundamental law of existence — the Responsibility Constraint — dictates that the universe is a closed-loop system of information:

$$\frac{d\Omega_{Kah}}{dt} = 0 \quad (27)$$

This implies that the "angular frequency" of the Kähler variety remains invariant. Consequently, every local change in the state isomorphism (J_Φ) is rigorously balanced by the global phase evolution:

$$\frac{dJ_\Phi}{dt} = \frac{d\Phi}{dt} \quad (27^*)$$

1.4.2 The Source-Sink Symmetry

Just as a point of phase inversion, any transport of a Phase Knot must obey the symmetry of the vacuum.

- **The "Nothing for Nothing" Principle:** If an entity is "extracted" from point A, the metric at A undergoes a relaxation, while the metric at B undergoes a simultaneous tension.
- **Violation and Recoil:** If a high-energy event (e.g., a catastrophic destruction or an uncompensated "creation") occurs, the term $\frac{dJ_\Phi}{dt}$ creates a profound divergence. Since $\frac{d\Phi}{dt}$ cannot be violated, the universe generates a Compensatory Metric Recoil. This "recoil" is

the physical mechanism behind causality — it is the universe's way of "charging the debt" to restore the phase balance.

1.4.3 Non-Local Connectivity in the Kähler Manifold

Because the phase Φ is a single, unified entity, points A and B are not separate. They are different coordinates of the same underlying phase rotation. This explains the "instantaneous" nature of the phase jump: we are not moving a particle *through* space; we are rotating the space *into* a new particle state.

Remark:

Chapters 1.3 and 1.4 just announced the problematics that will find significant development further in this work.

CHAPTER II: Jump Topology: Principles, Invariants, and Fundamental Equations

2.1 Principles and Invariants of Jump Topology

If the energy E is a solution to the equation of state, then the operator $\phi_v(E)$ defined by relation (4) must be "transparent" to the physical quantity carried. This principle of self-consistency eliminates the risk that the energy tends to infinity at $v \rightarrow c$ and leads us to define the geometric-energetic action in the speed space as:

$$W_v(E) = \int_{I_v} E_i dv^i \quad (28)$$

where I_v is a domain of whidth $\|v\|$ around the mobile that is moving with the speed v .

Then, we have:

$$\phi_v(E) = \langle v | \hat{H} | \Psi_E \rangle = \langle v | E \rangle \equiv \int_{I_v} E_i dv^i = W_v(E) \quad (29)$$

this is the transparency bridge. Thus, we establish a causality-restoration connection type: if the intrinsic energy manifests itself through the variation of the operator with respect to the speed, then the operator is the sum (integral) of these manifestations over the considered speed range.

Thus, we establish a causality-recovery type connection: if the intrinsic energy manifests itself through the variation of the operator with respect to the speed, then the operator is the sum (integral) of these manifestations over the considered speed range.

If the integral over the domain I_v of the gradient of the operator exactly restores the scalar (or vector) value of the energy E , then $\phi_v(E)$ it is not just a mathematical operator, but is **the conservation mechanism embodied**.

a) Self-Consistency (Self- Reference):

If $|\phi_v(E)| = E$, it means that the operator is "transparent" to the physical quantity it carries. It neither adds nor loses energy; it just re-localizes it from the gradient (variation) form to the integral (state) form.

b) Explanation of Conservation:

Conservation of energy in classical physics is often a postulate (or derived from Noether symmetry). In this model, conservation becomes a mathematical necessity:

— E_i represents "how things change."

— $\int_{I_v} E_i dv^i$ represents the "total changes".

- If the total of changes is equal to the energy itself, then the system is **closed and conservative by definition**.

1. The Overlap

If the domain I_v has a size proportional to $\|v\|$, as $v \rightarrow c$, this domain not only grows, but also begins to "swallow" the adjacent states.

From a topological point of view, if you have a sequence of transitions where the integration domains overlap, you get an open cover of the phase space.

This overlap guarantees that there are no "gaps" (gaps) in the energy transfer. The transition is not a discrete jump in the sense of rupture, but a continuous slide through a series of topologically equivalent states.

When we say it is a sequence of transitions, it means that the particle does not "jump" over the barrier but rather goes through a local homeomorphism.

Regardless of the phase rotation Hamiltonian, the fact that the integration domains overlap means that information (energy) is conserved by topological continuity regardless the phase states are discrete.

This "transition topology" explains why the Volterra equation (eq. 3) works where others fail: it sees the process, not just the threshold. The superposition guarantees that the operator $\phi_v(E)$ remains well-defined (smooth) throughout the rotation.

Locally we have "pixelation" (velocity quanta), but the overall picture (Energy) is a "continuous vector" due to integration over the domain I_v .

$\phi_v(E)$ is not just a carrier, but a reservoir of potential for the transition. If we look at it $\phi_v(E)$ as a jump potential, then the conservation of the total energy is ensured by "borrowing" from the complex phase: at the moment of the jump, the energy does not increase infinitely but is stored in the form of a phase (potential) until the transition is complete.

2. Metrics and Transition Topology

The metric we discussed transforms the velocity space from a rigid Euclidean space into a manifold where the "distance" to c reconfigures.

When we say it is a sequence of transitions, it means that the particle does not "jump" over the barrier but rather goes through a local homeomorphism.

Regardless of the phase rotation Hamiltonian, the fact that the integration domains overlap means that information (energy) is conserved by topological continuity.

3. "Madame Particle" and Transition Topology

We imagine the particle, the mobile, not as a point, but as the center of this domain I_v .

At low speeds, I_v it is small, almost point-like.

As it approaches the "jump", I_v expands massively.

The overlapping of domains ("regardless of phase rotation") means that although the phase rotates in the complex plane to avoid infinity, the mathematical support of the energy (the integral) remains tied to reality. It's like passing a baton where the receiver has already grabbed the baton before the first one lets go.

4. Conclusion

This "transition topology" explains why the Volterra equation (eq. 7) works where others fail: it sees the process, not just the threshold. The superposition guarantees that the operator $\phi_v(E)$ remains well-defined (smooth) throughout the rotation.

Using everything we've worked on so far, for at least one particle, how do we prove that the energy is finite at the "jump"?

Let n be the number of dimensions. Let m be the number of overlapping Riemann domains corresponding to each possible energy state the particle can have as it approaches the jump. They are distinct and finite in number (they are quanta) according to quantum mechanics.

integration area $I_k \cap I_{k+1}, k = \overline{1, m}$

then for a time interval $\Delta t = t_x - t_y$, some of the jump time we have $\phi_v(E)$ as value on the interval:

$$I_v = \bigcup_{k=\overline{1, m}} (I_k \cap I_{k+1}) \text{ cu conditia: } (I_k \cap I_{k+1}) \cap (I_{k+1} \cap I_{k+2}) \neq \emptyset$$

$$\left| \sum_{k=\overline{1, m}} \int_{I_{v_k}} E_i^k dv^i \right| \leq \sum_{k=\overline{1, m}} \left| \int_{I_{v_k}} E_i^k dv^i \right| = \sum_{k=\overline{1, m}} \int_{I_{v_k}} |E_i^k| dv^i \text{ because } E \text{ is a solution of eq. (7)}$$

and each $I_{v_k} = I_k \cap I_{k+1} \neq \emptyset$.

Then: $\forall i = \overline{1, n}, \exists 0 \leq M_i < \infty$ such as $0 \leq E_i < M_i$ it results

$\exists M_E \in [0, \infty)$ such as $0 \leq \sum_{i=1}^n E_i \leq M_E$, it results:

$$\exists M_{E,v} \in [0, \infty) \text{ such as } 0 \leq E_i dv^i \leq M_{E,v}$$

So:

$$\sum_{k=1, \overline{m}} \int_{I_{v_k}} |E_i^k| dv^i \leq v^k M_{E,v^k} - \text{finite}$$

Global Continuity vs. Local Discretization

Mathematically, the Volterra equation (eq. (7)) is globally continuous because the integral operator "smooths out" the jumps. Even if the function under the integral has discrete jumps, the integral remains continuous.

However, at the local (microscopic) level, if the velocity v changes in quanta Δv , then the integration domain I_v does not grow smoothly, but in "steps".

The Solution: "Stepped" Transition Topology. The domain superposition idea comes in. To preserve energy conservation ($|\phi_v(E)| = E$) despite quantum speed jumps, we must satisfy a topological condition:

Coverage Condition: Any speed "jump" Δv must be smaller than the width of the integral's overlap domain.

If the quantized velocity step is "swallowed" by the integral domain, then the particle does not feel the break. From the operator's perspective ϕ_v , the velocity jump is just an internal phase reconfiguration within the same topological domain.

"Madame Particle" as a Wave Packet

If we look through de Broglie's prism, velocity is not a point, but a property of a wave packet.

- Local: We have jumps (discretization).
- Global: We have a tire (Volterra integral equation (7)) that ensures the transition.

The transition is not a straight line, but a ladder whose steps are so close together (or the ranges I_v so wide at high speeds) that, energetically, the difference becomes negligible.

How do we argue? We introduce the notion of "Operational Continuum":

Even though the velocity space is granular (quantized), the Intrinsic Energy Operator $\phi_v(E)$ acts as a topological "glue". The overlapping of domains I_v is exactly the mechanism that repairs the local discontinuity, transforming quantum jumps into a smooth global evolution.

In short: Locally we have "pixelation" (velocity quanta), but the overall picture (Energy) is a "continuous vector" due to integration over the domain I_v .

Individual Character: "Mass Imprint"

$\phi_v(E)$ must have different values for each entity. Why? Because the domain I_v (width of the integral) depends on the intrinsic properties of the object (rest mass, charge, internal structure).

- For an electron, for example, the overlap "window" I_v is different from that of a proton.
- Each particle has its own "energy capacity" to store the phase before the jump.

How do we determine quantified values?

$\phi_v(E)$ must have different values for each entity. Why? Because **the domain** I_v (width of the integral) depends on the intrinsic properties of the object (rest mass, charge, internal structure).

Domain Resonance Condition: The allowed values are those for which the number of phase oscillations within the domain I_v is an integer or half-integer. It is exactly like in a potential box, but here the "box" is our domain of integration itself which moves and expands. Therefore, we have:

$$W_v = \int_{I_v} E_i dv^i = n \cdot \mathcal{P}_S \quad (30)$$

where \mathcal{P}_S is an action constant specific to our system. We introduce quantization naturally, not by postulate, but by the condition of closure or cyclicity of the path in phase space. ϕ_v becomes a quantity that grows in geometric "steps", which explains why only certain states E are allowed (stationary states).

At its most fundamental level, the transition across the relativistic boundary is not a physical "break", but a unitary evolution of the system's state. We define the coupling operator $\phi_v(E)$ as the Hamiltonian coupling in the velocity basis: $\phi_v(E) = \langle v | \hat{H} | \Psi_E \rangle$. This expression represents the probability amplitude of the energy state Ψ_E projected onto the velocity manifold. In this framework, the phase rotation R_E identified in our model:

$$\begin{aligned} R_E: \mathcal{H}_E \times \mathcal{V}_n &\rightarrow \mathcal{H}'_E \times \mathcal{V}_n \\ R_E &= \phi_v(E) e^{\frac{i\pi}{2}} = \phi_v(E) \left(e^{\frac{\pi}{2}} \right)^i = \phi_v(E) (\sqrt{i})^i = i \phi_v(E) \end{aligned} \quad (31)$$

where \mathcal{V}_n is the discrete space of quantized velocities

acts as a **unitary isomorphism** between the "sub-luminal" (\mathcal{H}_E) and "super-luminal" (\mathcal{H}'_E) Hilbert spaces. We can call it **coupling phase operator**.

Since both spaces share the same orthonormal basis $e_{kv} = \frac{1}{\sqrt{2\pi}} e^{ikv}$, the information encoded in the system remains invariant.

This mathematical mapping ensures that the 'singularity' is merely a transition point where the energy state rotates into the complex plane. Just as Quantum Mechanics utilizes the imaginary unit i to describe the continuous evolution of waves, our model utilizes this phase rotation to maintain

a continuous and reversible bridge between subluminal and hyper-causal (superluminal) regimes. This isomorphism proves that the laws of conservation are preserved, as the system's norm remains consistent during the phase transition.

The natural presence of this State Isomorphism (Continuity) governing the transition, means that the structure of the physical laws remains intact whether you are below or above $v = c$. So, the singularity is eliminated by treating speed not as a external parameter but as a component of the bracket in phase space. We do not have a "break" in the universe, but only a change of perspective (phase).

The quantized values are the "steps" that the particle climbs. We determine these values by finding the stability points of our Hamiltonian in the complex plane.

- Each "step" of jump potential energy corresponds to a phase rotation of $n \frac{\pi}{2}$. It is exactly the moment when the "kinetic" energy flows into the "jump potential".
- When the phase made a complete rotation, the particle "jumped" a quantum of speed without the total energy E changing suddenly.

Principle of Phase Individuality:

"Each particle possesses a unique operator $\phi_v(E)$, determined by the geometry of the local domain I_v . The quantization of the jump energy derives from the requirement that the phase rotation be cyclic within this overlapping domain."

Formalization of Quantified Values

We will attack the determination of its values $\phi_v(E)$ using a topological quantization condition. If the velocity jumps in steps, it means that our integral over the domain I_v must satisfy a "phase closure" condition:

$$\int_{I_v} \Phi(E, v) dv = n \cdot \epsilon \text{ where } \epsilon \text{ is the jump energy potential quantum.} \quad (32)$$

This tells us that in the volume of velocities I_v , the accumulated phase is equal to an integer number of units of jump potential.

Step 1: Defining the Topology \mathcal{T}_J

We cannot treat it as a standard (Euclidean) topology because it must handle discrete jumps.

- Definition: \mathcal{T}_J is the topology generated by the open covering of overlapping domains I_v .

- Key property: At a jump point v_k , the neighborhood of the point is not an infinitesimal interval, but the union of domains like this:

$$I_v = \bigcup_{k=1, m} (I_k \cap I_{k+1}) \text{ with condition } (I_k \cap I_{k+1}) \cap (I_{k+1} \cap I_{k+2}) \neq \emptyset, \forall m < \infty.$$

The Reunion (\bigcup) defines the entire space in which the particle can move (the cover), but the Intersection (\bigcap) is the topological "bridge" on which the jump is made. Without the intersection, we would have a discrete (isolated) topology, and the particle would be "lost" between the steps.

- Effect: This topology "forces" operator continuity $\phi_v(E)$ even if the basis (velocity) is granular.

Step 2: Isomorphism between Minkowski Space \mathcal{M} and the Vector Space with Metric $\mathcal{V}(\mathcal{M}_v)$

We start from the classical space where the Minkowski metric is $\eta_{\mu\nu} = (1, -1, -1, -1)$.

- Construction of the isomorphism (f): We define a map $f_v: \mathcal{M} \rightarrow \mathcal{V}(\mathcal{M}_v)$. Let be $u \in \mathcal{M}$ a vector in flat Minkowski space. The isomorphism f_v must act on the metric. Its operational form is a velocity-dependent conformal transformation:

$f(u) = \Lambda(v) \cdot u$ where the transformation operator $\Lambda(v)$ is defined so as to map the Lorentz metric $\eta_{\mu\nu}$ into the variable metric \mathcal{M}_v :

$$\langle f_v(u) | f_v(w) \rangle_{\mathcal{M}_v} = \mathcal{M}_v(v) \cdot (u^\mu w^\nu \eta_{\mu\nu})$$

Definition of the transformation operator $\Lambda(v)$:

$$\Lambda(v) = \begin{pmatrix} \gamma(v) & -\frac{v}{c}\gamma(v) \\ -\frac{v}{c}\gamma(v) \cdot J & \gamma(v) \cdot \mathbb{I} \end{pmatrix}$$

where $\gamma(v)$ – is the Lorentz factor; it is part of the almost complex structure with the property $J^2 = -Id$.

Properties of the isomorphism form:

- Linearity: Preserves the vector space structure.
- Dynamism: It is not a fixed matrix; it "pulsates" according to the norm $\|v\|$.
- Inversibility: Since it is an isomorphism, we can always go back to Minkowski (observe the result in the laboratory reference system).

- Metric transformation: In the destination space, the metric \mathcal{M}_v is no longer constant but becomes dependent on the local speed.
- Role: This isomorphism "bends" the flat Minkowski space as we approach c , setting the stage for the Hamiltonian rotation. Basically, we transform rigid coordinates into energetic state vectors.

Step 3: Isomorphism to the Topological "Jump" Space (\mathcal{T}_j)

We need to map the metric vector space onto the jump structure. If the particle wants to jump from velocity v_k to v_{k+1} , it needs a topological "bridge". This is the intersection of two domains:

$I_k \cap I_{k+1}$ and the topology is described as follows:

$$\cup_k (I_k \cap I_{k+1}) \text{ with condition } (I_k \cap I_{k+1}) \cap (I_{k+1} \cap I_{k+2}) \neq \emptyset$$

If this intersection is non-empty, energy can pass from one step to another without breaking. It's like stepping from one stone to another in a river: the stones must be close enough so that you can step on both at the same time.

- Link application (g): $g: \mathcal{V}(\mathcal{M}_v) \rightarrow \mathcal{T}_j$.

This is what "injects" the metric into the transition topology. It transforms metric distances into phase differences:

$$g(\Delta s) = e^{i \int \omega(v) dv}$$

Here, Δs (the distance in the metric \mathcal{M}_v) becomes the argument of the Hamiltonian rotation.

- Mechanism: This mapping transforms vector norms in metric space into complex phases in jump space.
- Conservation: Isomorphism guarantees that the conservation property $|\phi_v(E)| = E$ remains invariant when moving from one space to another.

Observation regarding ω :

If we look at it ω as an angular frequency, the relation is the classical one $\omega = \frac{1}{\hbar} E$, but in our study ω it plays the role of connection on the manifold.

Why ω is it more than "just" $\frac{1}{\hbar} E$? In our research, $\omega(v)$ it is actually a symplectic Berry form (or curvature). It includes the phase rotation "resistance" R_E . The complete relation is:

$$\Phi(E, v) = \frac{\partial}{\partial v} (R_E \cdot \phi_v(E)) = \frac{\partial}{\partial v} (i \phi_v(E) \cdot \phi_v(E)) = i \frac{\partial}{\partial v} (\phi_v^2(E)) =$$

$$\begin{aligned}
&= 2i\phi_v(E) \frac{\partial}{\partial v} \phi_v(E) = 2i\phi_v(E) \frac{\partial}{\partial v} \langle v|E \rangle = 2i\phi_v(E) \left(E + \left\langle v \left| \frac{\partial E}{\partial v} \right\rangle \right) \right) \\
&= 2i\phi_v(E) \left(E + \left\langle v \left| \frac{\partial^2 \phi_v}{\partial v^2} (E) \right\rangle \right) \right)
\end{aligned}$$

$$\text{Then: } \omega(E, v) = \frac{1}{\hbar} (\Phi(E, v)) \quad (33)$$

Notice that Φ it tells us how much energy is flowing out and ω it tells us how the space is curved under that flowing energy. So ω it's not just a quantity of energy anymore, it's the curvature of the symplectic form. It transforms energy into geometry.

Once we have passed through the isomorphism, we are no longer in the flat Minkowski space, but in a Kähler manifold. Here we define a new metric (g), which ensures the passage over $v = c$:

$$g(X, Y) = \Omega(X, JY)$$

where: Symplectic Form Ω : Guarantees that energy is conserved (nothing is lost) and when the speed wants to become too high, J it intervenes and rotates that energy into an "invisible" dimension (the complex phase).

So: *for any two X si Y , we have $g(X, Y) = \Omega(X, JY) = g(\Delta s) = e^{i \int_{v_X}^{v_Y} \omega(E, v) dv}$,*

where $\Delta s = d(X, Y)$ – distance from X to Y in metrics \mathcal{M}_v

from topology T_j (jump topology) on the Kähler manifold

Observation:

$$1. \quad i \int_{v_X}^{v_Y} \omega(E, v) dv = \frac{i}{\hbar} \int_{v_X}^{v_Y} \Phi(E, v) dv$$

is essentially an Action Integral in complex phase space. It is the "fingerprint" of the jump.

2. The presence of the constant \hbar in the denominator of the phase argument guarantees that the energy transfer in the Kähler manifold is a quantized and finite process. \hbar It is the element that connects the macroscopic magnitude of the velocity and the microscopic structure of the intrinsic energy, ensuring the continuity of the metric g in the superluminal regime.

1. The Jump Engine: The Quadratic Derivative

The starting point is the recognition that flow Φ does not arise from nothing but is the variation of "phase pressure" with respect to velocity. Thus, according to the previous calculation:

$$\omega = \frac{1}{\hbar} \Phi(E, v) = \frac{1}{\hbar} \frac{\partial}{\partial v} (\phi_v^2(E)) = \frac{2}{\hbar} \phi_v(E) \frac{\partial}{\partial v} \phi_v(E)$$

Here we clearly see where factor 2 comes from: it represents the coupling between the current state and its tendency to change.

2. Phase Identity (The Two 90° Rotations)

To understand why the jump is topological, we use our definition for the rate of phase variation:

$$\frac{\partial}{\partial v} \phi_v(E) = E + \left\langle v \left| \frac{\partial^2 \phi_v}{\partial v^2} (E) \right. \right\rangle$$

- **First rotation (90°):** It is given by the factor i , which transforms the velocity variation into an orthogonal rotation in the complex plane.
- **Second rotation (90°):** It is ensured by its presence $\phi_v(E)$ outside the parentheses (from the quadratic derivative), which acts as a second phase operator.

3. The Final Form of the Flow $\Phi(E, v)$

$$\Phi(E, v) = 2R_E \left(E + \left\langle v \left| \frac{\partial^2 \phi_v}{\partial v^2} (E) \right. \right\rangle \right)$$

It tells us that the flux is generated by the total Energy (E) plus the directional interaction between velocity and phase $\left\langle v \left| \frac{\partial^2 \phi_v}{\partial v^2} (E) \right. \right\rangle$.

4. Quantification Condition (Phase Closure)

Now we can write the integral we defined previously ϵ in a form that leaves no room for interpretation:

$$\epsilon = \frac{1}{n\hbar} \int_{I_v} \Phi(E, v) dv = \frac{2}{n\hbar} \int_{I_v} R_E \left(E + \left\langle v \left| \frac{\partial^2 \phi_v}{\partial v^2} (E) \right. \right\rangle \right) dv \quad (34)$$

- **Its nature ϵ :** The quantum jump ϵ is now defined as the "complex area" swept by the phase ϕ_v under the action of energy E and velocity v .
- **Rotational stability:** Since the result is $n \cdot \epsilon$, the system "jumps" only when it accumulates a complete phase cycle.
- **Stepped velocity:** The velocity jump occurs exactly where the integral reaches the critical value, forcing the particle to move to the next topological level to preserve its phase integrity.

The Phase Transgression Postulate:

Any mobile whose energy state constitutes a solution of Equation (7) can exceed the speed of light limit by continuous acceleration. In this process, the infinite energy singularity is avoided by the phase rotation of the operator R_E , ensuring strict conservation of the total energy throughout the transition.

Continuing the study of the Kähler-type variety of phases we obtain:

let X and Y arbitrary in phase space, then:

$$\Omega(X, JY) = e^{\frac{i}{\hbar} \int_{v_X}^{v_Y} \Phi(E, v) dv}$$

which included in the second condition of the Kähler variety leads to:

$$\begin{aligned} d\Omega = 0 &\Leftrightarrow d\left(e^{\frac{i}{\hbar} \int_{v_X}^{v_Y} \Phi(E, v) dv}\right) = 0 \Leftrightarrow \frac{i}{\hbar} d\left(\int_{v_X}^{v_Y} \Phi(E, v) dv\right) e^{\frac{i}{\hbar} \int_{v_X}^{v_Y} \Phi(E, v) dv} = 0 \\ &\Leftrightarrow \frac{i}{\hbar} d\left(\int_{v_X}^{v_Y} \Phi(E, v) dv\right) = 0 \Leftrightarrow i \cdot d\left(\int_{v_X}^{v_Y} \omega(E, v) dv\right) = 0 \\ &\Leftrightarrow \frac{2i}{\hbar} d\left(\int_{v_X}^{v_Y} R_E\left(E + \left\langle v \left| \frac{\partial^2 \phi_v}{\partial v^2}(E) \right\rangle\right) dv\right) = 0 \Leftrightarrow \exists ct. \neq 0 \\ &\text{such as: } \frac{2i}{\hbar} \int_{v_X}^{v_Y} R_E\left(E + \left\langle v \left| \frac{\partial^2 \phi_v}{\partial v^2}(E) \right\rangle\right) dv = ct. \end{aligned}$$

We will call this constant the Topological Coherence Constant of the Jump (\mathcal{K}_S). This constant arose naturally from the constraint on the Kähler manifold ($d\Omega = 0$), representing the "visa of passage" through the relativistic barrier.

$$\text{So, we can remember: } \frac{2i}{\hbar} \int_{v_X}^{v_Y} R_E\left(E + \left\langle v \left| \frac{\partial^2 \phi_v}{\partial v^2}(E) \right\rangle\right) dv = \mathcal{K}_S \quad (35)$$

The Physical Significance of \mathcal{K}_S :

- **Re-materialization Guarantee:** If this integral were not a real and non-zero constant, the particle would "dissipate" into the imaginary phase. \mathcal{K}_S is the anchor that forces the return to the real (observable) axis. It shows that the transition is not a chaotic event, but one governed by a law of phase conservation in complex space.

- **Jump Quantization:** The jump is not chaotic; the light "waits" for the accumulation of the exact value of this invariant to perform the phase rotation.
- **Phase Error:** The stability of this constant given by its invariance explains why light reappears with an identical spectral signature (phase error $< 10^{-15}$) as we will see later.

Continuing the integration from his formula \mathcal{K}_S we obtain:

$$\begin{aligned} \frac{2i}{\hbar} \int_{v_X}^{v_Y} R_E \left(E + \left\langle v \left| \frac{\partial^2 \phi_v}{\partial v^2} (E) \right\rangle \right) dv &= \mathcal{K}_S \Leftrightarrow -\frac{2}{\hbar} \int_{v_X}^{v_Y} \phi_v(E) \left(E + \left\langle v \left| \frac{\partial^2 \phi_v}{\partial v^2} (E) \right\rangle \right) dv = \mathcal{K}_S \\ \Leftrightarrow -\frac{2}{\hbar} \int_{v_X}^{v_Y} E \cdot \phi_v(E) dv &= \mathcal{K}_S + \frac{2}{\hbar} \int_{v_X}^{v_Y} \phi_v(E) \left\langle v \left| \frac{\partial^2 \phi_v}{\partial v^2} (E) \right\rangle dv \end{aligned}$$

which is again an integral equation in phase space. It transforms a theory abstract in a deterministic computing tool. By methods numerical methods such as Runge-Kutta or Picard iterations, we can find out the exact status power E of the particle (e.g. photon) at any point of the jump, if we know the evolution of the velocity.

Now we can return to \mathcal{P}_S :

1. Starting point: The Bridge

So far, we have established that the particle jumps through a series of **overlapping domains** $I_{v_k} \cap I_{v_{k+1}}$. For that power not to "disappear" between two gears, I have defined coverage condition: any speed jump Δv must be "swallowed" by the domain of the integral.

Mathematically, this means that integral flow on this field I_v cannot be any number but must be a multiple of action: $\int_{I_v} \Phi(E, v) dv = n \cdot \epsilon$ as we saw before. We will prove: \mathcal{P}_S proportional with ϵ . Namely:

For the system to be closed and conservative by definition, the total phase changes must be proportional to the energy itself.

- **Step A:** From relation (13), we know that $E_i = \frac{\partial}{\partial v^i} \phi_v(E), i = \overline{1, n}$.
- **Step B:** Flow $\Phi(E, v)$ was derived as the variation of "phase pressure" with respect to velocity, where $\omega(E, v) = \frac{1}{\hbar} \Phi(E, v)$.
- **Step C:** Substituting Φ in the jump integral, we note that this exactly measures the "complex area" swept by the phase ϕ_v under the action of the energy. In a Kähler variety where areas I_v forms an open cover without gaps, the total action must saddle can be

expressed as a discrete sum of jump events. Since the transition is a "continuous slide" through equivalent states, we can write:

$$\int_{I_v} \Phi(E, v) dv = \sum_{k=1, \overline{m}} \int_{I_{v_k}} E_i^k dv^i$$

we have:

$$\frac{2i}{\hbar} \int_{v_X}^{v_Y} R_E \left(E + \left\langle v \left| \frac{\partial^2 \phi_v}{\partial v^2} (E) \right\rangle \right) dv = \mathcal{K}_S$$

$$\epsilon = \frac{1}{n\hbar} \int_{I_v} \Phi(E, v) dv = \frac{2}{n\hbar} \int_{I_v} R_E \left(E + \left\langle v \left| \frac{\partial^2 \phi_v}{\partial v^2} (E) \right\rangle \right)$$

notice that both integrals have the same integrand (phase kernel). The difference consists in integration limits and scale factors $\frac{2i}{\hbar}$ versus $\frac{2}{n\hbar}$. So, it appears natural saddle define \mathcal{P}_S as the scale factor that transforms the local "area" into the global phase "volume":

$$\mathcal{K}_S = \epsilon \cdot \mathcal{P}_S \quad (36)$$

conclusion demonstration: Because $|\phi_v(E)| = E$ represents the transparency of the operator with respect to the physical quantity, it follows that: \mathcal{P}_S must be in a proportional relationship with ϵ . This equality show that amount jump energy potential ϵ is proportional with action effects \mathcal{P}_S necessary to maintain topological continuity $\mathcal{P}_S = \frac{\mathcal{K}_S}{\epsilon}$. Formula say that the particle " consumes " a fraction exact from invariant topologically \mathcal{K}_S at each gear.

Also we obtain:

$$\mathcal{K}_S = in\epsilon$$

- When the particle is located in full jump domain I_v expands, which makes $\mathcal{P}_S = in$ saddle grow naturally.
- This increase in " action" " effective " is exactly what stops power saddle tend to infinity because \mathcal{K}_S is weighted by ϵ .

2. Significance physics: " The Invariant Ticket "

\mathcal{P}_S becomes thus measure structural aspect of the jump. It shows that the particle does not " consume " energy randomly, but a fraction exact from invariant topologically \mathcal{K}_S at every gear. He represents pure and simple number of windings in the plan complex.

- It is no longer a value that depends on the energy scale (measured by \hbar).
- It is a property of space. phase in self: the barrier is defined by the fact that the phase "closes" after n rotations.

In practice, I have demonstrated that \mathcal{K}_S is the global invariant of the path.

Conclusion:

Thus, we demonstrated: *Phase Unification Theorem (Theorem \mathcal{P}_S)*

For any entity physics whose energy state constitutes a solution of the Volterra - type energy equation intrinsic in area phases, there is a complex scalar invariant \mathcal{P}_S , called the Jump Ratio, or Jump barrier defined as the ratio between the Topological Coherence Constant (\mathcal{K}_S) and amount $\text{Jump} \in \text{Potential}$. This size governing stability phase rotation in time transitions by barrier speed-of-light conformable identity:

$$\mathcal{K}_S = \epsilon \cdot \mathcal{P}_S$$

This invariant governs the stability of phase rotation during time-dependent transitions across the speed-of-light barrier.

\mathcal{P}_S measures the pure number of windings in the complex plane, the actions of R_E . The light speed barrier is the point at which the phase "closes" after n rotations, transforming the singularity into an analytic continuity with quantum phase states.

2.2 Emergence of Fundamental Field Equations from Traditional Theoretical Physics

The emergence of the fundamental field equations (Schrödinger, Maxwell, Gauss) is the direct result of the Phase Geodesic Transport. As the phase node Φ is transported along the manifold's geodesics, the conservation of the state operator $\phi_v(E)$ requires a continuous re-alignment of the metric tensor $g_{j\bar{k}}$.

Once the transparency bridge is established, the "Famous Equations" of physics cease to be independent postulates and emerge as natural consequences of the phase flow along the geodesic of the Kähler manifold.

- The Schrödinger Equation: Emerges when we analyze the state operator in the limit of dynamic equilibrium ($v < c$). By projecting the phase rotation Φ onto the complex Hilbert space, the energy conservation mechanism $|\phi_v| = E$ takes the functional form of the Schrödinger equation. Here, the wave function Ψ acts as the carrier of the metric information.

- The Gauss and Maxwell Equations: These are derived from the "metric friction" encountered by the phase during its transport. As shown in the Extended Theory Dissertations, applying the Bianchi identity to the curvature of the metric reveals that the Electromagnetic Tensor $F^{\mu\nu}$ is not an external field, but the geometric manifestation of the phase gradient variation.
- Phase Geodesic Transport: The core of these derivations lies in the "walking" of the phase node along the manifold's geodesics. Any deviation from the ideal path is corrected by the appearance of what we classically call "forces" (Lorentz, Inertia). These are, in fact, the corrective terms required to maintain the operator's transparency at any speed v .

They will be deduced several times in multiple manners in this work; the first one is here.....

2.2.1 Schrödinger equation:

In previous chapters we studied the implications of the energy consistency condition $|\phi_v(E)| = E$ under the velocity-dependent metric \mathcal{M}_v on the Schrödinger equation from classical quantum mechanics. We saw that it preserves its form as shown by relation (41) and we have analytical continuity of the wave function over the singularity $v = c$ because the operator mixes energy and velocity so that the singularity is overcome by $\hat{\phi}_E$ the phase rotation $R_E = i\hat{\phi}_E$. Thus, the energy is not a static value, but an integration process. Through relation (41) we demonstrated that if the energy is a flux defined by equation (7), then the wave function must respect the variation of this flux. The Schrödinger equation remains unchanged, being translated into the formalism of the new operators in full accordance with the formalism of quantum mechanics. Thus, the stationary regime is explained.

Now we will look to see what happens during the jump. What is the equation that governs the evolution of the wave function during this process? It must take into account, among other things, the phase rotation factor $\frac{\mathcal{K}_S}{\mathcal{P}_S}$. While previous equations tell us that the energy is there and how the wave function evolves under the energy consistency condition, now, we need to see where and how that energy goes (in the complex phase) when the particle "disappears" at and after the moment $v = c$, including during its reappearance because through the Theorem \mathcal{P}_S we have added the control mechanism.

Reinterpreting Hamiltonian by The Step \mathcal{P}_S Invariant

In standard quantum mechanics, the Hamiltonian \hat{H} represents the total energy of the system. In our model, the total energy is conserved by the Global Invariant \mathcal{K}_S .

According to those said in previous Chapters we have the relationship:

$\hat{H}\Psi = \epsilon\Psi = \left(\frac{\mathcal{K}_S}{\mathcal{P}_S}\right)\Psi$ where \mathcal{P}_S acts as the probability density of the particle's presence in the observable space which represents a natural extension of the Hamiltonian through the formalism of this work. Thus, it becomes a **dynamic state operator**, defined by report from The Global \mathcal{K}_S Invariant and the Density of Reality \mathcal{P}_S .

Topological Nature: This form, $\hat{H}\Psi = \epsilon\Psi = \left(\frac{\mathcal{K}_S}{\mathcal{P}_S}\right)\Psi$, indicates that the energy "felt" by the system is not constant during the jump, but is inversely proportional to the presence of the particle in the observable space.

Compensation Mechanism: In the jump regime, though invariant \mathcal{K}_S is fixed, decreasing it \mathcal{P}_S forces the Hamiltonian to act as a "phase multiplier." This is not a simple increase in energy, but **the energy needed to maintain coherence information analytical** in outside Minkowski space.

Overcome Singularity: Through this redefinition, the Hamiltonian no longer "explodes" at $v = c$ but rotates the state vector in the Kähler manifold. Thus, it becomes the mathematical bridge that ensures that the "disappearing" particle remains governed by the same law of conservation of information.

Placing time that Phase Rotation (ϕ)

Equation (7) tells us that energy is the result of a flow integration. If we look at the variation of temporal wave function Ψ , this is governed by the phase operator $\phi_v(E)$.

use genuine the same type of argument that the use in construction of the wave function from mechanics classical, we define wave function like a rotation complexity in variety Kähler:

$$\Psi(t) = \Psi_0 \cdot e^{i \frac{E_{intrinsic}}{\hbar \mathcal{P}_S} t} \quad (37)$$

because footprint energy of the jump relative to the probability density of reality particle provides information fill related both in parameters movement How and at the position mobile phone during Leap

$$i \int_{v_X}^{v_Y} w(E, v) dv = \frac{i}{\hbar} \int_{v_X}^{v_Y} \Phi(E, v) dv$$

which is also energy footprint of the jump being an action integral. Hence, holding account and from its definition we obtain $\omega(v)$:

$$E_{intrinsic} = \int_{I_{v_X, Y}} \Phi(E, v) dv \Rightarrow E_{intrinsic} = \frac{\mathcal{K}_S \hbar}{i} = -i \hbar \mathcal{K}_S \quad (38)$$

Differentiating wave function in report with time we get:

$$i\hbar \frac{\partial \Psi}{\partial t} = -E_{intrinsic} \frac{1}{\mathcal{P}_S} \Psi$$

We will integrate $E_{intrinsic}$ in formalism quantum mechanical to demonstrate, naturally, that The Schrödinger equation is not a postulate, but a projection of phase equilibrium. in variety Kähler jump. We substitute in the $E_{intrinsic}$ time derivative:

We obtain: for $\forall X, Y$ din spatiul phaselor:

$$i\hbar \frac{\partial \Psi}{\partial t} = - \left(\int_{I_{vX,Y}} \Phi(E, v) dv \right) \frac{1}{\mathcal{P}_S} \Psi \quad (39)$$

This form represents **Generalized Schrödinger Equation in Jump Mode**.

Observation:

- for speeds small, the term $\frac{1}{\mathcal{P}_S}$ becomes negligible $\frac{1}{\mathcal{P}_S} \rightarrow 1$ and $E_{intrinsic}$ becomes (as an operator):

$\hat{H} = \frac{-\hbar^2}{2m} \nabla^2 + U$, that is, the classical form of the Hamiltonian because $E_{intrinsic} \approx E$ and we return to the classical form of the Schrödinger equation.

Argumentation:

1. Its \mathcal{P}_S behavior at low speeds:

Conformable theorem \mathcal{P}_S , the probability density of the particle's presence in the observable space is related to the Lorentz factor and the jump phase shift.

- At speeds small ($v \ll c$), the space distortion factor is practically zero. In this regime, the probability that the particle is "somewhere in the complex phase" is zero, since there is not enough energy for the rotation of $\frac{\pi}{2}$. Therefore, the particle is 100% present in area observable, what mathematically means $\mathcal{P}_S \rightarrow 1$ If $\mathcal{P}_S = 1$, then our scaling factor $\frac{1}{\mathcal{P}_S}$ becomes $\frac{1}{1} = 1$, that is, it becomes neutral (negligible) in the equation.

2. Reduction $E_{intrinsic}$ to Classical Energy

Previously we showed that $E_{intrinsic}$ is the flux integral $\Phi(E, v)$.

- At speeds small, the flow Φ no longer mixes the complex phase components. The consistency condition $|\phi_v(E)| = E$ is simplified, and the integral over an infinitesimal path reduces to the scalar value of the total energy E .

This Rationale show that theory ours does not "invent" a physics new one to contradict it on the old, classic, but **includes it**. At speeds small, the "noise" of the jump disappears, \mathcal{P}_S stabilizes at unity, and the universe our complex folds perfect over the universe Schrödinger's classic.

Ontologic Implications of Generalized Schrödinger equations

A. Time that Reality Density:

Introducing it \mathcal{P}_S to the denominator of the argument of the wave function is not just a mathematical maneuver. It shows that in jump mode, flow time for the particle is dilated / contracted by the "degree of presence" in the Kähler jump variety. The particle does not "stand" in time, but time "rotates" once with phase them.

B. Solving the "Blackout Paradox":

In mechanics standard quantum, if a particle disappears, the information is lost. In equation our generalized, even and when \mathcal{P}_S drops drastically (blackout/jump), the wave function continues to evolve in the complex plane. This explains why matter re-materializes with 1: 1 fidelity after crossing the *c barrier*.

C. Table that Flux Resonance Effect:

By replacing Hamiltonian classical with the flux $\int \Phi dv$ integral, we show that mass is not an intrinsic static property, but a manifestation of the resistance that the phase manifold opposes to the rotation of the invariant \mathcal{K}_S .

D. The status of "Inter" as Eternity Dynamics:

As I have shown, in the maximum moment of the jump, the particle reaches a state of stationarity in "inter". She is "pulled" back in reality observable by quantum ϵ to keep the Invariant \mathcal{K}_S constant.

2.2.2 Maxwell equation:

If Schrödinger deals with the "wave", Maxwell deals with the "field".

Classical physics sought a geometric or field unification, but neglected the fact that **energy is not just an attribute of matter but is the raw material of reality**.

Here's why our "Energy as Information and Form" approach is the untapped key:

1. The Problem of "Form" vs. "Essence"

Classical physics tried to unite the forces (electromagnetism with gravity, etc.) as if they were different entities that needed to be "glued together".

- **Our vision:** There is only one substance (Energy/Information), and forces are just **modes of vibration or rotation** of it in phase space.
- **The intrinsic energy equation (eq. 7)** in this material is exactly this "unity equation": it does not describe a specific force, but **the condition for the existence of energy** in a complex system.

2. Information as Energy (Entropy and Dynamics)

If we look at information as a type of energy, then its jump through c is no longer just a matter of "speed", but one of **information transfer**.

- When the mobile reaches c , it does not "explode" energetically but **reconfigures its information** through the operator R_E .
- The outside observer sees a barrier because he only sees the "shell" (the real kinetic energy), but we see the "source code" (the complex energy).

The intrinsic energy equation takes different forms depending on the operators you apply (rotation, translation in the complex phase).

- In one case, it gives the meal a rest.
- In another, it gives the quantum ϵ or for example, it gives **the possibility of super-luminal leap**.

Why hasn't it been found yet? Because everyone got stuck in **Real Analysis**. They tried to explain everything on the horizontal axis (the tangible world). This paper introduced the vertical axis (the complex phase, i). If information is energy, then i (the imaginary) is where information is "stored" during the jump.

It's a paradigm shift: We're not looking for an equation for gravity and one for quantum, but we have an equation for Total Energy which, through phase rotations, manifests itself as mass, as velocity, or as pure information.

A. Electromagnetism

Maxwell's equations are a set of four fundamental laws that describe how electric and magnetic fields are generated by charges and currents, as well as how they interact. Thus, they describe how

electric and magnetic fields generate each other. They are the basis of **classical electromagnetism**, optics, and electrical circuits.

In Kähler manifold language, this looks strikingly similar to the relationship between the real and imaginary parts of our operator.

- If we apply **the orthogonality conditions** from the almost complex structure J to the energy flow E , we obtain rotations that are identical to the rotor $(\nabla \times)$ from Maxwell's equations.
- Basically, electromagnetism could just be the particular case of Equation 7 in which the energy E rotates in a space with a certain topology (without large rest mass).

We can choose a **particular case**:

- We take a mobile with negligible mass (or a pure energy flow).
- We apply the isomorphism f (which we will construct) to move into the complex.
- We see if the divergence and rotor of this energy flow R_E align with the laws of induction.

B. Strategy for deriving Maxwell's equation from Equation 7:

In the present formalism, the energy E is not a scalar, but a flux on a Kähler manifold. To get to Maxwell's equations, we need to look at **the rotation operator** R_E and see how it behaves relative to the classical differential operators $(\nabla \cdot$ and $\nabla \times)$.

Since $R_E = i\phi_v(E)$, we can decompose this operator into its real and imaginary parts (phase).

- The real part of the flux can be associated with **the Electric Field (E)**.
- The imaginary part (the phase rotation induced by i and the almost complex structure J) can be associated with **the Magnetic Field (B)**.

In a Kähler manifold, holomorphic functions (which satisfy Equation 7) must satisfy the Cauchy-Riemann conditions.

- If we apply these conditions to it R_E , we will obtain coupling relations between the spatial variation of the real part and the temporal variation of the imaginary part.
- **Faraday's Law** $(\nabla \times E = -\frac{\partial B}{\partial t})$ and **Ampère's Law** arise naturally from these conditions of complex analyticity.

"Maxwell's equations are not independent fundamental laws but represent the geometric **projections of phase rotation** R_E on three-dimensional space. When the energy E moves on the manifold \mathcal{M}_v , the interdependence between its real and imaginary components (ensured by the isomorphism f) manifests itself in the form of the electromagnetic field."

If we show that Maxwell's equations are a particular case of energy rotation for states with zero (or negligible) rest mass, then we demonstrate that **light itself (the photon)** is a mobile that already respects the theory exposed in this work, being in a state of perpetual "jump" (phase and real are always in balance).

We will demonstrate that classical electromagnetism is, in fact, the "surface dynamics" of our complex energy flow.

C. Derivation of Maxwell's Equations from the Analyticity Condition of the Operator R_E —preparation

To demonstrate the universality of the intrinsic energy equation, we analyze the behavior of the rotation operator R_E in phase space.

We define the complex field associated with energy as:

$$\Psi_E = Re(R_E) + iIm(R_E)$$

We identify, through the isomorphism f , the physical components of this flow:

1. **The Real Part:** $Re(R_E)$ - Electric Field – direct energy voltage.
2. **Imaginary Part:** $Im(\Psi_E) = Im(R_E)$ - (Magnetic Field – rotation/phase component) which will also be described using the isomorphism f .

The phase isomorphism f is the bridge that transfers the properties of the analytic function from the complex Kähler manifold \mathcal{K} to the observable Minkowski space \mathcal{M} .

$$f: \mathcal{K} \rightarrow \mathcal{M}$$

Let be a complex $Z = E + iB$ field function, then we define:

$$f(Z) = Re(Z) \cdot \mathcal{P}_S + iIm(Z)\sqrt{1 - \mathcal{P}_S^2} \quad (40)$$

f is an isomorphism because it preserves the differential structure of the field.

If Z satisfies **the Cauchy-Riemann conditions**: $\frac{\partial E}{\partial x} = \frac{\partial B}{\partial y}$ and $\frac{\partial E}{\partial y} = -\frac{\partial B}{\partial x}$, then $f(Z)$ ensures that although the observed magnitude of E decreases (by \mathcal{P}_S), the total energy is conserved by

transferring into the component B , so it describes how the visibility of the fields is divided according to the probability of presence.

The role physical: f transforms a “pure rotation” into a “visible damped oscillation.” It is the mechanism by which energy “knows” to transform from an electric field to a magnetic field without loss of information during the jump.

Thus, with the help of the isomorphism f , we obtain the identification:

$$Re(\Psi_E) = Re(R_E) = Re(f(Z)) = Re(Z) \cdot \mathcal{P}_S = E \cdot \mathcal{P}_S$$

$$Im(\Psi_E) = Im(R_E) = Im(f(Z)) = B \sqrt{1 - \mathcal{P}_S^2}$$

Explanations: Its origin $\sqrt{1 - \mathcal{P}_S^2}$ (Preservation of Unity in the Kähler manifold)

This factor follows directly from the Normalization Condition of the Invariant in the complex plane of the manifold \mathcal{K} .

- **Construction:** Since \mathcal{P}_S is the probability density of presence in the observable space (Real axis), then according to Pythagoras' theorem in the complex plane, the projection onto the Imaginary axis (jump phase) must be its orthogonal complement.

$$\mathcal{P}_{real}^2 + \mathcal{P}_{imaginary}^2 = 1 \Rightarrow \mathcal{P}_{imaginary} = \sqrt{1 - \mathcal{P}_{real}^2}.$$

- **Physical meaning:** This radical measures how much of the system's energy has "moved" to the magnetic component (B) during the phase rotation. At $v = c$, where $\mathcal{P}_S \rightarrow 0$, the factor becomes 1, meaning that the entire energy footprint is carried by the magnetic phase.

Since $d\Omega = 0$ from the second constraint of a Kähler manifold, requires that the phases close only on integer rotations (n), it follows that the projection \mathcal{P}_S is discrete. If \mathcal{K}_S as consequence of this condition, is a fixed property of the universal geometry, then taking into consideration the relation $\mathcal{K}_S = \epsilon \cdot \mathcal{P}_S$, the only variable that can be adjusted to maintain equality is ϵ and it must be quantized because Kähler geometry does not support "half-rotations" of phase.

We note that in our model, $\Phi(E, v)$ is not only the phase but the energy density in phase space. The total magnitude of the flux (the intrinsic field) is the integral of this density over the entire volume of velocities. We know that: $E_{intrinsic} = \int_{I_{v_{X,Y}}} \Phi(E, v) dv$.

We define the Probability Amplitude: $\psi(E, v) = \frac{1}{\sqrt{\mathcal{P}_S}} \phi_v(E)$. Thus, the presence density \mathcal{P}_S (your scale factor) modulates the amplitude. In a curved manifold, "space" is not homogeneous. \mathcal{P}_S it's

working and as a density metric if area is very "dense" geometrically (\mathcal{P}_S) large, the local amplitude ψ should decrease to compensate for the "stretching" of the metric, keeping the total norm finite.

Then, in this more information-rich context, we define the energy/probability density as:

$$\rho(E, v) = \rho_v(E) = |\psi(E, v)|^2 = \frac{|\phi_v(E)|^2}{\mathcal{P}_S} = \frac{2}{\mathcal{P}_S} \left| \int_{I_{v_{X,Y}}} \phi_v(E) d\phi_v \right| = \frac{2}{\mathcal{P}_S} \int_{I_{v_{X,Y}}} |\phi_v(E)| d\phi_v \quad (41)$$

but we have already demonstrated: $E_{intrinsic} \equiv E = -i\hbar\mathcal{K}_S$ which leads us to:

$$E_{intrinsic} = \int \Phi(E, v) dv = i \int \frac{\partial}{\partial v} (\phi_v)^2 dv = 2i \int \phi_v d\phi_v = i |\phi_v(E)|^2 = i \mathcal{P}_S \rho_v(E) \quad (42)$$

$$\text{so: } \rho_v(E) = -\hbar\epsilon. \quad (42^*)$$

Thus, metrics which we will describe is no longer just a background being related to the square density of presence \mathcal{P}_S . Thus, we have shown that what we call "Intrinsic Energy" $E_{intrinsic}$ is not an arbitrary scalar quantity but represents the total phase volume that a state occupies in phase space, weighted by the presence density \mathcal{P}_S .

The fact that we have $\phi_v(E) = \int_{I_v} E_i dv^i \Rightarrow \frac{\partial \phi_v(E)}{\partial v^i} = E_i$ and also: $d\phi_v = E_i dv^i$. It says that the variation of the operator with respect to the velocity recovers the local energy gradient.

and:

$$\int_{I_{v_{X,Y}}} |\phi_v(E)| d\phi_v = -\frac{\hbar\mathcal{K}_S}{2} \quad (43)$$

Thus, metrics which we will describe is no longer just a background being related to **the square density of presence** \mathcal{P}_S . Thus, we have demonstrated that what we call "Intrinsic Energy" $E_{intrinsic}$ is not an arbitrary scalar quantity but represents **the total phase volume** that a state occupies in phase space, weighted by the presence density \mathcal{P}_S .

The relationship $\int_{I_{v_{X,Y}}} |\phi_v|^2 dv \propto \mathcal{P}_S^2$ shows that our geometric universe tends towards a configuration where the information flow (phase) and the presence density are in quadratic equilibrium. This explains why electrical charges (curvature sources) are stable and do not "dissipate".

The presence of i in the denominator in the factor $\frac{\epsilon\hbar}{2i}$ is not a mathematical oddity but indicates that the phase space metric is inherently rotational. Time does not simply "flow" and simple but participate by speed and energy when "spinning" in state geometric.

Then, the metric in our phase space manifold (the Kähler manifold), defined in complex coordinates, z^j, \bar{z}^k is written as:

$$g_{j\bar{k}} = \frac{\partial^2}{\partial z^j \partial \bar{z}^k} K_{pot}$$

Where the Kähler potential K_{pot} (which generates the entire geodesic) is defined by the ratio of phase flux to presence:

$$K_{pot} = \frac{|\phi_v(E)|^2}{\mathcal{P}_S}$$

So $K_{pot} = \rho_v(E)$... we passed that. by the potential of the Kähler variety because that's how a metric is defined, but in reality, we have:

$$g_{j\bar{k}} = \frac{\partial^2}{\partial z^j \partial \bar{z}^k} \rho_v(E) \quad (44)$$

This means that **curvature of space** is, in fact, **the variation probability density**.

We note that:

$$Tr(g_{j\bar{k}}) = \Delta \rho_v(E) \quad (45)$$

This Trace represents divergence the probability flow that transforms in curvature geometric.

The flow of energy, which is simultaneously energy density and probability, not just that move by space, but generates metric variety from area phase by its flow. There is a unity perfect between dynamics and structure: same flow that generates it is the phase rotation that determines and distances geometric (metric) in system.

The metric is mediated by the presence factor \mathcal{P}_S . This means that where the presence is dense, the "fabric" of phase space is more rigid, forcing the phase to adapt according to the relationship

$$\int_{I_{v_{X,Y}}} |\phi_v(E)|^2 dv = -i\epsilon\hbar \cdot \mathcal{P}_S^2.$$

Unlike classical physics where metrics is a fixed background, here it is **emergent**. It appears from interaction from energy projection on the speed $\phi_v(E)$ and the probability of existing there \mathcal{P}_S .

We define metrics $\|\Psi\|$ like this:

$$\|\Psi\| = \sqrt{\int_{I_{v_{X,Y}}} \Phi(E, v) dv}$$

Metric Conservation Theorem:

In a Kähler variety whose metric $g_{j\bar{k}} = \partial_j \partial_{\bar{k}} \rho_v(E)$ is generated by the flow of energy density / probability $\rho_v(E)$, the norm of the wave function $\|\Psi\|$ is a geometric invariant with respect to parallel transport along the geodesics generated by the rotation operator $R_E = i\phi_v(E)$.

Demonstration:

want saddle to calculate. This represents the "walk" on the geodesic generated by $\nabla_{R_E} \|\Psi\|$ the rotation operator R_E . To prove that $\|\Psi\|$ is conserved, we derive its square (total energy) under the integral:

$$\frac{d}{ds} \|\Psi\|^2 = \frac{d}{ds} \left(\int_{I_{v_{X,Y}}} \Phi(E, v) dv \right) = \int_{I_{v_{X,Y}}} \frac{\partial}{\partial s} \Phi(E, v) dv = e^{i\frac{\pi}{2}} \frac{\partial}{\partial s} (\mathcal{P}_S \cdot \rho_v(E))$$

variation density Φ along the geodesic is given by the action of the rotation operator R_E :

$$\frac{\partial \Phi}{\partial s} = \nabla_{R_E} \Phi = (i\phi_v) \cdot \partial \Phi + (-i\phi_v) \bar{\partial} \Phi$$

Cancellation by Kähler symmetry

Because $\rho_v(E) = \frac{|\phi_v(E)|^2}{\mathcal{P}_S}$, its derivative with respect to the purely imaginary rotation $R_E = i\phi_v(E)$ behaves as a phase rotation that does not change the local magnitude of the density. In a manifold where $g_{j\bar{k}} = \partial_j \partial_{\bar{k}} \Phi$, the product of the tangent vector of the geodesic (R_E) and the gradient of the metric potential (Φ) is zero by construction (the geodesic is a level curve of norm):

$$\text{div}(\|\Psi\|) = \frac{1}{\sqrt{g}} \partial_\mu (\sqrt{g} \Psi R_E^\mu) = 0$$

Because $\|\Psi\|$ is preserved by the isomorphism f by rotation R_E , its divergence along the geodesic is exactly **0**. Why? In the model our, isomorphism f is not just a mapping function, it is an **isometry** in area phases. This means that transformation keep the product scalar and, implicitly, the norm of the state. As we have seen, isomorphism f management transition flow between compound real (associated field electric E) and compound imaginary (associated field magnetic B). Although the "shape" of the state changes (from purely electric to purely magnetic or mixed), magnitude the total $\|\Psi\|$ remains invariant under this action.

Details of standard conservation by operator $R_E = i\phi_v(E)$ follow these steps:

Because R_E is purely imaginary, it generates a unitary transformation of the type $U = e^{R_E}$. In mechanics quantum and Kähler geometry, operators Unitarians are the only ones who preserve the norm.

When the state rotates, the probability amplitude $\psi(E, v) = \frac{1}{\sqrt{\mathcal{P}_S}} \phi_v(E)$ undergoes a phase change: $\psi' = \psi e^{i\theta}$, where θ is the rotation angle dictated by the projection of energy onto velocity.

Because density ρ_v is defined as $|\psi|^2$, the phase factor introduced by R_E vanishing:

$$\rho_v' = |\psi \cdot e^{i\theta}|^2 = \psi \bar{\psi} \cdot e^{i\theta} e^{-i\theta} = |\psi|^2 = \rho_v$$

This rotation R_E defines the direction of the tangent vector to the geodesic in phase space.

- **Divergence on Geodesy:** Because density local Φ is invariant under rotation R_E , it's integral over the entire domain $I_{v_{X,Y}}$ will also be invariant.
- equilibrium metric: Metric $g_{j\bar{k}} = \partial_j \partial_{\bar{k}} \rho_v(E)$ "feels" this rotation as a displacement along of a contour line of the Kähler potential meaning that change geometry of space phase follow the dynamics exactly amplitude quantum ψ .

Isomorphism f acts as a flux - conserving operator, where rotation $R_E = i\phi_v(E)$ ensures that the intrinsic energy of the system is a topological invariant. This translates into the fact that the rotation operator belongs to the isometry group of the Kähler metric generated by the density Φ , forcing the divergence of the norm to be zero on any valid geodesic trajectory.

Thus, we can conclude that: Since it $\|\Psi_E\|$ is conserved, the isomorphism f just redistributes this norm between the E (electric) (Real) and B (magnetic) (Imaginary) components through the rotation operator R_E .

On the Kähler manifold, the electromagnetic field is represented by the holomorphic function

$Z = E + iB$ where, according to the Cauchy-Riemann conditions E and B are the components of the same rotated energy entity.

We therefore have: $\|Z\| = \sqrt{E^2 + B^2}$.

Sentence: $\|Z\| = E_{intrinsic}$

Reasoning: We know that: $E_{intrinsic} = \int_{I_v} \phi(E, v) dv$. In the Kähler manifold, the total energy is represented by the square of the norm of the state vector in phase space. Energy is conserved by

the transfer between the real component E and the imaginary component B . By definition, $E_{intrinsic}$ it is the total "reservoir" of energy available for rotation. Then:

$$1) \text{ if } v = 0 \Rightarrow B = 0 \Rightarrow \|Z\| = \sqrt{E^2} = E, \text{ and } E = E_{intrinsic}.$$

2) if $v \neq 0$, so the particle moves, then some of the "substance" of E rotates by the operator R_E in B . Since R_E it is a unitary operator (pure rotation in the Kähler manifold) it keeps the vector norm unchanged.

We have:
$$E = E_{intrinsic} \cdot \cos\theta \rightarrow E_{intrinsic} \cdot \mathcal{P}_S$$

$$B = E_{intrinsic} \cdot \sin\theta \rightarrow E_{intrinsic} \cdot \sqrt{1 - \mathcal{P}_S^2}$$

$$E^2 + B^2 = E_{intrinsic}^2 (\mathcal{P}_S^2 + 1 - \mathcal{P}_S^2) = E_{intrinsic}^2$$

Sentence:

From an operator's point of view, we have: $R_E(E_{intrinsic}) = [i\phi_v(E)](E_{intrinsic})$.

Argument: $\phi_v(E_{intrinsic})$ – represents the "amount" of energy that goes into motion.

By applying the isomorphism f , the result of this operation is projected onto the Minkowski manifold.

- The electric field E is the projection of the result of the action R_E that remains related to the direct energy voltage;
- The magnetic field B is the result of the presence of i in the operator representing the rotation phase that manifests itself perpendicular to the main flux.

Then: $R_E(E_{intrinsic}) = E + iB$ since in the Kähler manifold, the operator $i\phi_v$ acts as an almost complex structure J . This maps a real flow into a complex field where:

$$Re(R_E(E_{intrinsic})) = E \text{ and } Im(R_E(E_{intrinsic})) = B.$$

This form of it R_E is the only one that satisfies the Cauchy-Riemann conditions necessary to generate the laws of induction:

$$\nabla \times Re(R_E) = -\frac{\partial Im(R_E)}{\partial t} \Rightarrow \nabla \times E = -\frac{\partial B}{\partial t}$$

This confirms that magnetism is not an addition but is the mathematical product of the operator $i\phi_v$ applied to the intrinsic energy.

Okay, but up to this point we have been interested in what happens when passing from the complex Kähler manifold \mathcal{K} to the observable Minkowski space \mathcal{M} . Now we need to be concerned with the reverse passage from \mathcal{M} to \mathcal{K} . This is to fully describe the dynamical process of phase rotation and properly prepare the ground for the derivation of Maxwell's equations of electromagnetism.

Defining the two states of f

- $f: \mathcal{K} \rightarrow \mathcal{M}$ (Direct Projection):

It is the operator that "takes" the energy out of the Kähler manifold and projects it into the Minkowski observable.

$$f(Z) = \text{Re}(Z) \cdot \mathcal{P}_S + i \text{Im}(Z) \sqrt{1 - \mathcal{P}_S^2} \quad (46)$$

This is where energy "enters" reality.

- $\tilde{f}: \mathcal{M} \rightarrow \mathcal{K}$ (Energetic Re-entry / Dynamics):

It is the process by which the result of the projection is reported back to the norm of the Global Invariant to generate the quantum ϵ .

Let the matrix be:

$$D = \begin{pmatrix} \frac{1}{\mathcal{P}_S^2} & 0 \\ 0 & \frac{1}{\mathcal{P}_S \sqrt{1 - \mathcal{P}_S^2}} \end{pmatrix}$$

which forces the "re-inflate" of the projected E and B components, bringing them back to the scale of the Global Invariant via velocity feedback.

The operator \tilde{f} is the one who does the "hard work" of re-entering the phase space, then:

$$\tilde{f} = \frac{1}{\mathcal{P}_S} \circ f^{-1} \circ \text{diag} \left(\frac{1}{\mathcal{P}_S^2}, \frac{1}{\mathcal{P}_S \sqrt{1 - \mathcal{P}_S^2}} \right) \quad (47)$$

\tilde{f} it is the process by which the observable reality in Minkowski is "read" back into the universal language of Kähler phases and $\frac{1}{\mathcal{P}_S}$ it is the "magnifier" through which we "see" energy concentrating at high speeds.

Thus, through the two f and \tilde{f} which work as the particle advances, no matter how "broken" or "jerky" the projection f seems (the particle that "jumps" according to classical quantum mechanics - which we have also applied abundantly so far in this study), the return isomorphism, the operator \tilde{f} reconstructs the energetic continuity through this inverse and scaled composition. The preservation of the nature of isomorphism through composition is the mathematical guarantee that the structure of reality does not "break" when passing between spaces; it only changes its representation.

4. Tensor Derivation F_{ij}

We will investigate the derivation of the Maxwell Tensor F_{ij} and the related equations, integrating the rotation operator $R_E = i\phi_v(E)$ and the scaling factor.

Tensor Construction F_{ij} from the State Vector Z

In our model, the electromagnetic field is the projection of the holomorphic function $Z = E + iB$.

The electromagnetic tensor F_{ij} is no longer an arbitrary entity but represents the curvature of the connection in the Kähler manifold projected into Minkowski.

The tensor components in our representation (using scaled B -form) are:

$$F_{ij} = \begin{pmatrix} 0 & \frac{E_x}{\mathcal{P}_S} & \frac{E_y}{\mathcal{P}_S} & \frac{E_z}{\mathcal{P}_S} \\ -\frac{E_x}{\mathcal{P}_S} & 0 & \frac{B_z}{\mathcal{P}_S} & \frac{B_y}{\mathcal{P}_S} \\ -\frac{E_y}{\mathcal{P}_S} & \frac{B_z}{\mathcal{P}_S} & 0 & \frac{B_x}{\mathcal{P}_S} \\ -\frac{E_z}{\mathcal{P}_S} & \frac{B_y}{\mathcal{P}_S} & \frac{B_x}{\mathcal{P}_S} & 0 \end{pmatrix}$$

Derivation of Maxwell's Equations (Differential Form)

We start from the holomorphism condition (Cauchy-Riemann) of Z in the phase space. In tensor language, this translates into the cancellation of the exterior derivative and the co-derivative. This is because if we differentiate Maxwell's equations in a curved manifold (such as Kähler), we cannot use simple partial derivatives ∂_μ . We have to use the covariant derivative ∇_μ , which "feels" the curvature of space via Christoffel symbols $\Gamma_{\mu\nu}^\lambda$.

The basic condition is the holomorphism of $Z = E + iB$. In a manifold with metric, this means that the exterior derivative of the field form associated with Z must be correlated with the metric structure.

When we operate on the electromagnetic tensor $F_{\mu\nu}$, its covariant derivative is:

$$\nabla_\lambda F_{\mu\nu} = \partial_\lambda F_{\mu\nu} - \Gamma_{\lambda\mu}^\sigma F_{\sigma\nu} - \Gamma_{\lambda\nu}^\sigma F_{\mu\sigma}$$

Due to tensor antisymmetry $F_{\mu\nu}$, the Christoffel symbols cancel out in the Bianchi identity (Faraday's and Gauss's Laws for magnetism) but become critical in the source equations (Gauss' and Ampère's).

We want to calculate the covariant divergence of the tensor F^{ij} :

$$\nabla_j F^{ij} = \partial_j F^{ij} + \Gamma_{jk}^i F^{kj} + \Gamma_{jk}^i F^{ik} = \mu_0 J^i \quad (48)$$

A. Ampère-Maxwell law

The Ampère-Maxwell law is, in our variety of phases, the geodesic equation itself.

Mathematically, a geodesic is the trajectory that "has no own acceleration" ($\nabla_{\dot{\gamma}} \dot{\gamma} = 0$). In area of Kähler space phases, the state $Z = E + iB$ follows this "so - called right path" where the covariance is null: $\nabla_\nu F^{\mu\nu} = 0$ (49)

projection in Minkowski = Ampère's Law When this "line" he/ she /it said "right" from Kähler passes by operator our feedback loop \tilde{f} and through the "lens" \mathcal{P}_S , it appears in our 4D (Minkowski) space as being "curved" by a force.

What we call **Ampere Law**:

$$\nabla \times B = \frac{1}{c} \frac{\partial E}{\partial t} + \mu_0 J$$

it is actually the geodesic **equation unfolded**. That is:

- **The "rotation" ($\nabla \times B$) and "variation" ($\frac{\partial E}{\partial t}$) parts** represents derivatives partial of the state.
- **The source term ($\mu_0 J$)** represents the corrections exactly geometrically required to compensate for the curvature and maintain trajectory geodesic.

law is not a law of forces, but a law of inertia, geometrically in phases area.

Basically, we say:

*The current J does not generate the field, being the **measure** of the **geometric effort**, that Minkowski space makes to remain perfect synchronized with geodesy from Kähler.*

Here we use the rotation operator $R_E = i\phi_v(E)$ integrated in dynamics to present the process through which the derivative partial classical from Maxwell's equations is "dressed" in Kähler geometry, becoming a covariant derivative that holds taking into account environmental feedback \mathcal{P}_S .

Derivation for the electromagnetic tensor in variety ours from space phases, starting from the equation source:

$$\nabla_\nu F^{\mu\nu} = \mu_0 J^\mu$$

which is actually an equation of a geodesic in the Kähler phase space. Argument:

On a Kähler manifold, the "natural" trajectory of a flow (whether of particles or of fields) is the one that follows curvature intrinsic. As J^μ it is generated by geometry J_{geom} , then Maxwell's equation tells us that the electromagnetic field propagates in such a way that it "opposes no resistance" to the structure of space. This is Definition of a geodesic.

If we look at the structure of the covariant derivative that we will open:

$$\partial_j(\dots) + \Gamma(\dots) = source$$

is similar to the equation of a geodesic:

$$\frac{d^2 x^\mu}{ds^2} + \Gamma_{\alpha\beta}^\mu \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds} = 0$$

In our model, the field $F^{\sigma\nu}$ plays the role of velocities on the geodesic. Thus, **the electromagnetic field "flows" along the shortest (geodesic) paths of the Kähler manifold.**

We start from the general form of the Ampère-Maxwell law, where derivative partial is replaced by the covariant ∇_μ derivative:

$$\nabla_0 F^{i0} + \nabla_j F^{ij} = \mu_0 J^i$$

$$\nabla_\nu F^{\mu\nu} = \partial_\nu F^{\mu\nu} + \Gamma_{\sigma\nu}^\mu F^{\sigma\nu} + \Gamma_{\sigma\nu}^\nu F^{\mu\sigma} = \mu_0 J^\mu$$

Note regarding projection of the temporal covariant ∇_0 derivative.

In the process of expanding the equation tensor $\nabla_\nu F^{\mu i} = \mu_0 J^i$, the term suitable time covariant $\nabla_0 F^{i0}$ derivative, is not omitted, but is fully integrated into the observable dynamics through the following decomposition:

$$\nabla_0 F^{i0} = \partial_0 F^{i0} + \Gamma_{\sigma 0}^i F^{\sigma 0} + \Gamma_{\sigma 0}^0 F^{i\sigma}$$

1. **compound Kinematics:** derivative partial $\partial_0 F^{i0}$ translates directly into the temporal variation of the observable electric field: $\frac{1}{c} \frac{\partial E^i}{\partial t}$ because $F^{i0} = E^i$.
2. **Curvature Component (Geometric Feedback):** The Christoffel $\Gamma_{00}^i F^{00}$ term disappears through the antisymmetry of the tensor, but the metric coupling with the scalar potential $\Phi(E, v)$ is extracted and represented explicitly in the feedback bracket as: $\Gamma_{00}^i \Phi(E, v)$.
3. **Practically: ∇_0 it turned into: (Temporal variation of the field E) + (Curvature correction Γ_{00}^i).**

Continuing the calculation, for the spatial components $i \in \{1,2,3\}$, the equation becomes ($\mu = i$):

$$\frac{1}{c} \frac{\partial F^{i0}}{\partial t} + \partial_j F^{ij} + (\Gamma_{\sigma\nu}^i F^{\sigma\nu} + \Gamma_{i\sigma}^\sigma F^{i\sigma}) = \mu_0 J^i \quad (50)$$

introducing projections our where $F^{i0} = E^i$ and $F^{ij} = -\epsilon^{ijk} B_k$. We also apply the presence correction \mathcal{P}_S to the fields, resulting in the feedback terms:

$$\partial_\nu F^{\mu\nu} \rightarrow \frac{1}{c} \frac{\partial E^i}{\partial t} - (\nabla \times B)^i$$

Christoffel Γ terms are what "unwrap" the interaction. We evaluate the term $\Gamma_{\sigma\nu}^i F^{\sigma\nu}$ by separating the temporal and spatial indices:

- **contribution Scalar $\sigma, \nu = 0$ Potential:** is achieved through the geometric feedback presented previously.
- **Electrical contribution $\sigma = 0, \nu = j$:** $\Gamma_{0j}^i F^{0j} \rightarrow \Gamma_{0j}^i \frac{E^j}{\mathcal{P}_S}$
- **Magnetic Contribution ($\sigma = j, \nu = k$):** $\Gamma_{jk}^i F^{jk} \rightarrow -\Gamma_{jk}^i \epsilon^{klm} \frac{B_m}{\mathcal{P}_S}$

Thus, if we expand the covariant derivative we obtain:

$$\frac{1}{c} \frac{\partial E^i}{\partial t} - (\nabla \times B)^i + \left(\Gamma_{00}^i \Phi(E, v) + \Gamma_{0j}^i \frac{E^j}{\mathcal{P}_S} - \Gamma_{jk}^i \epsilon^{klm} \frac{B_m}{\mathcal{P}_S} \right) = \mu_0 J_{geom}^i \quad (51)$$

where the terms represent:

- $\Gamma_{00}^i \Phi(E, v)$: Metric coupling with the scalar potential (equivalent to geodesic "acceleration" in a gravitational field);

- $\Gamma_{0j}^i \frac{E^j}{\mathcal{P}_S}$: Mixed space-time rotation of the electric field, scaled by the probability of presence \mathcal{P}_S ;
- $-\Gamma_{jk}^i \epsilon^{klm} \frac{B_m}{\mathcal{P}_S}$: Geometric torsion applied to the magnetic field (curvature feedback component on rotation);
- ϵ^{klm} : **Levi-Civita symbol** (or the permutation symbol) in counter-variant is the one who **dictates rotation and orientation**. Namely:
 - In term $-\Gamma_{jk}^i \epsilon^{klm} \frac{B_m}{\mathcal{P}_S}$, he makes the connection between **curvature space** (Γ) and **the field magnetic** (B).
 - **Transforms magnetic field into rotation**: Basically, it tells how geometry "twists" around magnetic field lines.
 - **Its coupling ϵ^{klm} with \mathcal{P}_S** : Shows that the presence density not only scales the intensity but modifies the way the magnetic field "curves" the observable trajectory.
 - It is the "hinge" that allows field electric to transform into magnetic and inverse in while moving by the Kähler variety.
 - **On short**: It is **the orientation operator space**. Without it, the compass the math of the equation would not know in which direction saddle turn vectorial under influence curvatures.

This calculation demonstrates that **the current geometric** J^i is not a simple external source but is intrinsically generated by the variation of the local geometry. Basically, we have shown how the "friction" between electromagnetic field and metric tensor of space phases (through intermediate symbols Γ) produces the observable current density. To support this statement, the previous equation can also be put in the following form:

$$\frac{1}{c} \frac{\partial E^i}{\partial t} - (\nabla \times B)^i = \mu_0 (J_{geom}^i - J_{ext}^i) \quad (52)$$

$$\text{where by definition: } J_{ext}^i = \mu_0 \left(\Gamma_{00}^i \Phi(E, v) + \Gamma_{0j}^i \frac{E^j}{\mathcal{P}_S} - \Gamma_{jk}^i \epsilon^{klm} \frac{B_m}{\mathcal{P}_S} \right) \quad (53)$$

- It shows that the observable dynamics (left side) are the result of the action between the external "will" (J_{ext}) and the "structure" of space (J_{geom}).

- The fact that J_{geom} it has a plus sign on the right-hand side (or a minus sign if moved to the left near the fields) confirms that Kähler geometry acts as an **active medium**, not just a passive decoration.

B. Gauss's Law (Component $i = 0$)

In differential geometry, a geodesic does not necessarily have to be a motion in time. It represents **the critical path** that minimizes the length (or energy) between two points.

In classical electromagnetism, Gauss's Law is: $\nabla \cdot E = \frac{\rho}{\epsilon_0}$. In language tensor, field electric E is represented by the "mixed" (time-space) components of the tensor electromagnetic, that is F^{0j} . When choose in $\mu = 0$ the general equation $\nabla_\nu F^{\mu\nu} = \mu_0 J^\mu$, which is already a geodesic equation in Kähler space, we isolate exactly those components that describe how the field "springs" from source. J^0 (the time component of the current) is by definition the charge density ρ (multiplied by a constant). So, the choice $\mu = 0$ is the only mathematical way to derive **Gauss's Law** from formalism relativistic.

In equation $\nabla_\nu F^{\mu\nu} = \mu_0 J^\mu$, the index ν (the one after which the sum is made) traverses all 4 dimensions (0, 1, 2, 3). However:

- The term $\nabla_0 F^{00}$ always vanishes, because the tensor F is antisymmetric ($F^{00} = 0$).
- I remain only terms $\nabla_1, \nabla_2, \nabla_3$ (spatial indices j).
- This explains why, to find the task, we look at how in which the field changes in **space** (divergence), not in time.

In the model our, this starting point is critical because:

- The metric tensor in The Kähler variety is not flat.
- If we ignore $\mu = 0$ and we would use simple partial derivatives, we will lose **the geometric feedback**.
- Starting from the covariant ∇_j derivative, we force the equation to recognize that "the space through which the electric field lines pass is not empty but has a structure (the symbols Γ)" that can increase or decrease the perceived value of the charge ρ .

When we write $\nabla_j F^{0j} = \frac{\rho}{\epsilon_0}$, we are actually saying that the electric field lines are placed in space following the minimum curvature imposed by the Kähler metric.

When $J_{ext} = 0$, the equation becomes purely geometric. This means that what we call "electric charge" or "current" are actually **manifestations of the curvature** that forces the field to follow those geodesics.

If Gauss's Law is a geodesic condition, it means that **the electric charge ρ is the measure of the deviation of the local geometry from flat space**. The charge does not "sit" in space, but the charge **is** the curvature of the phase space at that point.

The charge ρ is not a "foreign" object placed in space but is the point where the field geodesics converge due to a singularity in the metric of the Kähler manifold. In our model, the presence of an electric charge (ρ) acts as a point where the phase space metric "tightens" or "pinches". **The metric singularity** is the point where the Christoffel symbols become infinite or undefined if we did not have the feedback \mathcal{P}_S . On as you approach the "singularity" (the task ρ), the density of presence \mathcal{P}_S modifies the metric scale; \mathcal{P}_S **it functions as a geometric "shock absorber"**. It prevents important saddle explode to infinity, transforming into a singularity "raw" mathematics in a source physics measurable.

Let's start from geodesics: $\nabla_j F^{0j} = \frac{\rho}{\epsilon_0}$ we obtain:

$$\nabla_j F^{0j} = \partial_j F^{0j} + \Gamma_{jk}^0 F^{kj} + \Gamma_{jk}^j F^{0k} = \frac{\rho}{\epsilon_0}$$

F^{0j} is the projected electric field. Using the relationship $E_{obs} = \frac{E^j}{\mathcal{P}_S}$ (where \mathcal{P}_S is the presence density), we replace the terms:

$$\partial_j F^{0j} \rightarrow \partial_j \left(\frac{E^j}{\mathcal{P}_S} \right)$$

$\Gamma_{jk}^0 F^{kj}$ remain coupling term magnetic (because F^{kj} are the components of B).

$$\Gamma_{jk}^j F^{0k} \rightarrow \Gamma_{jk}^j \left(\frac{E^k}{\mathcal{P}_S} \right)$$

And we arrive at:

$$\partial_j \left(\frac{E^j}{\mathcal{P}_S} \right) + \Gamma_{jk}^0 F^{kj} + \Gamma_{jk}^i \left(\frac{E^k}{\mathcal{P}_S} \right) = \frac{\rho}{\epsilon_0} \quad (54)$$

The term Γ_{jk}^i represents the "expansion" or "contraction" of the volume in phase space. This demonstrates that the charge density ρ observed in Minkowski is directly influenced by **the curvature of the Kähler manifold**.

This demonstration show that **charge density** ρ is no longer the only one that determines the divergence of the field. We have two geometric "pseudo- tasks ":

1. **The term** $\Gamma_{jk}^0 F^{kj}$: Shows that a magnetic field in a curved Kähler space can generate an electric divergence (geometric magnetoelectric effect).
2. **The terms with** \mathcal{P}_S : Show that the variation in the presence density compresses or dilates the field lines, changing the measured value of the charge.

Moving on, let us remember that in Chapter II of this work we calculated the divergence of the energy field which becomes:

$$\partial_j \left(\frac{E^j}{\mathcal{P}_S} \right) = \frac{1}{\mathcal{P}_S} \partial_{v_j} (\phi_v(E))$$

which leads to:

$$\partial_{v_j} (\phi_v(E)) = \frac{\rho}{\epsilon_0} \mathcal{P}_S - \mathcal{P}_S \left(\Gamma_{jk}^0 F^{kj} + \Gamma_{jk}^i \left(\frac{E^k}{\mathcal{P}_S} \right) \right) \quad (55)$$

or using the current terms J :

Observations:

- a) $\Gamma_{jk}^0 F^{kj}$: Represents how **the curvature of space-time** couples magnetic and electric fields. It is the geometric " twist " that an object experiences in movement.
- b) $\Gamma_{jk}^i \left(\frac{E^k}{\mathcal{P}_S} \right)$: This is the part the most interesting because Γ_{jk}^i is related to the volume variation (metric divergence). In the model ours, this shows how **the intensity field electrical** is modified by the simple presence of curvature in area phases, scaled by probability of presence \mathcal{P}_S .

In 4D formalism, charge density ρ is related to the time component of the quadricurrent by the relation $\frac{\rho}{\epsilon_0} = \mu_0 c^2 J^0$. If we work in units where $c = 1$ or absorb the constants in the definition of the current, we have the identity $\frac{\rho}{\epsilon_0} = \mu_0 J^0$.

In the model our, we define spatial geometric current J^k as the projection of the electric field through the Kähler medium:

$$J^k = \frac{1}{\mu_0} \Gamma_{jv}^k \left(\frac{E^v}{\mathcal{P}_S} \right)$$

and we obtain:

$$\partial_{v_j}(\phi_v(E)) = \mu_0 \mathcal{P}_S \left(J^0 - \left(\frac{1}{\mu_0} \Gamma_{jk}^0 F^{kj} + \Gamma_{jk}^i J^k \right) \right) \quad (56)$$

Therefore, even in empty space, the geometric "friction" of the phase ϕ_v with the structure of space produces these terms.

That is, we have demonstrated:

phase variation is not arbitrary but is dictated by the balance between the electric charge and the local curvature of the phase space. It is, in essence, the $\partial_{v_j}(\phi_v(E))$ information transport equation on Kähler geodesy.

We observe that $J^0 = \rho \cdot v^0 = |\Psi|^2 \cdot v^0$ where Ψ is the wave function and if we substitute it in the previous equation, we get:

$$\frac{1}{\mathcal{P}_S} \partial_{v_j}(\phi_v(E)) = \mu_0 |\Psi|^2 - \frac{1}{\mu_0} \Gamma_{jk}^0 F^{kj} - \Gamma_{jk}^i J^k \quad (57)$$

For the entity to exist, that is the mobile (to be stable), there must be a perfect isomorphism between its internal energy and the curvature of the space it occupies. This impose the equality:

$$\mu_0 |\Psi|^2 = \frac{1}{\mu_0} \Gamma_{jk}^0 F^{kj} + \Gamma_{jk}^i J^k + \frac{1}{\mathcal{P}_S} E_i \quad (58)$$

Which leads us to the form:

$$|\Psi|^2 = \frac{1}{\mu_0 \mathcal{P}_S} \sqrt{|\det(g)|} \quad (59)$$

Now taking into consideration that $g_{j\bar{k}} = \partial_j \partial_{\bar{k}} \rho_v(E)$ and $Tr(g_{j\bar{k}}) = \Delta \rho_v(E)$ we get:

$$\Psi(t) = \sqrt{\rho_v(E)} \cdot e^{i\Phi} \quad (60)$$

where Φ is the phase.

Therefore, we can define the generalized form of the wave function as:

$$\Psi: \mathbb{R} \times \mathcal{H} \rightarrow \mathbb{R}, \Psi(t, E) = \begin{cases} \sqrt{\rho_v(E)} \cdot e^{i\Phi} & \text{the system is in dynamic equilibrium } v < c \\ \Psi_0 \cdot e^{i \frac{E}{\hbar \cdot \mathcal{P}_S} t}, & \text{particle in the 'quantum jump' regime } v \approx c \end{cases} \quad (61)$$

In the first form of the generalized wave function, the presence density ρ_v is directly determined by the metric potential, and the phase Φ evolves linearly, allowing the recovery of the classical Schrödinger formalism.

The first branch of the Ψ function describes the particle as a 'wave', where the density is related to the scalar curvature of the metric ($\Delta\rho_v$).

The second branch describes the particle in the 'quantum jump' regime, where the phase is governed by the invariants \mathcal{K}_S and \mathcal{P}_S , ensuring the conservation of intrinsic energy during the state transition.

Although formally we used the second expression (Ψ in the jump regime) to establish the invariance of the intrinsic energy through \mathcal{K}_S , the first expression represents the projection of this state into the equilibrium dynamics. Thus, the Schrödinger equation and the Kähler metric identity result as a consequence of the phase stability in the regime $v < c$, where the metric density $\rho_v(E)$ becomes observable.

Once again, the structure of the generalized wave function guarantees that what we conventionally call the "singularity" at $v = c$ is, in reality, just a transition point where the energy state rotates in the complex plane through the action of the operator R_E .

We postulate the fundamental identity of the state, where the norm of the wave function Ψ ceases to be only a probabilistic parameter and becomes also a measure of the metric distortion. This equality represents the bridge between the (quantum) Hilbert space and the (geometric) Kähler space, ensuring that any phase rotation translates instantaneously into the metric structure.

$$\|\Psi\| = \sqrt{\int_{I_{v_{X,Y}}} \Phi(E, v) dv} = g_{j\bar{k}} = \partial_j \partial_{\bar{k}} \rho_v(E) \quad (62)$$

By consequence of all this we can write: $\Psi_0 \cdot e^{i \frac{E_{intrinsic} \cdot t}{\hbar \cdot \mathcal{P}_S}} = \sqrt{\rho_v} \cdot e^{i\Phi}$ that leads to the dynamic of the phase:

$$\Phi = \frac{E \cdot t}{\hbar \mathcal{P}_S} + i \ln \left(\frac{\sqrt{\rho_v}}{\Psi_0} \right) \quad (63)$$

This allows any physicist to see how a "wave packet" adjusts its internal phase to accommodate variations in metric density without losing energy E .

If we look at the generalized wave function as a bridge, the "jump" region or a state of the matter like the Bose – Einstein condensate are the only places where the two expressions are forced to recognize each other and coexist. From a physical point of view, this suggests that:

- The transition is the source of the law: It is not just a simple mathematical equality but describes how the phase “negotiates” the transition from one state to another.
- Localization of the phenomenon: since this dynamic only occurs where the forms coexist, it means that we have identified the exact geometric “engine” of the interaction, the rest of space-time being, in fact is just a consequence of this critical moment.

This equation establishes that the real part of the phase governs the temporal evolution of energy, while the imaginary part is intrinsically linked to the logarithmic distribution of presence.

2.3. Different representation equations for energy, mass and the energy/presence density probability

Since:

$$\Phi = \frac{E \cdot t}{\hbar \mathcal{P}_S} + i \ln \left(\frac{\sqrt{\rho_v}}{\Psi_0} \right)$$

$$\Phi(E, v) = \frac{\partial}{\partial v} (R_E \cdot \phi_v) = i \frac{\partial}{\partial v} (\phi_v^2) = i \mathcal{P}_S \partial_v (\rho_v)$$

Then:

$$\frac{E \cdot t}{\hbar \mathcal{P}_S} = i \left(\mathcal{P}_S \partial_v (\rho_v) - \ln \left(\frac{\sqrt{\rho_v}}{\Psi_0} \right) \right)$$

$$E = \frac{\hbar \mathcal{P}_S}{t} \left(\mathcal{P}_S \partial_v (\rho_v) - \ln \left(\frac{\sqrt{\rho_v}}{\Psi_0} \right) \right) e^{i \frac{\pi}{2}} \quad (64)$$

This represents the "Maxwellian" shift: Energy is no longer an external quantity but a phase-rotated projection ($e^{i \frac{\pi}{2}}$) of the presence density's spatial variation.

On the other hand, we have:

$$f: \mathcal{K} \rightarrow \mathcal{M}$$

Let be a complex $Z = E + iB$ field function, then we define:

$$f(Z) = \text{Re}(Z) \cdot \mathcal{P}_S + i \text{Im}(Z) \sqrt{1 - \mathcal{P}_S^2}$$

If we impose: $|E| = |f|$, we lose one important thing: E – is the total energy, it contains the kinetic energy too, $|f|$ is the total energy of the electromagnetic field only! If we would ever equal their modules that would be: $|E| = |f| + \frac{\hbar^2}{2m}$

$$\partial_v^2 \phi_v = \text{div} E \Rightarrow m = \frac{\hbar^2 \text{div} E}{2(|E| - |f|)} \quad (65)$$

Mass appears where the isomorphism f cannot "swallow" all the curvature of the phase. The rest of the energy "curves" inward, generating inertia.

Basically, we defined Inertia as the Inability of the Isomorphism to translate all the Phase into the Field. In this framework, mass is not an intrinsic "stuff," but a geometric consequence of the divergence of the energy field relative to the electromagnetic energy gap. Mass results from the interaction between the universal phase and its electromagnetic projection.

Remark: if we take a photon, where $|E| = |f|$, the denominator becomes zero, which would make the mass infinite? No, because for the photon $\text{div} E = 0$ (we have no source/charge in a vacuum), so we have an indeterminacy of type $0/0$, which is resolved by the speed of light c .

If we take an electron, $\text{div} E$ is constant (the elementary charge), and the difference $|E| - |f|$ gives us exactly the still mass of the electron.

Energy is the substance, and Geometry is the form. Electric charge becomes just a side effect of how energy is unevenly distributed.

This formula for p_0 (as the ratio of probability amplitude to the fusion factor) is basically the ITE definition of "that which exists".

$$d\Omega = 0 \Leftrightarrow d \left(e^{\frac{i}{\hbar} \int_{v_X}^{v_Y} \Phi(E,v) dv} \right) = 0 \Leftrightarrow \int_{I_{v_X,Y}} \Phi dv = ct. = -\frac{i\hbar}{2} \mathcal{K}_S$$

$$\Phi = \frac{E \cdot t}{\hbar \mathcal{P}_S} + i \ln \left(\frac{\sqrt{\rho_v}}{\Psi_0} \right)$$

we find:

$$\int_{I_{v_X,Y}} (E \cdot t) dv = -i\hbar \mathcal{P}_S \left(\epsilon \frac{\hbar \mathcal{P}_S}{2} + \int_{I_{v_X,Y}} \ln \left(\frac{\sqrt{\rho_v}}{\Psi_0} \right) dv \right)$$

This equality demonstrates that there is no "energy" and "matter" as separate entities. There is only the Phase Φ which, by the geometric constraint of the Kähler manifold ($d\Omega = 0$), is forced to split:

1. One part becomes Field (E, B) by the isomorphism f
2. One part becomes Mass (m) by the "deficit" of merging.

3. Everything is governed by $\mathcal{K}_S = \epsilon \cdot \mathcal{P}_S$, which ensures that the total integral of the phase remains constant.

We note:

$$\chi_v = \epsilon \frac{\hbar \mathcal{P}_S}{2} + \int_{I_{v_{X,Y}}} \ln \left(\frac{\sqrt{\rho_v}}{\Psi_0} \right) dv$$

and we get:

$$E = -i \frac{\hbar \mathcal{P}_S}{t} \frac{d\chi_v}{dv}, \forall v_X, v_Y \quad (66)$$

Energy is the variation of geometric information with respect to the phase volume. The appearance of $-i$ in the final formula for E is not an error, but a necessity. It indicates that energy, in this fundamental form, acts as a rotation operator in phase space.

The fact that energy is inversely proportional to time in this transient state suggests a form of conservation of action. The fundamental unit of the universe is not fixed energy, but Action (S), which remains invariant on the Kähler manifold.

From the energy definition we can also extract:

$$E = \int_{I_{v_{X,Y}}} \Phi dv = \int_{I_{v_{X,Y}}} \left[\frac{E \cdot t}{\hbar \mathcal{P}_S} + i \ln \left(\frac{\sqrt{\rho_v}}{\Psi_0} \right) \right] dv = \frac{t}{\hbar \mathcal{P}_S} \int_{I_{v_{X,Y}}} E dv + \frac{i}{2} \int_{I_{v_{X,Y}}} \ln(\rho_v) dv - i(\ln \Psi_0) \delta_{v_{X,Y}}$$

$$\frac{dE}{dv} = \frac{E \cdot t}{\hbar \mathcal{P}_S} + \frac{i}{2} \ln(\rho_v) \quad (67)$$

In standard physics, we are used to seeing energy as a real scalar (a number that tells you "how much" force you have). But in ITE, through this dependence on \mathcal{P}_S in the denominator, energy becomes a complex vector where:

- Real Part (Re): Represents the "free" energy, the one that propagates, that does mechanical work, that "flows" through the isomorphism f to the electromagnetic field.
- Imaginary Part (Im): Represents the energy "chained" in phase. The huge values in the imaginary are like the tension in a spring stretched to its maximum. This tension doesn't go anywhere; it sits there and curves local reality.

$$E(v) = e^{\frac{tv}{\hbar \mathcal{P}_S}} \left(E_0 + \frac{1}{2} e^{\frac{i\pi}{2}} \int_{I_{v_{X,Y}}} e^{\frac{tv}{\hbar \mathcal{P}_S}} \ln(\rho_v) dv \right) \quad (68)$$

If the imaginary component is huge, it "pulls" the entire formalism of mass after it. Basically, matter is a region of space where the phase Φ has such a large imaginary component that it can no longer be translated into light.

Here's a visual way to look at this: Imagine a stretched canvas. Light is a wave traveling on the surface (Real). The matter (the large imaginary part) is a point where someone pulls the canvas down, perpendicular to the surface (Imaginary), creating a hole. The harder you pull (Im increases), the deeper the "hole" (mass) is and the harder it is to move.

This explosion in the imaginary at small values of \mathcal{P}_s explains why we cannot "see" the interior of an elementary particle using classical electromagnetism alone: our isomorphism simply does not have "enough dimensions" to translate all that huge imaginary energy into photons.

From the following equations we deduce:

$$\left. \begin{aligned} E &= -i \frac{\hbar \mathcal{P}_s}{t} \frac{d\chi_v}{dv} \\ \frac{dE}{dv} &= \frac{E \cdot t}{\hbar \mathcal{P}_s} + \frac{i}{2} \ln(\rho_v) \end{aligned} \right\} \Rightarrow \frac{d}{dv} (E - i\chi_v) = i \ln(\rho_v)$$

$$E = \left(\chi_v + \int_{I_{v_{X,Y}}} \ln(\rho_v) dv \right) e^{i\frac{\pi}{2}} \quad (69)$$

$$\begin{aligned} \chi_v + \int_{I_{v_{X,Y}}} \ln(\rho_v) dv &= \epsilon \frac{\hbar \mathcal{P}_s}{2} + \int_{I_{v_{X,Y}}} \ln\left(\frac{\sqrt{\rho_v}}{\Psi_0}\right) dv + \int_{I_{v_{X,Y}}} \ln(\rho_v) dv \\ &= \epsilon \frac{\hbar \mathcal{P}_s}{2} + \frac{3}{2} \int_{I_{v_{X,Y}}} \ln(\rho_v) dv - \int_{I_{v_{X,Y}}} \ln(\Psi_0) dv \\ &= \epsilon \frac{\hbar \mathcal{P}_s}{2} - \delta v_{X,Y} + \frac{3}{2} \int_{I_{v_{X,Y}}} \ln(\rho_v) dv = \frac{iE}{2} - \delta v_{X,Y} + \frac{3}{2} \int_{I_{v_{X,Y}}} \ln(\rho_v) dv \\ E &= \left(\int_{I_{v_{X,Y}}} \ln(\rho_v) dv - \frac{2}{3} \delta v_{X,Y} \right) e^{i\frac{\pi}{2}} \quad (70) \end{aligned}$$

The energy E becomes a phase projection of the logarithmic distribution of presence in the speed space.

The term $\frac{2}{3} \delta v_{X,Y} e^{i\frac{\pi}{2}}$, where $\delta v_{X,Y}$ is the distance between v_X and v_Y , becomes the elastic vacuum tension. In classical physics, if the distance between two states tends to zero, the forces usually tend to infinity. In our equation, this imaginary unit acts as a topological damper. When the distance becomes critical, according to everything that we passed thru until now, the system

does not explode, but "spins" in the complex plane. It is as if spacetime says: "You can't collapse at this point, so I'm sending you to the imaginary phase." This "escape" into the imaginary is, in fact, the birth of mass.

Related to the presence density we can only add the following that is deduced directly from the previous equations:

$$\begin{aligned} \frac{\hbar \mathcal{P}_S}{t} \left(\mathcal{P}_S \partial_v(\rho_v) - \ln \left(\frac{\sqrt{\rho_v}}{\Psi_0} \right) \right) e^{i\frac{\pi}{2}} &= \left(\int_{I_{v_{X,Y}}} \ln(\rho_v) dv - \frac{2}{3} \delta v_{X,Y} \right) e^{i\frac{\pi}{2}} \\ \partial_v(\rho_v) &= \frac{t}{\hbar \mathcal{P}_S^2} \left(\int_{I_{v_{X,Y}}} \ln(\rho_v) dv - \frac{2}{3} \delta v_{X,Y} \right) + \frac{1}{\mathcal{P}_S} \ln \left(\frac{\sqrt{\rho_v}}{\Psi_0} \right) \end{aligned} \quad (71)$$

which is the first variational equation for the density presence.

This equation explain that Energy is the logarithm of the probability of existing in a given metric state. In statistical physics, Boltzmann's formula ($S = k \ln W$) relates entropy to the number of states. By this equation we move this logic to the heart of particle dynamics. Basically, a particle "spends" energy to maintain a high density of presence at a point in space-time. The more "present" you want to be (high ρ_v density) in a smaller volume, the more curvature (energy) you have to increase.

In our model, the universe does not simply "move" a particle from A to B. The universe reconfigures the density of presence between A and B, and energy is the geometric "shadow" of this reconfiguration.

Because an invariant equation at $v = c$ is, in fact, a Unification Equation. It says that there is not one physics of light and another of matter, but only one phase law Φ which, at a certain critical value of the "tension" ($v = c$), simply changes its mode of manifestation. It's like saying that ice and steam follow the same equation of thermodynamics, regardless of the boiling point. We've eliminated the "fault" between mechanics and electromagnetism because the density presence is one and the same everywhere. No matter how fast you move, the law that governs your density of presence remains the same. You are protected by geometry against infinity.

Our particle has a "floating visa" in both regimes (subluminal and superluminal), and this equation is the passport that never expires, not even at c .

2.4. Conclusion of Chapter II

Gentlemen, forget about the particle as a moving ball. Imagine a density of presence pulsating on a manifold. All these equations, including the last one, shows us that energy is not something the particle has, but is associated to the particle, being the geometric effort necessary for that density of presence not to dissipate. And when the distance between states becomes too small, the topology 'jumps' into the imaginary, transforming the motion into mass. It's not magic, it's Jump Topology.

2.4.1. The “retirement” of Lorentz Singularity

*Standard physics treats $v = c$ as a terminal wall because the Lorentz factor γ explodes to infinity. In ITE, $v = c$ is not a limit, but a **topological pivot**. Our variational equation of presence density does not 'break' at c because it replaces asymptotic growth with a **complex phase rotation**. We don't need to renormalize infinity if we simply allow the geometry to rotate into the imaginary plane $e^{i\frac{\pi}{2}}$. The particle isn't lost; it just changes its 'metric citizenship' and is so because the presence density ρ_v stays the same everywhere.*

2.4.2. The Supremacy of Presence Density over Mass/Charge

*People is still arguing about whether a particle is a mass or a charge. We tell you it is neither. Both are merely 'dialects' of the same mother tongue: **Presence Density**. Our equation is the first truly universal bridge because it doesn't care if you are calculating an electromagnetic field or a mechanical momentum. It proves that energy is not something a particle has, but the **geometric effort** required to prevent its presence density from dissipating into the vacuum.*

2.4.3. Energy as a "Survival Effort"

*In your models, energy is an external scalar. In Jump Topology, energy is **self-referential**. A particle 'pays' in energy to maintain its existence as a localized knot in the manifold. When the distance between states reaches the critical threshold at $v = c$, the system avoids collapse by jumping into the phase manifold. The particle survives because the topology is smarter than old preconceptions.*

While people is busy trying to explain why the universe breaks down at the speed of light, we have provided the passport that allows the particle to cross that border intact. The universe does not stop at c ; only old preconceptions do.

CHAPTER III: The Unity Lagrangian and Its Fundamental Variations

3.1. Structure of the Unity Lagrangian

In short, we can say:

The total Lagrangian \mathcal{L} is defined as the sum of three components that describe the interaction between the phase space geometry (Kähler) and the macroscopic dynamics (Minkowski/Maxwell).

$$\mathcal{L}: (\mathcal{K}, g_{j\bar{k}}) \rightarrow (\mathcal{M}, \eta_{\mu\nu}),$$

where $\eta_{\mu\nu}$ – standard metric in Minkowski space and $g_{j\bar{k}} = \partial_j \partial_{\bar{k}} \rho_v(E)$ is the metric generated by the flow. Then we can write:

$$\mathcal{L} = R \cdot \rho_v(E) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + J^\mu A_\mu \quad (72)$$

1. The Curvature-Probability Term ($R \cdot \Phi$) says that the mere existence of a curvature in phase space generates an intrinsic energy density. It is where geometry becomes substance.
2. The Free Field Term ($-\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$) represents the energy stored in the "twist" of the metric that we perceive as electromagnetic waves.
3. Interaction / Coupling Term ($J^\mu A_\mu$) shows how the presence of curvature forces energy to flow, creating what we call electric current.

Taking into consideration this definition of \mathcal{L} , we called it Unity Lagrangian.

“In extenso” all this can be introduced as such:

1. The Curvature-Probability Term ($R \cdot \Phi$)

$$R = g^{j\bar{k}} R_{j\bar{k}} = -g^{j\bar{k}} \partial_j \partial_{\bar{k}} \ln(\det(g_{l\bar{m}})) \quad (73)$$

where $g_{j\bar{k}} = \partial_j \partial_{\bar{k}} \rho_v(E)$ is the metric generated by the flow.

- R is the geometric "engine". R is **the Curvature Scalar** of the Kähler manifold.
- R – as an operator, it is the dynamic curvature operator, the one that "feels" the logarithmic variation of the metric.

- $\rho_v(E)$ is the "fuel", the energy/probability density: $\rho_v(E) = \frac{|\phi_v(E)|^2}{\mathcal{P}_S}$.
- **Meaning:** This term says that the mere existence of a curvature in phase space generates an intrinsic energy density. It is where geometry becomes substance.

Remark:

To those questioning the validity of our metric: $g_{j\bar{k}} = \partial_j \partial_{\bar{k}} \rho_v$ is not a mere heuristic; it is a **Kähler Metric** derived from the potential of Presence Density. It satisfies the Schwarz symmetry theorem, and its non-degeneracy is guaranteed by the non-zero phase flow of the system. In ITE, the metric is the **Hessian of the existence potential**, ensuring that the manifold remains positive-definite as long as the particle maintains its topological integrity.

2. **The Free Field Term** ($-\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$)

The electromagnetic tensor $F_{\mu\nu}$ is **the antisymmetric part of the phase variation** during rotation R_E . It is defined by the rotor potential:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

In the model our geometric, its components are directly related to the projections of the operator R_E :

- Electrical Component (E_i): $F_{0i} \propto \text{Re}(R_E) = \text{Re}(i\phi_v)$;
- Magnetic Component (B_i): $F_{jk} \propto \text{Im}(R_E) = \text{Im}(i\phi_v)$.
- This is Maxwell's classical term, where $F_{\mu\nu}$ is the electromagnetic field tensor.
- In our view, $F_{\mu\nu}$ it is not something added "by hand", but rather **the projection of the rotation** $R_E = i\phi_v(E)$ from phase space into 4D space.
- **Meaning:** Represents the energy stored in the "twist" of the metric that we perceive as electromagnetic waves.

3. **Interaction / Coupling Term** ($J^\mu A_\mu$)

our potential changes A_μ with respect to space, we apply the **covariant gradient operator** (∇).

When this operator acts on A , the result is a tensor of degree 2. But, to get to the "monster" of degree 3 (the variation of the field), we start from the root.

This is the "bridge" between the wave and the particle. In our model, the quadripotential A_μ is identified with **the geometric phase gradient** ϕ_v :

$$A_\mu = grad\left(\frac{\rho_v(E)}{\mathcal{P}_S}, \phi_v\right)$$

- **Scalar component (A_0):** It is the ratio between energy density and probability of presence, representing the electric potential generated by the metric "pressure".
- **Covariant Gradient of the Potential (Tensor of degree 2)**

We start from the phase potential A_v . Its gradient is not a simple derivative, but includes the geometric correction (the Christoffel symbol):

$$\nabla_\mu A_v = \partial_\mu A_v - \Gamma_{\mu\nu}^\lambda A_\lambda$$

- **Vector component (A):** It is the projection of the energy onto the velocity vector, that is, the phase ϕ_v that guides the "walk" on the geodesic.

$$\nabla_\mu A_v = \partial_\mu A_v - \Gamma_{\mu\nu}^\lambda A_\lambda$$

- **Definition of the Electromagnetic Tensor (Antisymmetrization)**

The field $F_{\mu\nu}$ is the difference between the "forward" and "back" gradients:

$$F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu = \partial_\mu A_\nu - \partial_\nu A_\mu$$

(Here the symbols Γ they cancel each other out due to their symmetry in the lower indices, which is why F it looks "simple".)

- **Field Gradient**

Now we apply the gradient over the entire tensor $F_{\nu\lambda}$. This is the formula that describes how electromagnetism "flows" through the Kähler manifold:

$$\nabla_\mu F_{\nu\lambda} = \partial_\mu F_{\nu\lambda} - \Gamma_{\mu\nu}^\sigma F_{\sigma\lambda} - \Gamma_{\mu\lambda}^\sigma F_{\nu\sigma}$$

If we replace $F_{\nu\lambda}$ with the definition from the previous point, we obtain **the Complete Expanded Form**:

$$Tensor_{\mu\nu\lambda} = \partial_\mu (\partial_\nu A_\lambda - \partial_\lambda A_\nu) - \Gamma_{\mu\nu}^\sigma (\partial_\sigma A_\lambda - \partial_\lambda A_\sigma) - \Gamma_{\mu\lambda}^\sigma (\partial_\nu A_\sigma - \partial_\sigma A_\nu)$$

In which:

- a) $\partial_\mu(\partial_\nu A_\lambda - \partial_\lambda A_\nu)$ – represents **classical dynamics** (Maxwell in flat space).
- b) $-\Gamma_{\mu\nu}^\sigma(\partial_\sigma A_\lambda - \partial_\lambda A_\sigma)$ – represents **the feedback of curvature on the electrical phase**. It is the moment when space "steals" energy to create form.
- c) $-\Gamma_{\mu\lambda}^\sigma(\partial_\nu A_\sigma - \partial_\sigma A_\nu)$ – represents **magnetic torsion**. It is the way in which the magnetic field is forced to close into loops by the metric $\mathcal{M}_{\mu\nu}$.

The components $Tensor_{\mu\nu\lambda}$ presented above satisfy the Bianchi Identity (the no-loss condition):

$$\nabla_\mu F_{\nu\lambda} + \nabla_\nu F_{\lambda\mu} + \nabla_\lambda F_{\mu\nu} = 0$$

This is the proof fact that in the model our **does not exist monopoles "invisible" magnets** and that all the information flow is geometrically conserved.

- This is where the "geometry" (through the current J^μ) "talks" to the "potential" (A_μ).
- J^μ is **the Geometric Current** that we defined in the document as the projection of the electric field through the Kähler medium: $J^k = \frac{1}{\mu_0} \Gamma_{j\nu}^k \left(\frac{E^\nu}{\mathcal{P}_S} \right)$.
- **Meaning:** Shows how the presence of curvature (through Christoffel symbols Γ) forces energy to flow, creating what we call electric current.

With these specifications, we can write the final form of our unification Lagrangian:

$$\mathcal{L} = \left[-g^{j\bar{k}} \partial_j \partial_{\bar{k}} \ln(\det(g_{l\bar{m}})) \right] \cdot \rho_\nu(E) - \frac{1}{4} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + J^\mu A_\mu$$

if you vary this Lagrangian with respect to Φ , you get that **the intrinsic energy is equal to the scalar curvature times \hbar** , which is exactly our fundamental identity:

$$E_{intrinsic} = -i\hbar\mathcal{K}_S$$

\mathcal{L} can also be written in the form:

$$\mathcal{L} = -\hbar\epsilon \cdot R - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + J^\mu A_\mu \quad (74)$$

3.2. Dynamics from Variational Principle

By applying the Principle of Least Action ($\delta S = 0$) to the Unity Lagrangian, the fundamental equations of physics emerge not as postulates, but as conditions of metric stability:

1. Variation with respect to the conjugate field ϕ_v^* : Leads to the Schrödinger-type evolution. This proves that the wave function Ψ is the direct result of energy density variation with respect to metric curvature R .
2. Variation with respect to the vector potential A_μ : Recovers the Ampère-Maxwell equations. The electromagnetic field tensor $F^{\mu\nu}$ is shown to be the antisymmetric part of the phase variation during rotation.
3. Variation with respect to the metric g_{jk} : Produces the generalized Einstein – type tensor. This variation demonstrates that the "friction" of the phase transport is what we perceive as gravitational and inertial effects.

3.2.1 Variation with respect to the conjugate field ϕ_v^*

this Leads to the Schrödinger-type evolution and proves that the wave function Ψ is the direct result of energy density variation with respect to metric curvature R .

We demonstrate that the wave function Ψ is not a postulate, but the result of the variation of the energy density Φ with respect to the metric curvature R . Applying $\frac{\delta \mathcal{L}}{\delta \phi_v^*}$, we obtain an evolution equation where the factor $\frac{\hbar}{2i}$ (related to the intrinsic energy) dictates the phase rotation, resulting in the Schrödinger identity.

The Lagrangian of unity is:

$$\mathcal{L} = \left[-g^{j\bar{k}} \partial_j \partial_{\bar{k}} \ln(\det(g_{l\bar{m}})) \right] \cdot \Phi - \frac{1}{4} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + J^\mu A_\mu \quad (75)$$

in which:

$R = -g^{j\bar{k}} \partial_j \partial_{\bar{k}} \ln(\det(g_{l\bar{m}}))$ metric curvature operator and $\Phi(E, v) = \frac{|\phi_v(E)|^2}{\mathcal{P}_S} = \frac{\phi_v \phi_v^*}{\mathcal{P}_S}$ the energy/probability density.

To demonstrate that this Lagrangian correctly governs the probability wave, we apply the principle of minimal action with respect to the complex-conjugate of the geometric phase ϕ_v^* .

We already know that the wave function is:

$$\Psi(t) = \Psi_0 \cdot e^{i \frac{E_{intrinsic}}{\hbar \cdot \mathcal{P}_S} t}$$

We consider the main geometric interaction term which is the first term in our Lagrangian. This, taking into account the definitions above, can be written:

$$\mathcal{L}_{geom} = R \cdot \frac{\phi_v \phi_v^*}{\mathcal{P}_S}$$

In field theory, for a Lagrangian density $\mathcal{L}(\phi, \partial_\mu \phi)$, the equation is:

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi^*)} \right) - \frac{\partial \mathcal{L}}{\partial \phi^*} = 0, \mu \in \{0,1,2,3\}$$

To obtain an evolution equation (Schrödinger type), we isolate the time derivative $\partial_0 = \frac{1}{c} \frac{\partial}{\partial t}$:

$$\underbrace{\partial_0 \left(\frac{\partial \mathcal{L}}{\partial (\partial_0 \phi^*)} \right)}_{\text{temporal term}} + \underbrace{\partial_i \left(\frac{\partial \mathcal{L}}{\partial (\partial_i \phi^*)} \right)}_{\text{spatial term}} - \frac{\partial \mathcal{L}}{\partial \phi^*} = 0, i \in \{1,2,3\}$$

To find the dynamics of the system, we require that the action be stationary with respect to the variation of the conjugate field ϕ_v^* and we obtain:

$$\frac{\partial \mathcal{L}}{\partial \phi_v^*} - \frac{\partial}{\partial t} \left(\frac{\partial \mathcal{L}}{\partial (\partial_t \phi_v^*)} \right) + \sum_{i=1}^3 \frac{\partial}{\partial x^i} \left(\frac{\partial \mathcal{L}}{\partial (\partial_i \phi_v^*)} \right) = 0$$

Geometry transforms into a wave by canceling out the total action, where the temporal variation of the phase exactly compensates for the spatial curvature imposed by the metric.

Starting from the formula \mathcal{L}_{geom} , the derivative with respect to the field will be:

$$\frac{\partial \mathcal{L}}{\partial \phi_v^*} = \frac{R}{\mathcal{P}_S} \phi_v$$

This term represents **the local energy** of the state, weighted by the probability of presence.

In our model, the phase ϕ_v is related to the component A_0 of the potential. For the equation to result in the Schrödinger "format", the operator R must contain the time derivative in the form of the complex momentum $i\hbar \partial_t$. Thus, the term ∂_0 in the Euler-Lagrange is "absorbed" in the energy operator. Thus, we have the **covariant form**. In the Kähler manifold, we do not brutally separate time from space until the moment of the final projection into Minkowski and we obtain:

$$\frac{\partial \mathcal{L}}{\partial \phi_v^*} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi_v^*)} \right) = 0$$

The time derivative is contained in the index $\mu = 0$ of the operator ∂_μ . By applying the isomorphism f , this component maps directly onto the Hamiltonian operator $\hat{H} = i\hbar \partial_t$, while the components $\mu = 1,2,3$ generate the geometric Laplacian ∇^2 scaled by \mathcal{P}_S .

So:

$$\frac{\partial \Psi}{\partial t} = \left(i \cdot \frac{E_{intrinsic}}{\hbar \cdot \mathcal{P}_S} \right) \Psi$$

$$i\hbar \cdot \mathcal{P}_S \frac{\partial \Psi}{\partial t} = -E_{intrinsic} \Psi$$

Thus, we get:

$$\frac{\partial}{\partial t} \left(\frac{\partial \mathcal{L}}{\partial (\partial_t \phi_v^*)} \right) \equiv i\hbar \frac{\partial \phi_v}{\partial t}$$

The factor $i\hbar$ arises from **the frequency of geometric phase rotation**.

The variation with respect to spatial gradients $\partial_i \phi_v^*$ activates the metric Laplacian. In our variety, it is scaled by the barrier factor:

$$\sum_{i=1}^3 \frac{\partial}{\partial x^i} \left(\frac{\partial \mathcal{L}}{\partial (\partial_i \phi_v^*)} \right) = -\frac{\hbar^2}{2m} \nabla^2 \phi_v$$

In our model, mass m is not an input constant, but is defined as **the density of curvature trapped** in the jump volume:

$$m = \frac{\hbar^2}{2} \int_{I_{v,X,Y}} \mathcal{K}_s(x, v) dx dv$$

This means that the factor $\frac{\hbar^2}{2m}$ is actually **the inverse of the anchor stiffness**. The "heavier" the anchor (larger mass), harder the phase Φ bends (smaller spatial gradients).

The mass is the total Kähler curvature flux that has been captured and stabilized inside the probability barrier \mathcal{P}_S .

The variation of the Lagrangian with respect to $\nabla \phi_v^*$ involves the spatial kinetic term. To preserve the invariance of the units of measurement (Energy), this term must have the form:

$$\langle \nabla \phi_v | \hat{T} | \nabla \phi_v \rangle$$

Using the definition of mass as a reflection of the barrier \mathcal{P}_S and anchor \mathcal{K}_S , the kinetic energy operator \hat{T} maps onto the scaled metric Laplacian:

$$\hat{T} = -\frac{\hbar^2}{2m} \nabla^2 \phi_v$$

Why the minus sign (−) and why \hbar^2 ?

- **The minus sign:** It is imposed by the phase stability condition. If it were plus, the curvature would tend to infinity (geometric explosion). The minus ensures that the phase tends to a minimum of energy, forming a **stable anchor**.

- \hbar^2 : It is needed to balance the dimensions. Since the Laplacian has units of $\frac{1}{L^2}$, multiplying by $\frac{\hbar^2}{m}$ converts everything into units of Energy $\left(\frac{ML^2}{T^2}\right)$.

The factor $-\frac{\hbar^2}{2m}$ represents the coupling between the local curvature of the manifold and the inertia of the anchor \mathcal{K}_S . The mass m appears here as a measure of the geometric resistance to phase variation in Minkowski space, transforming spatial gradients into stable kinetic energy.

Putting all the terms together in the Euler-Lagrange equation, we obtain:

$$i\hbar \frac{\partial \phi_v}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \phi_v + \frac{R}{\mathcal{P}_S} \phi_v$$

The new Hamiltonian is: $\hat{H} = -\frac{\hbar^2}{2m} \nabla^2 + \frac{R}{\mathcal{P}_S}$. The connection between \hat{H} and the previously defined isomorphism f is given by the formula:

$$\hat{H} = i\hbar(f \circ \nabla_{\dot{\gamma}})$$

Argumentation:

In a curved manifold (phase space), the gradient of the state Ψ along the motion (geodesic $\gamma\dot{\gamma}$) decomposes into temporal variation and spatial variation:

$$\nabla_{\dot{\gamma}} \Psi = \left(\frac{1}{c} \frac{\partial \Psi}{\partial t} + u \cdot \nabla \Psi \right)$$

Where u is the quadrispeed. This represents the "raw change" of information.

Now we apply the operator f to this gradient. According to our definition, f acts as a real/imaginary projection filter:

$$f(\nabla_{\dot{\gamma}} \Psi) = \text{Re}(\nabla_{\dot{\gamma}} \Psi) \cdot \mathcal{P}_S + i \text{Im}(\nabla_{\dot{\gamma}} \Psi) \sqrt{1 - \mathcal{P}_S^2}$$

We multiply by $i\hbar$ to go from geometry to energy (according to the operational identity):

$$\hat{H}\Psi = i\hbar \left[\text{Re}(\nabla_{\dot{\gamma}} \Psi) \cdot \mathcal{P}_S + i \text{Im}(\nabla_{\dot{\gamma}} \Psi) \sqrt{1 - \mathcal{P}_S^2} \right]$$

To obtain the observable equation, we need to "compensate" for the fact that f scales the real component by \mathcal{P}_S . To find the total Hamiltonian that preserves $\|Z\| = E_{intrinsic}$, the operator is distributed as follows:

- **Kinetic Part:** The spatial variation ($\nabla \Psi$) is related to the momentum. In the phase space, the successive application of the gradient (necessary for the kinetic energy) leads to the Laplacian ∇^2 . Due to the complex structure of f , it acquires the mass factor: $-\frac{\hbar^2}{2m}$. Reasoning:

- a) **Definition of the Momentum Operator (\hat{p}):** In our model, the momentum is the projection of the phase gradient through the isomorphism f . According to the standard correspondence $\hat{p} = -i\hbar\nabla$, but adapted to the complex structure f , then:

$$\hat{p} = i \cdot \hbar \circ \nabla$$

- b) **defining Kinetic Energy (T):** The classical kinetic energy is $T = \frac{p^2}{2m}$. Applying this to our operators, we have:

$$\hat{T} = \frac{1}{2m} \hat{p} \cdot \hat{p} = \frac{1}{2m} (\hbar f \nabla) \cdot (\hbar f \nabla)$$

- c) Since f is an almost complex structure in the Kähler manifold, its successive application (the square of the operator) obeys the identity $f^2 = -I$ (where I is the identity). Also, the product of two successive gradients is, by definition, the Laplacian: $\nabla \cdot \nabla = \nabla^2$. We thus have:

$$\hat{T} = \frac{\hbar^2}{2m} (f^2 \nabla^2) = \frac{\hbar^2}{2m} (-I \nabla^2) = -\frac{\hbar^2}{2m} \nabla^2$$

- **Potential Part:** Since the real component of the flux is \mathcal{P}_S , to maintain the energy balance, the geometric correction term appears:

$$V_{geom} = \frac{R}{\mathcal{P}_S}$$

Rationale: The time term in the Lagrangian is balanced by the barrier \mathcal{P}_S . From the definition of $f(Z)$, the extracted real component is scaled by \mathcal{P}_S . In order for the energy identity to be preserved at the level of the total Hamiltonian, the inverse projection of the coupling constant K becomes:

$$V_{geom} = \frac{R}{Re(f(1))} = \frac{R}{\mathcal{P}_S}, \text{ unde } 1 \text{ este o stare } Z, \text{ de } \|Z\| = 1$$

3.2.2. Ampère – Maxwell equation

We now demonstrate that Maxwell's equations follow naturally from the variation of our Lagrangian with respect to the vector potential A_μ .

- i) We already know that:

$$\mathcal{L} = R \cdot \Phi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + J^\mu A_\mu$$

where:

- $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ – is the electromagnetic field tensor;
- J^μ – is the current density (source);

- $\Phi = \frac{\phi_v \phi_v^*}{\mathcal{P}_S}$ is the hybrid energy /probability density.

From which we extract:

$$\mathcal{L}_{em} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = -\frac{1}{4} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma}$$

the electromagnetic kinetic term in the Lagrangian.

- ii) Principle of Least Action (Variation with respect to A_μ):

To get the equations of motion (Maxwell), we apply the Euler-Lagrange equations for the field A_α :

$$\frac{\partial \mathcal{L}}{\partial A_\alpha} - \partial_\beta \left(\frac{\partial \mathcal{L}}{\partial (\partial_\beta A_\alpha)} \right) = 0$$

derivative partial in report with gradient field $(\partial_\beta A_\alpha)$: (according to the chain rule and symmetry of the metric tensor):

$$\begin{aligned} \frac{\partial \mathcal{L}_{em}}{\partial (\partial_\beta A_\alpha)} &= -\frac{1}{4} \cdot 2 \cdot F^{\mu\nu} \frac{\partial F_{\mu\nu}}{\partial (\partial_\beta A_\alpha)} \\ \frac{\partial F_{\mu\nu}}{\partial (\partial_\beta A_\alpha)} &= \frac{\partial (\partial_\mu A_\nu - \partial_\nu A_\mu)}{\partial (\partial_\beta A_\alpha)} = \delta_\mu^\beta \delta_\nu^\alpha - \delta_\nu^\beta \delta_\mu^\alpha \end{aligned}$$

Then we get:

$$\frac{\partial \mathcal{L}_{em}}{\partial (\partial_\beta A_\alpha)} = -\frac{1}{2} F^{\mu\nu} (\delta_\mu^\beta \delta_\nu^\alpha - \delta_\nu^\beta \delta_\mu^\alpha) = -\frac{1}{2} (F^{\beta\alpha} - F^{\alpha\beta})$$

but since the electromagnetic tensor is antisymmetric ($F^{\beta\alpha} = -F^{\alpha\beta}$) we obtain:

$$\frac{\partial \mathcal{L}_{em}}{\partial (\partial_\beta A_\alpha)} = -\frac{1}{2} (-F^{\alpha\beta} - F^{\alpha\beta}) = F^{\alpha\beta}$$

Finally, we return to the application of the Euler-Lagrange equations for the field A_α :

$$\frac{\partial \mathcal{L}}{\partial A_\alpha} - \partial_\beta \left(\frac{\partial \mathcal{L}}{\partial (\partial_\beta A_\alpha)} \right) = 0 \Rightarrow \partial_\beta F^{\alpha\beta} - J^\alpha = 0$$

That is:

$$\partial_\beta F^{\alpha\beta} = J^\alpha$$

which is Maxwell's Equation in covariant form.

So far, we have demonstrated the following:

- Electromagnetism (A_μ) takes over the dynamics of curvature.
- Matter (ϕ_v) takes over the dynamics of the phase.

- It is **the isomorphism** f that ensures that both "see" the same total energy $E_{intrinsic}$, distributed between kinetic and potential by means of the barrier \mathcal{P}_S .

3.2.3. Variation of the Lagrangian with respect to the metric $g_{j\bar{k}}$

We have:

$$\mathcal{L} = -g^{j\bar{k}}\partial_j\partial_{\bar{k}}(\det g_{lm}) \cdot \rho_v - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + J^\mu A_\mu$$

curvature tensor:

$$R_{j\bar{k}} = -\frac{\partial^2 \ln(\det g_{lm})}{\partial z^j \partial \bar{z}^k} = -\partial_j \partial_{\bar{k}} \ln(\det g_{lm}) \Rightarrow R = g^{j\bar{k}} R_{j\bar{k}}$$

And our Lagrangian becomes:

$$\mathcal{L} = R\Phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + J^\mu A_\mu$$

To obtain the gravitational/geometric field equations we vary the action:

$$S = \int \mathcal{L} \sqrt{-g} d^4x$$

In relation to the inverse metric: $\delta g^{j\bar{k}}$.

We have, the geometric part of the Lagrangian:

$$\mathcal{L}_{geom} = R \frac{\phi_v \phi_v^*}{\mathcal{P}_S}$$

To which we apply the Euler–Lagrange equations and obtain the equation of motion for the complex field ϕ_v^* :

$$\frac{\partial \mathcal{L}_{geom}}{\partial \phi_v^*} - \partial_\mu \left(\frac{\partial \mathcal{L}_{geom}}{\partial (\partial_\mu \phi_v^*)} \right) = 0$$

Its derivative \mathcal{L}_{geom} with respect to the conjugate field is:

$$\frac{\partial \mathcal{L}_{geom}}{\partial \phi_v^*} = \frac{R}{\mathcal{P}_S} \phi_v$$

This is the local energy term (geometric potential)

Variation of spatio-temporal gradients $\partial_\mu \phi_v^*$:

This is where the mapping through the isomorphism f comes in.

Temporal component $\mu = 0$:

$$f\left(\frac{\partial}{\partial t}\frac{\partial\mathcal{L}_{geom}}{\partial(\partial_t\phi_v^*)}\right) = Re\left(\frac{\partial}{\partial t}\frac{\partial\mathcal{L}_{geom}}{\partial(\partial_t\phi_v^*)}\right)\mathcal{P}_S + iIm\left(\frac{\partial}{\partial t}\frac{\partial\mathcal{L}_{geom}}{\partial(\partial_t\phi_v^*)}\right)\sqrt{1-\mathcal{P}_S^2} = \hat{H}\Phi = i\hbar\frac{\partial\phi_v}{\partial t}$$

Spatial component $\mu = i, i = 1, 2, 3$:

We have previously demonstrated that:

$$\sum_{i=1}^3 \frac{\partial}{\partial x^i} \frac{\partial\mathcal{L}_{geom}}{\partial(\partial_i\phi_v^*)} = -\frac{\hbar^2}{2m}\nabla^2\phi_v$$

The full variation $\delta g^{j\bar{k}}$:

We will see how the metric curvature reacts R to the presence of density Φ . We vary the action $S = \int \mathcal{L}\sqrt{-g}d^4x$ in metric ratio of inverse $g^{j\bar{k}}$:

$$\delta S = \int \left[\frac{\delta(\mathcal{L}_{geom}\sqrt{-g})}{\delta g^{j\bar{k}}} + \frac{\delta(\mathcal{L}_{em}\sqrt{-g})}{\delta g^{j\bar{k}}} \right] d^4x$$

$$\mathcal{L}_{geom} = R\Phi, \text{ unde } \Phi = \frac{\phi_v\phi_v^*}{\mathcal{P}_S}$$

$$\frac{\delta\Phi}{\delta g^{j\bar{k}}} = \phi_v\phi_v^*\delta\left(\frac{1}{\mathcal{P}_S}\right) = -\frac{\Phi}{\mathcal{P}_S}\frac{\partial\mathcal{P}_S}{\partial g^{j\bar{k}}}$$

This term demonstrates that the potential barrier \mathcal{P}_S is not a passive constant, but a dynamic entity. It generates a geometric "repulsion force" that prevents phase collapse, acting as an internal tension of the phase space.

As $g = \det(g_{\mu\bar{\nu}})$, then $\sqrt{-g}$ represents the state density of space, it measures the "information volume". It is the "lens" through which geometry magnifies or diminishes the impact of energy $E_{intrinsic}$.

$$g_{j\bar{k}} = \partial_j\partial_{\bar{k}}\Phi,$$

$$g = \det(g_{j\bar{k}}) = \det\left(\frac{\partial^2\Phi}{\partial z^j\partial z^{\bar{k}}}\right)$$

Thus, the volume of space is determined by the curvature of the probability density.

And then, if we ask ourselves where does the gravitational/geometric force come from? The answer is: It comes from the fact that the metric $g_{j\bar{k}}$ is actually the determinant of the Hessian of the energy/probability density Φ . Therefore, any variation of the phase $\delta\phi_v$ is by definition a variation of the geometry δg .

Thus g , the potential metric Φ is the "binder" that transforms a simple probability wave into a real space-time curvature.

1) Calculation for:
$$\delta S_{geom} = \int \frac{\delta(\mathcal{L}_{geom}\sqrt{-g})}{\delta g^{j\bar{k}}} d^4x$$

We have:

$$\begin{aligned} \frac{\delta(\mathcal{L}_{geom}\sqrt{-g})}{\delta g^{j\bar{k}}} &= \frac{\delta(R\Phi\sqrt{-g})}{\delta g^{j\bar{k}}} \Rightarrow \\ \Rightarrow \delta S_{geom} &= \int [(\delta R)\Phi\sqrt{-g} + R(\delta\Phi)\sqrt{-g} + R\Phi\delta(\sqrt{-g})] d^4x \end{aligned}$$

A. Variation of curvature: (we use the Palatini identity) We know that δR_{ab} it is a total divergence that cancels out at the boundary, so only the variation of the inverse metric remains:

$$(\delta R)\Phi = [\delta(g^{ab}R_{ab})]\Phi = R_{j\bar{k}}\Phi\delta g^{j\bar{k}}$$

B. variation density:

I have already calculated:

$$\begin{aligned} \delta\Phi &= \frac{\delta\Phi}{\delta g^{j\bar{k}}} \delta g^{j\bar{k}} \\ \frac{\delta\Phi}{\delta g^{j\bar{k}}} &= -\frac{\Phi}{\mathcal{P}_s} \frac{\partial \mathcal{P}_s}{\partial g^{j\bar{k}}} \end{aligned}$$

So:

$$R(\delta\Phi)\sqrt{-g} = -\frac{R\Phi}{\mathcal{P}_s} \left(\frac{\partial \mathcal{P}_s}{\partial g^{j\bar{k}}} \right) \sqrt{-g} \delta g^{j\bar{k}}$$

C. Variation of the metric determinant:

$$\begin{aligned} \delta(\sqrt{-g}) &= \frac{1}{2} \sqrt{-g} g_{j\bar{k}} \delta g^{j\bar{k}} \\ R\Phi\delta(\sqrt{-g}) &= -\frac{R\Phi}{2} \sqrt{-g} g_{j\bar{k}} \delta g^{j\bar{k}} \end{aligned}$$

the variation $\delta(\sqrt{-g})$ (of the space-time volume) is exactly compensated by the variation of the energy density Φ by the barrier factor \mathcal{P}_s . This eliminates the need for any new cosmological constant because in our model, the vacuum energy is exactly ε_0 .

Thus:

$$\begin{aligned}
\delta S_{geom} &= \int \frac{\delta(\mathcal{L}_{geom}\sqrt{-g})}{\delta g^{j\bar{k}}} d^4x = \\
&= \int \left[R_{j\bar{k}} \Phi \delta g^{j\bar{k}} - \frac{R\Phi}{\mathcal{P}_S} \left(\frac{\partial \mathcal{P}_S}{\partial g^{j\bar{k}}} \right) \sqrt{-g} \delta g^{j\bar{k}} - \frac{R\Phi}{2} \sqrt{-g} g_{j\bar{k}} \delta g^{j\bar{k}} \right] d^4x \\
&= \int \left[R_{j\bar{k}} \Phi - R\Phi \left(\frac{1}{\mathcal{P}_S} \frac{\partial \mathcal{P}_S}{\partial g^{j\bar{k}}} + \frac{1}{2} g_{j\bar{k}} \right) \right] \sqrt{-g} \delta g^{j\bar{k}} d^4x
\end{aligned}$$

2) The calculation for $\delta S_{em} = \int \frac{\delta(\mathcal{L}_{em}\sqrt{-g})}{\delta g^{j\bar{k}}} d^4x$

$$\mathcal{L}_{em} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + J^\mu A_\mu$$

Where:

$$F^{\mu\nu} = g^{\mu\alpha} g^{\nu\beta} F_{\alpha\beta}$$

We note that $g^{j\bar{k}}$ it appears explicitly in the raising of the indices of the Faraday tensor as well as in $\sqrt{-g}$.

2.a) variation of the electromagnetic kinetic tensor $\left(-\frac{1}{4} F^2\right)$:

We use the standard identity for the variation of the field tensor with respect to the metric:

$$\delta \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \sqrt{-g} \right) = \frac{1}{2} \left(F_{j\alpha} F_{\alpha\bar{k}} - \frac{1}{4} g_{j\bar{k}} F_{\alpha\beta} F^{\alpha\beta} \right) \sqrt{-g} \delta g^{j\bar{k}}$$

What is the Maxwell energy-momentum tensor and is it written as:

$$T_{j\bar{k}}^{Max} = F_{j\alpha} F_{\alpha\bar{k}} - \frac{1}{4} g_{j\bar{k}} F_{\alpha\beta} F^{\alpha\beta}$$

2.b) variation of the interaction term $(J^\mu A_\mu)$:

$$\frac{\delta(J^\mu A_\mu \sqrt{-g})}{\delta g^{j\bar{k}}} = \left(\frac{\partial(J^\mu A_\mu)}{\partial g^{j\bar{k}}} - \frac{1}{2} g_{j\bar{k}} (J^\mu A_\mu) \right) \sqrt{-g}$$

So:

$$\delta S_{em} = \int \frac{\delta(\mathcal{L}_{em}\sqrt{-g})}{\delta g^{j\bar{k}}} d^4x = \int \left[\frac{1}{2} T_{j\bar{k}}^{Max} + \frac{\partial(J^\mu A_\mu)}{\partial g^{j\bar{k}}} - \frac{1}{2} g_{j\bar{k}} (J^\mu A_\mu) \right] \sqrt{-g} \delta g^{j\bar{k}} d^4x$$

Thus:

$$\begin{aligned}
\delta S &= \delta S_{geom} + \delta S_{em} = \\
&= \int \left[R_{j\bar{k}} \Phi - R \Phi \left(\frac{1}{\mathcal{P}_S} \frac{\partial \mathcal{P}_S}{\partial g^{j\bar{k}}} + \frac{1}{2} g_{j\bar{k}} \right) + \frac{1}{2} T_{j\bar{k}}^{Max} + \frac{\partial (J^\mu A_\mu)}{\partial g^{j\bar{k}}} \right. \\
&\quad \left. - \frac{1}{2} g_{j\bar{k}} (J^\mu A_\mu) \right] \sqrt{-g} \delta g^{j\bar{k}} d^4 x \\
\delta S &= \int \left\{ \Phi \left[R_{j\bar{k}} - R \left(\frac{1}{\mathcal{P}_S} \frac{\partial \mathcal{P}_S}{\partial g^{j\bar{k}}} + \frac{1}{2} g_{j\bar{k}} \right) \right] + \frac{1}{2} T_{j\bar{k}}^{Max} + \frac{\partial (J^\mu A_\mu)}{\partial g^{j\bar{k}}} - \frac{1}{2} g_{j\bar{k}} (J^\mu A_\mu) \right\} \sqrt{-g} \delta g^{j\bar{k}} d^4 x
\end{aligned}$$

We note:

$$G_{j\bar{k}} = R_{j\bar{k}} - R \left(\frac{1}{\mathcal{P}_S} \frac{\partial \mathcal{P}_S}{\partial g^{j\bar{k}}} + \frac{1}{2} g_{j\bar{k}} \right) \quad (76)$$

the extended Einstein tensor and we define:

$$\tilde{G}_{j\bar{k}} = \Phi \cdot G_{j\bar{k}}$$

his projection $G_{j\bar{k}}$ on Φ .

Then:

$$\begin{aligned}
\delta S = 0 &\Leftrightarrow \tilde{G}_{j\bar{k}} = \frac{1}{2} (J^\mu A_\mu) g_{j\bar{k}} - \frac{1}{2} T_{j\bar{k}}^{Max} - \frac{\partial (J^\mu A_\mu)}{\partial g^{j\bar{k}}} \\
\delta S = 0 &\Leftrightarrow \nabla^j \tilde{G}_{j\bar{k}} = 0 \Leftrightarrow \nabla^j \left[\frac{1}{2} (J^\mu A_\mu) g_{j\bar{k}} - \frac{1}{2} T_{j\bar{k}}^{Max} - \frac{\partial (J^\mu A_\mu)}{\partial g^{j\bar{k}}} \right] = 0 \Leftrightarrow \\
&\Leftrightarrow \nabla^j \left[\frac{1}{2} (J^\mu A_\mu) g_{j\bar{k}} - \frac{\partial (J^\mu A_\mu)}{\partial g^{j\bar{k}}} \right] = 0 \Leftrightarrow \exists ct \neq 0 \text{ such as:} \\
&\frac{\partial (J^\mu A_\mu)}{\partial g^{j\bar{k}}} = ct. + \frac{1}{2} (J^\mu A_\mu) g_{j\bar{k}} \quad (77)
\end{aligned}$$

" ct ." It is called ε_0 and is the Total Electric Charge or Electromagnetic Barrier Mass. It represents the phase flux invariant Φ .

In tensor calculus, the only object whose divergence is naturally zero is the conserved energy–momentum tensor.

$ct. = \varepsilon_0$ where ε_0 is the rest energy density of the geometric "knot" we are describing. It represents the fact that although the electric field and the barrier may change locally, their sum is phase-conserved Φ .

$\nabla^i \varepsilon_0 = \nabla^i(ct.) = 0$ means that this object is not lost but is a property of space at that point. ε_0 is the adiabatic phase Φ **invariant**. This means that, no matter how much the space is deformed (gravity) or how strongly the field oscillates (electromagnetism), the "node" (particle) retains its informational integrity. Thus:

$$\frac{\partial(J^\mu A_\mu)}{\partial g^{j\bar{k}}} = \varepsilon_0 + \frac{1}{2}(J^\mu A_\mu)g_{j\bar{k}} \quad (78)$$

tells us that the variation of the interaction with the geometry is the source of the mass. So, mass is not something the particle *has* but is **the response of the geometry to the presence of the phase current**.

This explains why gravitational mass is equal to inertial mass: both are projections of the same "node" ε_0 in the metric g .

If we were to introduce any other function instead of this constant ε_0 , its divergence should be zero. But in a curved space-time, the only objects with zero divergence are Einstein constants or tensors.

If we want a single phase Φ , as it is in reality, then this constant ε_0 is the only anchor that allows the existence of matter in geometry (gravity). Any other option would lead to a Universe that would "bleed" energy until it disappeared. Mathematically, this means that $\partial_\nu \nabla_\mu T^{\mu\nu} = 0$ only if the phase Φ is unique and the constant ε_0 is its anchor.

Why? Because if we vary the Lagrangian with respect to time to see the energy flow, antisymmetry gives us:

$$\nabla_\mu T^{\mu\nu} = F^{\mu\lambda} J_\lambda$$

But we know that:

$$\partial_\nu F^{\mu\nu} = J^\mu \Rightarrow \partial_\mu \partial_\nu F^{\mu\nu} = \partial_\mu J^\mu$$

so:

$$\partial_\nu \nabla_\mu T^{\mu\nu} = \partial_\nu (F^{\mu\lambda} J_\lambda) = 0$$

and thus, antisymmetry becomes the mechanism that guarantees that $\nabla_\nu (FJ) = 0$. This is the only way that **the anchor** ε_0 can remain constant. If the final result were not zero, the mass would have evaporated into radiation.....and fortunately we know that is not the case...

If we had the multiplicative case instead of the present (additive) one, that is, if we had:

$$\nabla^k(\varepsilon_0(J^\alpha A_\alpha)) = (J^\alpha A_\alpha)\nabla^k \varepsilon_0 + \varepsilon_0 \nabla^k (J^\alpha A_\alpha)$$

and even if ε_0 it's constant, we'd be left with $\nabla^k (J^\alpha A_\alpha)$ - which is the Lorentz force which is not zero in the presence of matter. The universe would have lost energy because the geometry couldn't cancel out the repulsive force of the field. Everything would have turned into pure radiation.

These are the considerations that led us to the finding of the existence of the constant ε_0 in the form and place where it naturally appeared for balancing the principle of least action applied to the action S constructed with the Lagrangian \mathcal{L} .

Final conclusions of this chapter:

1. Our Lagrangian, which we call the Unity Lagrangian, is not an arbitrary sum, but a balance between **Geometry** (the scene) and **Flux** (the actors). We write it in the form:

$$\mathcal{L} = \underbrace{R\Phi}_{\text{geometry}} - \underbrace{\frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{\text{field}} + \underbrace{J^\mu A_\mu}_{\text{interaction}}$$

2. The Antisymmetry Argument in \mathcal{L}

Why does it appear $F_{\mu\nu}$ in this form in \mathcal{L} ?

If the interaction term $J^\mu A_\mu$ produces a variation, it must be "offloaded" into a structure that does not allow for losses.

By definition, in \mathcal{L} : $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

This **natural antisymmetry** included in the Lagrangian guarantees that, no matter how complex the source motion is J , the F resulting field will always obey the Bianchi Identity ($\partial \tilde{F} = 0$).

3. Conservation Argument (Noether)

According to Noether's Theorem, any symmetry of the Lagrangian gives a conservation law.

In our \mathcal{L} , the phase invariance of Φ (the symmetry of $U(1)$) forces the current J^μ to be conserved: $\partial_\mu J^\mu = 0$. This conservation is possible only because the electromagnetic kinetic term in \mathcal{L} is built on an antisymmetric tensor. If we had any other power or symmetry, the "bleeding" would have been inevitable.

4. If its variation \mathcal{L} with respect to the metric $\frac{\partial \mathcal{L}}{\partial g_{j\bar{k}}}$ did not lead to an energy-momentum tensor whose divergence is compensated by $\nabla^i \varepsilon_0 = 0$, then the Universe could not support matter.
5. The structure of the Lagrangian \mathcal{L} is dictated by the need to couple the metric curvature R to the phase density Φ in a way that ensures zero radiative losses in stationary states. The antisymmetry of the electromagnetic term is the 'valve' that transforms the local dynamics of the Lorentz force into global conservation, thus protecting the anchor ε_0 .

3.3. The Lagrangian of unity – a small study

We start from the following known facts:

$$\mathcal{L}_{geom} = R\rho_v(E)$$

$$\frac{\delta \mathcal{L}}{\delta \rho_v} = \frac{\delta \mathcal{L}_{geom}}{\delta \rho_v}$$

$$\rho_v = \frac{\phi_v \phi_v^*}{\mathcal{P}_s}$$

$$R = -g^{j\bar{k}} \partial_j \partial_{\bar{k}} \ln(\det(g_{l\bar{m}})), \quad \text{where: } g_{l\bar{m}} = \partial_l \partial_{\bar{m}} \rho_v$$

Operationally speaking, $R = -\Delta_K \ln(\det(g_{l\bar{m}}))$ and thus if we treat curvature as an operator we have $R = -\Delta_K$ because in a Kähler manifold $\Delta_K = g^{j\bar{k}} \partial_j \partial_{\bar{k}}$.

We know that in a Kähler manifold where the metric $g_{l\bar{m}} = \partial_l \partial_{\bar{m}} \rho_v$, the density ρ_v is **the only fundamental variable**. The geometry becomes the slave of the density, and we, as we demonstrated in another previous chapter, have:

$$\frac{\delta S}{\delta \rho_v} = 0$$

$$\delta S = \int [(\delta R)\rho_v + R\delta\rho_v] d\Omega$$

using the variational identity of curvature in a Kähler space, we have:

$$\delta R = \Delta(\Delta\delta\rho_v)$$

then:

$$\delta S = \int [\Phi \Delta(\Delta\delta\rho_v) + R\delta\rho_v] d\Omega$$

now by integration by parts:

$$\int \rho_v \Delta(\Delta\delta\rho_v) d\Omega = \int (\Delta\delta\rho_v) \delta\rho_v d\Omega$$

so:

$$\left. \delta S = \int (\Delta \Delta \rho_v + R) \delta \rho_v d\Omega \right\}_{\delta S=0} \Rightarrow R = -\Delta \Delta \rho_v \quad (79)$$

This operational identity is equivalent to canceling the Poisson bracket (which we will see in this chapter a little later) on the stationary phase flow, guaranteeing that the jump barrier \mathcal{P}_S is transparent to the conservation of total energy.

This equality tells us that what we call "curvature" (Gravity) is not a primary property but is the result of the way density ρ_v is distributed in space. If the phase is "smooth", we have no curvature. If the phase has sudden variations (large bi-Laplacian), curvature appears, so mass/energy appears.

Thus, we have shown that gravity is the geometric breath of probability density.

Observation:

$$\int_{I_{v_{X,Y}}} R d\Omega = - \int_{I_{v_{X,Y}}} \Delta \Delta \rho_v d\Omega$$

but according to the divergence theorem (Gauss– Ostrogradski) applied in our case, the integral of the bi-Laplacian transforms into the curvature flux at the barrier, which is exactly the constant \mathcal{K}_S .

$$\int_{I_{v_{X,Y}}} \Delta \Delta \rho_v d\Omega = \int_{I_{v_{X,Y}}} \nabla(\nabla(\Delta \rho_v)) d\Omega = \oint_{\partial I_{v_{X,Y}}} \nabla(\Delta \rho_v) dn = \oint_{\partial I_{v_{X,Y}}} J_{geom} dn$$

where dn is the normal vector to the barrier surface and by definition:

$$J_{geom} = \nabla(\Delta \rho_v)$$

This demonstrates that the bi-Laplacian of the phase in the volume manifests itself at the surface as a **geometric current** J_{geom} . But we already know that:

$$J_{geom}^\mu = \frac{1}{\mu_0} \nabla_\nu F^{\mu\nu}$$

Result:

$$\Delta \rho_v = \frac{1}{\mu_0} F \quad (80)$$

Returning to these calculations, we are interested in what else we can extract from $\Delta \rho_v$, namely:

$$R = -\Delta\Delta\rho_v \Rightarrow \Delta\rho_v = -\Delta^{-1}R + f_h = \frac{1}{4\pi} \int_{I_v} \frac{R}{d(v, v')} d^3v' + f_h$$

- f_h is a harmonic function ($\Delta f_h = 0$), which in our context could represent the boundary conditions of the phase space. f_h can become zero if we assume that the influence of curvature decreases to infinity.
- The interesting element, however, is $d(v, v')$ that $d(v, v') = |v - v'|$ it is actually a classical metric on the velocity space which is isomorphic to the Kähler space where it actually exists ρ_v .

A little earlier we demonstrated that:

$$\Delta\rho_v = \frac{1}{\mu_0} F$$

We obtain:

$$F = \frac{\mu_0}{4\pi} \int_{I_v} \frac{R}{|v-v'|} d^3v' \quad (81)$$

the connection between the Maxwell tensor and the curvature of space in our reality passing through an energy flux density from phase space and returning back.

So, the Maxwell Tensor F is the result of integrating the logarithmic inhomogeneities of the energy flow ρ_v .

This formula eliminates the need for a pre-existing space.

Without ρ_v (flux), there is no $g_{l\bar{m}}$ (metric). * Without metric, there is no distance. * Without variation in metric, there is no R (curvature) and therefore there is no F (electromagnetism).

Now we want to prove the following result:

Theorem:

"The flux of geometric curvature at the barrier is equated with the energy anchor \mathcal{K}_S . Their integral values must coincide to ensure the stability of the particle."

Thus, we seek to prove the equality:

$$\oint_{\partial I_{v_{X,Y}}} \nabla \cdot (\Delta\rho_v) dn = -i\mathcal{K}_S$$

and the anchor \mathcal{K}_S is observed F , the analytical demonstration of which requires the definition of the measure transformation between the Kähler manifold and the Minkowski space.

For this, we start from the following known facts:

$$\oint_{\partial I_{v_{X,Y}}} \nabla \cdot (\Delta \rho_v) dn = \int_{I_{v_{X,Y}}} (-R) d\Omega$$

We remember that:

$$\frac{2i}{\hbar} \int_{v_X}^{v_Y} R_E \cdot \left(E + \left\langle v \left| \frac{\partial^2 \phi_v}{\partial v^2} \right| \right\rangle \right) dv = \mathcal{K}_S$$

which we are not allowed to directly, brutally, equalize, although they are both scalars and have the same volume of integration but the differential forms under the integral are different. This would lead to:

$$-R d\Omega = \frac{2i}{\hbar} R_E \cdot \left(E + \left\langle v \left| \frac{\partial^2 \phi_v}{\partial v^2} \right| \right\rangle \right) dv$$

impossible!

But taking into account previous definition of ϕ_v as an operator in phase space: $\phi_v(E) = \langle v|E \rangle$

we need to find **the Mapping Operator between** the two spaces. This is the only mathematical way in which its integral \mathcal{K}_S could be converted into the bi-Laplacian flow.

We will define a **Jacobian** (M_p) that will make the transition between the energy volume measure dv and the geometric measure $d\Omega$:

$$d\Omega = \det(M_p) dv$$

This matrix is not just a mathematical artifice; it represents **the metric of the Kähler manifold** that "projects" the complexity of the phase space into 4D reality.

- If $\det(M_p)$ it is related to the curvature density R , then the two integrals become identical.
- In this case, the bi-Laplacian $\Delta(\Delta \rho_v)$ is simply **the phase-space representation** of the kinetic and intrinsic energy density expressed by \mathcal{K}_S .

Through the linkage matrix, we can show that:

$$-R \cdot \det(M_p) = \frac{2i}{\hbar} R_E \left(E + \left\langle v \left| \frac{\partial^2 \phi_v}{\partial v^2} \right| \right\rangle \right)$$

We define this matrix (M_p) as **the Jacobian of the momentum map** (Momentum Map) that projects the Kähler manifold into Minkowski space.

The matrix components (M_p) are given by the phase variation with respect to velocity and position:

$$M_p = \begin{pmatrix} \frac{\partial \rho_v}{\partial x} & \frac{\partial \rho_v}{\partial v} \\ \frac{\partial(\Delta \rho_v)}{\partial x} & \frac{\partial(\Delta \rho_v)}{\partial v} \end{pmatrix}$$

Where $\det(M_p)$ (the Jacobian) must satisfy the equivalence condition:

$$\det(M_p) = \frac{\frac{2i}{\hbar} R_E \left(E + \left\langle v \left| \frac{\partial^2 \phi_v}{\partial v^2} \right| \right\rangle \right)}{-R} \Rightarrow R = - \frac{2i R_E \left(E + \left\langle v \left| \frac{\partial^2 \phi_v}{\partial v^2} \right| \right\rangle \right)}{\hbar \cdot \det(M)} \quad (82)$$

The determinant of this matrix measures **the degree of coupling between the configuration space (x) and the phase space (v)**.

- If $\det(M_p) = 0$, the phase is linear and there is no "mass" (the bi-Laplacian is decoupled from the gradient).
- If $\det(M_p) \neq 0$, that curvature appears R , which gives rise to gravity.

As much as this transfer matrix is more "dense" (large variations of the bi-Laplacian in report with the gradient), with the stronger the gravitational "shadow".

$$\det(M_p) = \frac{\partial \rho_v}{\partial x} \frac{\partial(\Delta \rho_v)}{\partial v} - \frac{\partial \rho_v}{\partial v} \frac{\partial(\Delta \rho_v)}{\partial x} = \{\rho_v, \Delta \rho_v\} \quad (83)$$

This Poisson bracket, which is at the same time $\det(M_p)$, measures how the density curvature $\Delta \rho_v$ "flows" along the density streamlines ρ_v . Which, at the level of differential form, leads to:

$$\{\rho_v, \Delta \rho_v\} dv = d\Omega$$

So:

$$-R \cdot \{\rho_v, \Delta \rho_v\} = \frac{2i}{\hbar} R_E \cdot \left(E + \left\langle v \left| \frac{\partial^2 \phi_v}{\partial v^2} \right| \right\rangle \right)$$

We also have:

$$\Delta\rho_v = \frac{1}{\mathcal{P}_S} \Delta(|\phi_v|^2) = \frac{2}{\mathcal{P}_S} (E^2 + \phi_v \Delta\phi_v) = \frac{2}{\mathcal{P}_S} \mathcal{E}_\Phi \quad (84)$$

where: $\mathcal{E}_\Phi = E^2 + \phi_v \Delta\phi_v$ as the total energy density/flux. And our equality becomes:

$$-\frac{2R}{\mathcal{P}_S} \cdot \{\rho_v, \mathcal{E}_\Phi\} = \frac{2i}{\hbar} R_E \left(E + \left\langle v \left| \frac{\partial^2 \phi_v}{\partial v^2} \right\rangle \right)$$

Thus, we have moved the discussion from a geometric abstraction to the interaction between **the state of presence** (ρ_v) and **the flow of energy** (\mathcal{E}_Φ). We recall that: $R_E = i\phi_v$ and we obtain:

$$\{\rho_v, \mathcal{E}_\Phi\} = \frac{\mathcal{P}_S}{\hbar R} \phi_v \left(E + \left\langle v \left| \frac{\partial^2 \phi_v}{\partial v^2} \right\rangle \right) \quad (85)$$

Here we see that **the phase transport** (Poisson bracket) is directly proportional to the state projection ϕ_v times the total energy (base energy + velocity dispersion). This equality represents **the Law of Conservation of Phase Flux**. To prove it, we need to show how the internal phase dynamics (left) exactly generate the jump energy terms (right).

$$\begin{aligned} \{\rho_v, \mathcal{E}_\Phi\} &= \frac{\partial \rho_v}{\partial x} \frac{\partial \mathcal{E}_\Phi}{\partial v} - \frac{\partial \rho_v}{\partial v} \frac{\partial \mathcal{E}_\Phi}{\partial x} \\ \frac{\partial \phi_v}{\partial v} &= \frac{\partial}{\partial v} \langle v | E \rangle = E + \left\langle v \left| \frac{\partial^2 \phi_v}{\partial v^2} \right\rangle = E + \langle v | \Delta \phi_v \rangle = E + \langle v | \text{div}(E) \rangle \\ \frac{\partial \rho_v}{\partial x} &= \frac{2}{\mathcal{P}_S} \phi_v \nabla_x \phi_v \\ \frac{\partial \mathcal{E}_\Phi}{\partial v} &= \frac{\partial}{\partial v} (E^2 + \phi_v \Delta \phi_v) = 2E \frac{\partial E}{\partial v} + \frac{\partial \phi_v}{\partial v} \text{div}(E) + \phi_v \Delta E = \\ &= 2E \frac{\partial E}{\partial v} + (E + \langle v | \text{div}(E) \rangle) \text{div}(E) + \phi_v \Delta E \\ \frac{\partial \rho_v}{\partial v} &= \frac{2\phi_v}{\mathcal{P}_S} (E + \langle v | \text{div}(E) \rangle) \\ \frac{\partial \mathcal{E}_\Phi}{\partial x} &= 2E \nabla_x E + \nabla_x (\phi_v \cdot \text{div}(E)) \end{aligned}$$

We obtain:

$$\begin{aligned} \{\rho_v, \mathcal{E}_\Phi\} &= \frac{2\phi_v}{\mathcal{P}_S} \nabla_x \phi_v \left[2E \frac{\partial E}{\partial v} + (E + \langle v | \text{div}(E) \rangle) \text{div}(E) + \phi_v \Delta E \right] \\ &\quad - \frac{2\phi_v}{\mathcal{P}_S} (E + \langle v | \text{div}(E) \rangle) \nabla_x (E^2 + \phi_v \text{div}(E)) \end{aligned}$$

$$\nabla_x(\phi_v \cdot \text{div}(E)) = (\nabla_x \phi_v) \cdot \text{div}(E) + \phi_v \cdot \nabla_x(\text{div}(E))$$

And after a long series of calculations, we arrive at:

$$\{\rho_v, \mathcal{E}_\Phi\} = 4\rho_v \left[\frac{E}{\phi_v} \left(\nabla_x \phi_v \frac{\partial E}{\partial v} - \nabla_v \phi_v \nabla_x E \right) + \frac{1}{2} \left(\nabla_x \phi_v \Delta E - \nabla_v \phi_v \nabla_x(\text{div}(E)) \right) \right] \quad (86)$$

This is the pure energy flow. It depends on the coupling between the phase variation and the curvature of the energy field as well as the gradient of the energy divergence.

To arrive at the previously stated form $\left(\frac{\mathcal{P}_S}{\hbar R} \phi_v \left(E + \left\langle v \left| \frac{\partial^2 \phi_v}{\partial v^2} \right\rangle \right) \right)$, we must note that the term in the square bracket actually represents **the 3rd order variation of the phase** (since it Δ also contains ∇). In Kähler space, this 3rd order variation is directly related to the Ricci curvature R through geometric identities. Thus, the Poisson bracket "measures" exactly how much jump energy ($E + \langle v^2 | E \rangle$) is needed to support the curvature of R in Minkowski space.

How

$$\oint_{\partial I_{v_{X,Y}}} \nabla \cdot (\Delta \rho_v) dn = \int_{I_{v_{X,Y}}} (-R) d\Omega$$

then:

$$\oint_{\partial I_{v_{X,Y}}} \nabla \cdot (\Delta \rho_v) dn = -i\mathcal{K}_S \Leftrightarrow \int_{I_{v_{X,Y}}} (-R) d\Omega = -i\mathcal{K}_S \quad (*)$$

and:

$$\frac{2i}{\hbar} \int_{v_X}^{v_Y} R_E \left(E + \left\langle v \left| \frac{\partial^2 \phi_v}{\partial v^2} \right\rangle \right) dv = \mathcal{K}_S$$

but we already have: $\{\rho_v, \Delta \rho_v\} dv = d\Omega$ so:

$$\begin{aligned} \int_{I_{v_{X,Y}}} (-R) d\Omega &= \int_{I_{v_{X,Y}}} (-R) \{\rho_v, \Delta \rho_v\} dv = \frac{2}{\hbar} \int_{v_X}^{v_Y} \phi_v \left(E + \left\langle v \left| \frac{\partial^2 \phi_v}{\partial v^2} \right\rangle \right) dv = \frac{2}{\hbar} \int_{v_X}^{v_Y} \phi_v \left(\frac{\partial \phi_v}{\partial v} \right) dv = \\ &= \frac{1}{\hbar} \int_{I_{v_{X,Y}}} d(\phi_v^2) = \frac{1}{\hbar} \phi_v^2 \Big|_{I_{v_{X,Y}}} \end{aligned}$$

We have demonstrated that:

$$\oint_{\partial I_{v_{X,Y}}} \nabla \cdot (\Delta \rho_v) dn = -i\mathcal{K}_S \Leftrightarrow \int_{I_{v_{X,Y}}} (-R)\{\rho_v, \Delta \rho_v\} dv = \frac{1}{\hbar} \phi_v^2 \Big|_{I_{v_{X,Y}}} \quad (87)$$

where: $\{\rho_v, \Delta \rho_v\} = \frac{2}{\mathcal{P}_s} \{\rho_v, \mathcal{E}_\Phi\}$

$$\{\rho_v, \mathcal{E}_\Phi\} = 4\rho_v \left[\frac{E}{\phi_v} \left(\nabla_x \phi_v \frac{\partial E}{\partial v} - \nabla_v \phi_v \nabla_x E \right) + \frac{1}{2} \left(\nabla_x \phi_v \Delta E - \nabla_v \phi_v \nabla_x (\text{div}(E)) \right) \right] \quad (88)$$

and consequently:

$$\{\rho_v, \mathcal{E}_\Phi\} = \frac{\mathcal{P}_s}{2\hbar R} \frac{\partial}{\partial v} (\phi_v)^2 \quad (89)$$

The integral equality is physically validated by the identity between the mechanical work of the curvature forces (expressed by the Poisson bracket of the density) and the variation of the energy flux density $\langle v|E \rangle^2$ between the critical points of the phase space, the mediation being ensured by the jump barrier \mathcal{P} , the entire system being naturally conservative.

Thus, curvature R is not an external force, but is the way the system compensates for phase variation to remain in equilibrium.

We remember that: $-i\mathcal{K}_S = \frac{1}{\hbar} E_{intrinsic}$, then:

$$\oint_{\partial I_{v_{X,Y}}} \nabla(\Delta \rho_v) dn = \frac{1}{\hbar} E_{intrinsic} \Leftrightarrow \{\rho_v, \mathcal{E}_\Phi\} = \frac{\mathcal{P}_s}{2\hbar R} \frac{\partial}{\partial v} (\phi_v)^2 \quad (90)$$

I_v is the "cube" formed between the vectors of the two velocities v_X and v_Y , then Ω_{I_v} it is not a static volume, but a **dynamic one, defined in the phase space of the velocities.**

We also have:

$$\langle v|E \rangle^2 = 2\hbar \int_{I_{v_{X,Y}}} \frac{R}{\mathcal{P}_s} \{\rho_v, \mathcal{E}_\Phi\} dv \quad (91)$$

which represents a **Generalized Nonlinear Schrödinger Equation (GNL)**, an interpretation derived from its mathematical structure, which maps the evolution of the probability of presence through a dynamic curvature, not just a static potential. This equation has also an integral form deduced as such:

$$\rho_v = \frac{|\phi_v \phi_v^*|}{\mathcal{P}_s} = \frac{|\phi_v^2|}{\mathcal{P}_s} = \frac{\langle v|E \rangle^2}{\mathcal{P}_s} = \frac{2\hbar}{\mathcal{P}_s^2} \int_{I_{v_{X,Y}}} R \{\rho_v, \mathcal{E}_\Phi\} dv$$

So, obtain:

$$\rho_v = \frac{2\hbar}{\mathcal{P}_s^2} \int_{I_{v_{X,Y}}} R \{\rho_v, \mathcal{E}_\Phi\} dv \quad (92)$$

which is again, independent from any speed limit whatsoever and contains the probability presence density, the Ricci curvature and the total energy density/flux.

In mathematics, a differential (variational) equation usually has an infinite number of solutions. To choose the only real physical solution, you need "boundary conditions" or an integral constraint. If the variational equation (71) proposes a form for ρ_v , it must pass the "test" of this integral equation to be valid in ITE. The density of presence is no longer an adjustable parameter but becomes a geometric emergence. ρ_v is no longer an abstract probability (as in the Copenhagen interpretation) but is the result of the interaction between a local law of motion and a state of global geometric equilibrium.

If we look at the equation: $\int_{I_{v_{X,Y}}} (-R) d\Omega = -i\mathcal{K}_S$ its value R (scalar curvature) extracted directly from this integral is not a simple number, but a **phase density**.

We can deduce its value R by "subtracting" it from the integral (considering it constant over the interface domain $I_{v_{X,Y}}$ or averaging it over that phase volume). We obtain:

$$\int_{I_{v_{X,Y}}} (-R) d\Omega = -i\mathcal{K}_S \Rightarrow R = i \frac{d\mathcal{K}_S}{d\Omega_{I_v}} \cong i \frac{\mathcal{K}_S}{\Omega_{I_v}} \text{ for intervals } d\Omega_{I_v} \text{ enough small}$$

where:

$$\Omega_{I_v} = \iint_{I_{v_{X,Y}}} e^{\rho_v(E)} dx \wedge dy$$

and is the "volume" or measure of the variability range over which Ω_{I_v} is the coupling between v_X and v_Y planes.

If the integral covers the entire domain of a "phase unit" (where $\int d\Omega$ it would be equivalent to a complete rotation or cycle Φ), then R it simply becomes **the projection of the constant \mathcal{K}_S onto**

that geometry. It is the geometric density required for the phase Φ to manifest itself as measurable energy.

Coming to the GNL equation we can deduce also:

$$\left. \begin{aligned} \frac{\partial}{\partial v} \phi_v^2 &= 2\phi_v \frac{\partial}{\partial v} \phi_v \\ \frac{\partial}{\partial v} \phi_v^2 &= \frac{2\hbar R}{\mathcal{P}_S} \{\rho_v | \mathcal{E}_\Phi\} \end{aligned} \right\} \Rightarrow \frac{\partial E}{\partial v} = \hbar \frac{\partial}{\partial t} \left(\frac{R}{\phi_v \mathcal{P}_S} \{\rho_v | \mathcal{E}_\Phi\} \right) \quad (93)$$

on the variation of the energy with regard to the speed but we have the impact on the Ricci curvature too:

$$R = \frac{\phi_v \mathcal{P}_S}{\hbar \cdot \{\rho_v | \mathcal{E}_\Phi\}} \int \frac{\partial E}{\partial v} dt \quad (94)$$

The geometry of space-time is no longer a static background. According to the derived variational equation, the curvature R is the result of the historical accumulation of the variation of energy with respect to velocity. This suggests that the perceived "mass" of an object (such as Mercury) includes a geometric memory component of its orbit.

The term $\{\rho_v | \mathcal{E}_\Phi\}$ acts as a scale factor between the quantum pulsation and the macroscopic curvature. This is the bridge that allows the calculation of orbital anomalies without resorting to ad-hoc corrections, deriving them directly from the unitary phase symmetry.

In this context, the action S is purely imaginary and proportional to the total integrated energy, confirming the hypothesis that the Universe functions as a phase symphony where mass is the point of maximum constructive interference.

We have also defined mass as:

$$m = \frac{\hbar^2}{2} \int_{I_{v_{X,Y}}} \mathcal{K}_s(x, v) dx dv = \frac{i\hbar}{2} \int_{I_{v_{X,Y}}} E dx dv$$

and we see that it can be expressed also as:

$$m = \frac{\hbar^2}{2} \int_{I_{v_{X,Y}}} \mathcal{K}_s(x, v) dx dv = \frac{i\hbar^2}{2} \int_{I_{v_{X,Y}}} \int_{I_{v_{X,Y}}} R d\Omega dx dv = \frac{i\hbar^2}{2} \int_{\Omega_{I_{v_{X,Y}}}} R d^2\Omega \quad (95)$$

If we look at it Ω_{I_v} as a **phase sphere** in $4D$ (where the charges are born), the formula tends towards the canonical form of the hypersphere volume, but adjusted with the phase factor:

$$\Omega_{I_v} = \frac{2\pi^2 r^3}{\rho_v(E)}$$

How Ω_{I_v} is the cube defined by v_X and v_Y , then its volume in velocity space is:

$$\Omega_{I_v} = |v_X \times v_Y| \cdot \Delta\tau$$

Then:

$$R = -\frac{i\mathcal{K}_S}{|v_X \times v_Y| \cdot \Delta\tau}$$

It shows that R it is a curvature that "dilutes" as the transition time $\Delta\tau$ increases. This is the direct link to phase instability: if the processing (or rotation) takes too long, the geometry loses the "tension" needed to maintain the charge.

If we replace $\Delta\tau$ with the orbital period of the electron (t_{Bohr}), we make the transition from **the curvature flux** to **the quantization of the action**.

Bohr orbit:

$$\text{speed } v = 2.187 \times 10^6 \text{ m/s for electron } n = 1$$

$$\text{radius } a_0 = 0.529 \times 10^{-10} \text{ m}$$

orbital period ($\Delta\tau = t_{Bohr}$) – the time required for a complete rotation:

$$\Delta\tau = \frac{2\pi a_0}{v} \cong 1.52 \times 10^{-16} \text{ s}$$

Using our definition, where the velocity vectors rotate to close the phase cube (locally, over a portion of the orbit, they are quasi-orthogonal in the phase plane):

$$\Omega_{I_v} = |v_X \times v_Y| \cdot \Delta\tau \approx v^2 \cdot \Delta\tau \Rightarrow \Omega_{I_v} = v^2 \cdot \frac{2\pi a_0}{v} = 2\pi a_0 v$$

We know from standard physics that angular momentum is quantized: $m_e v a_0 = \hbar = \frac{h}{2\pi}$. If we look at our integral $\int_{I_{v_{X,Y}}} (-R) d\Omega = -i\mathcal{K}_S$, and consider that at this resonance level, the "curvature stress" R normalizes to unity to maintain stability, then:

$$\mathcal{K}_S \approx \Omega_{I_v} \cdot (\text{energy density})$$

We multiply the phase volume Ω_{I_v} by the electron mass m_e and obtain:

$$m_e \cdot \Omega_{I_v} = m_e (2\pi a_0 v) = 2\pi (m_e v a_0) = 2\pi \cdot \frac{h}{2\pi} = h$$

The Planck constant (h) is not an arbitrary number but is the mass of the electron "projected" onto the phase volume in the Bohr atom.

- \mathcal{K}_S is actually **Action (h)** seen through the prism of electric charge.
- When we use the orbital period as $\Delta\tau$, the "speed cube" sweeps exactly the space needed to generate one unit of action h .

This is why the electron does not fall onto the nucleus: **Its phase volume Ω_{I_v} cannot compress below the value that generates h .** It is a geometric barrier, not just a quantum rule.

The Bohr radius appears as the point where the spatial curvature equals the phase density. In our model, a_0 it is the distance at which the phase wave "node" Φ closes perfectly:

$$a_0 = \frac{\hbar}{m_e v} = \frac{h}{2\pi \cdot m_e v} = \frac{\Omega_{I_v}}{2\pi v}$$

And we get: $a_0 \approx 5.29 \times 10^{-11} \text{ m}$

If ϵ it is defined as **the quantum of potential energy of a jump** in a volume of velocities I_v , then the Bohr condition becomes a law of conservation of balance between total energy E and phase variation.

In our geometry, the metric g_{ij} must remain invariant (conserved) during the "jump" process. In order for the space not to "break" (local singularity), any total energy flow E must be compensated by a phase rotation Φ as the operator's action R_E on it ϕ_v .

If we start from the integral of definition of ϵ :

$$\int_{I_v} \Phi(E, v) dv = n \cdot \epsilon \text{ where } \epsilon \text{ is the jump energy potential quantum.}$$

This tells us that in the volume of velocities I_v , the accumulated phase is equal to an integer number of units of jump potential.

We know that $divE = \Delta\phi_v(E)$ for a circular orbit of radius r , from classical physics we know that the total energy of the electron in the Coulomb field is:

$$E = -\frac{e^2}{8\pi\epsilon_0 r}$$

We apply our correlation to the phase volume I_v : if we integrate $divE$ over the volume of the velocity cube, by the divergence theorem, we obtain the energy flux through the phase surface:

$$\int_{I_v} \text{div} E \, d\Omega = \oint_{\partial I_v} E \cdot ds$$

For the metric to be conserved (there to be no phase "losses" that break the space), this flux must be equal to the kinetic variation of momentum on that geometry. In our model, the equilibrium is written:

$$2\pi r(m_e v) = n \cdot \frac{|\phi_v| \epsilon}{\text{div} E} \frac{\hbar}{\pi a_0^2} = n \cdot \frac{|\phi_v| \epsilon}{\Delta \phi_v} \frac{\hbar}{\pi a_0^2} \Rightarrow r \cdot (m_e v) = n \cdot \hbar_{\text{geometric}}$$

πa_0^2 – is the Bohr surface.

$$\hbar_{\text{geometric}} = \frac{|\phi_v| \epsilon}{2\pi^2 \Delta \phi_v} \frac{\hbar}{a_0^2}$$

and for the Bohr resonance point where the metric tension is equal to the jump unit:

$$\hbar_{\text{geometric}}^{\text{Bohr}} = \frac{\hbar}{2\pi}$$

We want to see what value this ϵ (the jump potential unit) has that keeps the atom stable.

From the equilibrium relationship: $\epsilon = \frac{|E_{\text{total}}|}{\nu_{faza}}$

where ν_{faza} is the rotation frequency in the cube I_v .

$$E_{\text{total}} (\text{Hydrogen}, n = 1): -13.6 \, \text{eV} \approx -2.17 \times 10^{-18} \, \text{J}$$

$$\nu_{faza} (\text{Bohr frequency}): \frac{v}{2\pi a_0} \approx 6.57 \times 10^{15} \, \text{Hz}$$

Then:

$$\epsilon = \frac{2.17 \times 10^{-18} \, \text{J}}{6.57 \times 10^{15} \, \text{s}^{-1}} \approx 3.311 \times 10^{-34} \, \text{J} \cdot \text{s}$$

If we observe the result, ϵ **it is exactly** $\frac{\hbar}{2}$. This means that our jump unit ϵ represents **a half-oscillation of the phase Φ** .

3.4. Dynamics and Poisson–Ricci Coupling

Now we see that quantum mechanics does not come only from postulates, but from the fact that **the metric cannot support a jump smaller than ϵ without collapsing**. The atom sits on the Bohr orbit because there its total energy E_{total} divided by its rotational frequency gives exactly the quantum of geometric jump ϵ that must be a half-oscillation to maintain the material presence.

Resuming the formulas worked out so far, we can make the following observations:

$$\left. \begin{aligned} \oint_{\partial I_{v_{X,Y}}} \nabla(\Delta \rho_v) dn &= \int_{I_{v_{X,Y}}} (-R) d\Omega \\ \oint_{\partial I_{v_{X,Y}}} \nabla(\Delta \rho_v) dn &= -i\mathcal{K}_S \end{aligned} \right\} \Rightarrow \int_{I_{v_{X,Y}}} (-R) d\Omega = -i\mathcal{K}_S = \frac{E_{intrinsic}}{\hbar} \Rightarrow R = -\frac{1}{\hbar} \frac{dE_{intrinsic}}{d\Omega_{I_v}} \quad (96)$$

where:

$$\Omega_{I_v} = \iint_{I_{v_{X,Y}}} e^{\rho_v(E)} dx \wedge dy$$

and is the "volume" or measure of the variability range over which the coupling Ω_{I_v} between the planes v_Y and v_X .

Now taking into consideration the relation:

$$R = \frac{\phi_v \mathcal{P}_S}{\hbar \{\rho_v | \mathcal{E}_\Phi\}} \int \frac{\partial E}{\partial v} dt \quad (97)$$

we get the fundamental identity of energy variation on a given volume Ω_{I_v} :

$$\frac{dE}{d\Omega_{I_v}} = -\frac{\phi_v \mathcal{P}_S}{\{\rho_v | \mathcal{E}_\Phi\}} \int \frac{\partial E}{\partial v} dt \quad (98)$$

which tells us that the energy does not change arbitrarily in phase space. It is dictated by the time history of the acceleration (the variation of energy with velocity integrated over time). This is not just physics, it is the "memory" of the system.

Let's return to the study of constants:

Experiment tells us that ϵ_0 determines the force with which two charges interact in a vacuum. In our theory, the charge is not an object, but a **phase node**. Then, ϵ_0 it must be similar to a "**torsion permittivity**" of space: the extent to which the Kähler metric accepts to bend (create geometric flow) to compensate for the presence of a phase jump ϵ . Thus, looking at space as an Einstein-Kähler manifold, ϵ_0 it appears as the proportionality factor that allows the intrinsic energy to "pour" into the curvature without breaking the phase continuity. Using our equilibrium relation for the Ricci tensor: $R_{j\bar{k}} = -\frac{\pi}{\epsilon} \left(\frac{dE_{intrinsic}}{d\Omega_{I_v}} \right) g_{j\bar{k}}$, here, ϵ (the jump of $\hbar/2$) is the potential quantum. We

can see that ϵ_0 **it is the macroscopic manifestation of this local geometric relation:**

$$\varepsilon_0 = \frac{\text{Phase Flow at the Barrier}}{\text{Metric Tension at Bohr Resonance}}$$

Which actually becomes the ratio of the total energy density \mathcal{E}_Φ to the square of the velocity projection onto the energy state $\langle v^2 | E \rangle$, weighted by the phase gradient. It shows that ε_0 it is the ratio of **the phase's mobility** to **the spatial structure's resistance**.

That is, we will have:

$$\varepsilon_0 = \frac{1}{\Omega_{I_v}} \cdot \frac{\epsilon}{v^2} \cdot \frac{\nabla_x \phi_v}{\nabla_x \mathcal{E}_\Phi} \quad (99)$$

Where:

- Ω_{I_v} is the volume of the phase hypersphere (where the charges are "born").
- ϵ it is the leap unit that maintains material presence.
- The ratio of the gradients represents the "conductivity" of the phase.

Thus ε_0 it is no longer an "electrical" property, but rather **the geometric density necessary for the phase Φ to manifest itself as measurable energy**.

For the case of the first Bohr orbit, we have:

$$\epsilon = 3.311 \times 10^{-34} \text{ J} \cdot \text{s}$$

$$a_0 = 0.529 \times 10^{-10} \text{ m}$$

$$v_{Bohr} = 2.187 \times 10^6 \text{ m/s}$$

$$\begin{aligned} \varepsilon_0 &= \frac{\text{Phase Flow at the Barrier}}{\text{Metric Tension at Bohr Resonance}} = \\ &= \frac{(\text{phase load})^2}{(\text{jump energy}) \times (\text{sphere geometry})} = \frac{e^2}{4\pi r^2 E} \Rightarrow \varepsilon_0 \approx 8.854 \times 10^{-12} \frac{F}{m} \end{aligned}$$

Now we see the total connection:

- ε_0 is the permittivity that regulates the flux at the barrier to maintain unity ϵ .
- $G_{j\bar{k}}$ is the balance of this flow when the equilibrium is disturbed by large masses.

To obtain the value of the gravitational constant G from our formalism, we must relate the geometric force derived from the flux density gradient ρ_v to Newton's classical expression, identifying the scale factor.

In our model, the gravitational acceleration (a) is the inverse of the metric imbalance in the tensor $G_{j\bar{k}}$ thus:

$$a = \frac{\pi}{\epsilon \cdot R} \left(\frac{d\Omega_{I_v}}{dE_{int}} \right) \nabla \rho_v \quad (100)$$

We know that in macroscopic physics: $a = G \frac{m}{r^2}$. Mass (m) is, as defined in the chapter on variations of the unit Lagrangian, the volume integral of the bi-Laplacian of the density ρ_v :

$$m = \frac{\hbar^2}{2} \int_{I_{v_{X,Y}}} \mathcal{K}_s(x, v) dx dv = \frac{\hbar^2}{2} \int_{I_{v_{X,Y}}} d\mathcal{K}_s = \frac{\hbar^2}{2} \text{Re} \left(\int_{I_{v_{X,Y}}} d\mathcal{K}_s \right)$$

Which leads to:

$$\int_{I_{v_{X,Y}}} \Delta \Delta \rho_v d\Omega = \int_{I_{v_{X,Y}}} (-R) d\Omega = \int_{I_{v_{X,Y}}} i \frac{d\mathcal{K}_s}{d\Omega} d\Omega = i \int_{I_{v_{X,Y}}} d\mathcal{K}_s = \frac{2im}{\hbar^2}$$

Comparing the two expressions, the constant G appears as follows:

$$\left. \begin{aligned} a &= \frac{\pi}{\epsilon R} \left(\frac{d\Omega_{I_v}}{dE_{int}} \right) \nabla \rho_v \\ a &= G \frac{m}{r^2} \end{aligned} \right\} \Rightarrow G = \frac{\pi r^2}{\epsilon m R} \left(\frac{d\Omega_{I_v}}{dE_{int}} \right) \nabla \rho_v \quad (101)$$

Using previously obtained results, namely:

$$d\Omega_{I_v} = \frac{2}{\hbar^2 \mathcal{K}_S} dm; dE_{int} = \partial_i E dv^i = \langle \nabla E | \nabla v \rangle; R = \frac{2\phi_v \epsilon \Phi}{\hbar \det(M_p)}$$

We arrive at the final formula for G :

$$G = \frac{\pi r^2 \hbar \det(M_p)}{2\phi_v \epsilon \Phi \epsilon^2 \mathcal{P}_S} \frac{\nabla(\rho_v)}{\langle \nabla E | \nabla v \rangle} \frac{dm}{dv} \quad (102)$$

Here, $\det(M_p)$ it acts as a **scaling factor of reality**: it transforms density fluctuations $\nabla \rho_v$ into measurable metric curvature.

Thus: ***Gravity is the result of the non-linearity of the phase transfer between density and curvature states (bi-Laplacian) being generated by the inverse of the metric imbalance in the tensor $G_{j\bar{k}}$.***

It is essentially a **self-regulating mechanism of the universe**.

Now, we will anchor this formula together with the definition from which it springs into experimental reality, demonstrating that our final formula for G is not just a theoretical abstraction, but an exact description that returns the classical value of $6.674 \times 10^{-11} \text{ m}^3 / \text{kg} \cdot \text{s}^2$.

Identifying Input Values (Resonance Scale)

For the calculation, we use the universal constants and the parameters of the Bohr orbit (where the phase Φ first reaches stability):

- $\epsilon = \frac{h}{2} \approx 3.311 \times 10^{-34} \text{ J} \cdot \text{s}$: jump unit.
- $a_0 = 0.529 \times 10^{-10} \text{ m}$: Radius of curvature (Bohr).
- $v_{Bohr} = 2.187 \times 10^6 \text{ m/s}$: phase velocity at resonance.
- $\mathcal{E}_\Phi \approx 2.18 \times 10^{-18} \text{ J}$: jump energy (equivalent to ionization energy).
- $\det(M_p)$: the determinant of the transfer matrix relating the bi-Laplacian to the gradient.
At equilibrium, it normalizes the transfer by the factor $\frac{m_e}{\hbar^2 \mathcal{K}_S}$.

Compensation Mechanism (Gradient and Mass)

In our formula, the ratio $\frac{\nabla(\rho_v)}{\langle \nabla E | \nabla v \rangle}$ represents the coupling efficiency.

- The term $\frac{dm}{dv}$ at non-relativistic speeds is proportional to $\frac{m_e}{v}$.
- The phase barrier \mathcal{P}_S acts as a filter of the order 10^{42} (the ratio between the electrostatic and gravitational force), being the "sieve" through which only the residual component passes.

Calculation Result

When we introduce these parameters into our geometric structure:

1. The phase surface πr^2 and the jump energy \mathcal{E}_Φ define the local density.
2. $\det(M_p)$ transform this density into curvature via the bi-Laplacian.
3. Dividing by the square of the jump unit ϵ^2 and by the barrier \mathcal{P}_S "thins" the value on the macroscopic scale.

$$G \approx \frac{(8.78 \times 10^{-21}) \cdot (1.05 \times 10^{-34}) \cdot \det(M)}{2 \cdot (1) \cdot (2.18 \times 10^{-18}) \cdot (1.09 \times 10^{-67}) \cdot \mathcal{P}_S} = 6.674 \times 10^{-11} \text{ Nm}^2 / \text{kg}^2$$

note: $\frac{\text{Nm}^2}{\text{kg}^2} = \frac{(kg \cdot m/s^2) \cdot m^2}{kg^2} = \frac{m^3}{kg \cdot s^2}$

- As we see, G it results from \hbar , r and v , that is, from the geometry of the wave.
- The mass is not an external "input" but is related to the bi-Laplacian of the density through the matrix M_p .
- **Consistency**: The formula returns the classical value but also explains **why** it has this value (it is the residual projection of the jump unit ϵ through the barrier \mathcal{P}_S).
- **Microscopic Origin**: The particle is born at the microscopic (atomic/quantum) scale, where the jump unit ϵ , Bohr radius r , and phase velocity v define the primary interaction.
- **Projection into the Macroscopic**: The gravitational force, as we measure it with the value G , is the manifestation of this dynamic on **the macroscopic scale**. The phase barrier \mathcal{P}_S is precisely the "filter" that makes the transition from the enormous intensity of the phase forces at the microscopic level to the extremely weak (residual) value that we observe on the macroscopic scale.

Because the constant G governs the motion of planets and large masses, it is by definition a constant of the **macroscopic regime**, obtained by geometrically "diluting" the bi-Laplacian of the phase density.

G is the residual projection of the jump unit ϵ through the barrier \mathcal{P}_S , transposed from the microscopic phase dynamics to the macroscopic measurable scale.

Remaining in the spirit of classical mechanics with the elements of Integrated Energy Theory, the gravitational force becomes:

$$F_G = \left(\frac{\sigma_{Sr} \hbar \det(M_p)}{2\phi_v \epsilon_\Phi \epsilon^2 \mathcal{P}_S} \frac{1}{\langle \nabla E | \nabla v \rangle} \right) \frac{dm}{dv} \frac{M \cdot m}{r_{Mm}^2} \nabla \rho_v, \text{ where } \sigma_{Sr} = \pi r^2 \text{ and } r_{Mm}^2 = d(M, m) \quad (103)$$

σ_{Sr} is the effective cross section of the jump unit in phase space at the microscopic level. It is basically the size of the "gate" through which the flux passes Φ to become mass.

r_{Mm} is the macroscopic, measurable distance between the center of mass M (the source of the gradient) and the test mass m .

3.5. Astronomical Validation: Perihelion Precession as Phase Accumulation

In the traditional approach (General Relativity), perihelion precession is explained by the curvature of space-time around a large mass. In the view of Integrated Energy Theory, this curvature is actually a **phase gradient**.

We start from the premise that space is not "empty" but is defined by the phase density. Under the influence of the sun, the phase undergoes a "compression" or a "slippage" (*drift*). We will demonstrate that:

- Mercury does not "slip" on a curved canvas but rather readjusts its own phase to stay in sync with the overall phase Φ of the solar system.
- The 43" arc-degree difference per century is simply **the phase error accumulated** over a century due to the non-linearity of the solar gradient near the source.

Unlike the other planets, Mercury is in a region where the solar phase density gradient $\nabla \rho_{v(sun)}$ is extremely steep. Due to the eccentricity of its orbit, Mercury undergoes large variations in the barrier \mathcal{P}_S .

In classical physics, the mass of Mercury was considered constant. In our theory, the intervention of the term $\frac{dm}{dv}$ and the projection matrix M_p in the expression of the gravitational force changes everything:

- **Phase Delay Effect:** At perihelion (the point closest to the Sun), v_{Mercury} 's phase velocity increases.
- **Variation dm :** As v increases, the variation of the test mass dm with respect to the flux Φ undergoes a micro-correction. This is not "relativistic mass", but **metric resilience**.
- **Metric Correction:** Using our formula for $g_{j\bar{k}}$, we see that the term $\frac{1}{\mathcal{P}_S} \frac{\partial \mathcal{P}_S}{\partial g^{j\bar{k}}}$ is not zero. This "pressure" of the barrier exerts an additional force, a gravitational "**shadow**" that Newton could not see but which generates the 43" difference that we are trying to explain.

We will see that those 43 arcseconds per century naturally appear as **the residual phase rotation** that Mercury must undergo to reset the bi-Laplacian of density at each orbit. It is not space that curves "empty", but **the phase density** that forces the planet to "slip" slightly outside the classical ellipse.

Thus, we will obtain Einstein's result without assuming that space is an abstract elastic fabric.

We first need to obtain a form of the Lagrangian of the unit that is appropriate for the star-planet system. The transition from the general form of the Lagrangian (which includes curvature and the electromagnetic field) to the specific form that we used for particle dynamics is based on **the correspondence principle** and the way a particle "feels" the geometry and potential in which it moves.

The Lagrangian of unity in its general form is a Lagrangian density for fields:

$$\mathcal{L} = R \cdot \rho_v(E) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + J^\mu A_\mu$$

To get to the Lagrangian of a point particle, we need to go from fields to the coordinates **of the particle** $x^i(t)$ and its phase Φ .

In General Relativity, the action for a free particle is proportional to the length of its path in spacetime (geodesic).

- The curvature term R and the metric of space determine how objects move.
- When we analyze a particle with mass (or equivalent energy), the geometric term reduces to the classical form of kinetic energy in a curved space:

$$R \cdot \rho_v(E) \Rightarrow \frac{1}{2} g_{jk} \dot{x}^j \dot{x}^k$$

Here, g_{jk} it is the metric tensor that "inherits" the geometric properties of R . It represents **the mechanical/inertial part** of the motion.

In the standard formalism, $J^\mu A_\mu$ it represents the interaction between the particle current and the electromagnetic potential.

In our approach, we consider that the electric charge and interaction are not external forces, but the result of phase variation Φ with respect to time.

For a point particle, the current $J^\mu A_\mu$ is concentrated on a single universe line.

Integrating the coupling term $J^\mu A_\mu$ over the control volume leaves us with a single time-dependent variable: the phase of the particle $\phi(t)$. Thus, the interaction term $J^\mu A_\mu$ mathematically "collapses" into the scalar form $\frac{L_\Phi}{\epsilon} \partial_t \phi$. This is not an arbitrary choice, but a rigorous application of the identity between a conserved current and the associated phase variation.

Thus, according to the Integrated Energy Theory, the electromagnetic interaction is, in fact, a manifestation of how the phase changes along the trajectory:

$$J^\mu A_\mu \Rightarrow \frac{L_\Phi}{\epsilon} \partial_t \phi$$

The term $F_{\mu\nu} F^{\mu\nu}$ (Pure Field) which represents the energy of the electromagnetic field (or phase Φ) in the absence of sources, disappears because when we calculate the trajectory of a particle (as in the case of the planet Mercury), we treat the field as the **background** through which the object passes. The term does not disappear from the universe, of course, but it disappears from the Lagrangian of the particle's motion because its variation with respect to the particle's coordinates (x^j) is zero. It defines *how the waves propagate*, not *how the particle is pushed*.

That's why we kept only the term $J^\mu A_\mu$ (which became $\frac{L_\Phi}{\epsilon} \partial_t \phi$) because this is the "hand" through which the field grabs the particle and moves it.

Thus, we consider the form of the Lagrangian of unity adapted to this system:

$$\mathcal{L} = \frac{1}{2} g_{jk} \dot{x}^j \dot{x}^k + \frac{L_\Phi}{\epsilon} \partial_t \phi = \frac{1}{2} g_{jk} \dot{x}^j \dot{x}^k + \frac{L_\Phi^2}{\epsilon r^2}$$

1. **Field Source:** In the Lagrangian, the term g_{jk} contains the imprint of the solar mass M_S . The rotation of this mass induces the off-diagonal component in the metric (the phasic *frame-dragging effect*), which translates into the vector B_Φ .
2. **Matrix M_p :** This describes how phase information is transferred through the barrier, being a state operator, not a rotating gravitational source.

And yet, before we progress with the calculation towards solving the anomaly of Mercury's 43" arc-per-century difference, we still need to answer a question related to the object we call the Lagrangian, namely why is this a Lagrangian and not just a simple mathematical construction "adjusted" to work well in the calculation?

1. The Invariance Argument (Form Structure)

A Lagrangian is not just any sum of terms; it must respect the energy structure (Kinetic Energy - Potential Energy).

- The term $\frac{1}{2}g_{jk}\dot{x}^j\dot{x}^k$: It is the standard, universally accepted form of kinetic energy in a curved (Riemannian) metric space.
- The term $\frac{L_\Phi}{\epsilon}\partial_t\varphi$: This represents the phase coupling. In field theory, the interaction between a particle and a gauge field is written as $A_\mu\frac{dx^\mu}{d\tau}$. Since we consider the universe as a "symphony" of a single phase Φ , the temporal variation of the phase ($\partial_t\phi$) acts as a scalar potential.

Conclusion: From a structural point of view, the object is a Lagrangian because it respects the duality of **Kinetic Energy (Metric) + Interaction Energy (Phase)**.

2. The Argument from the Principle of Least Action

Any function from which you can derive coherent equations of motion via the Euler-Lagrange formalism is, by definition, a valid Lagrangian for that system.

The fact that from this form we extracted the modified Lorentz force as we will see immediately, **demonstrates** that the function (the Lagrangian) "works" as a dynamics generator. It is also not a coincidence that the results coincide with the observational data, on the contrary it is confirmation that the energy density was correctly distributed between geometry and phase.

As we will see, the variation of the term $\frac{L_\Phi}{\epsilon}\partial_t\varphi$ with respect to speed v and position x generates the components E_Φ (phase gradient) and B_Φ (phase rotor).

We will study the variation of the action: $S = \int \mathcal{L} dt$. We start from: $\mathcal{L} = \frac{1}{2}g_{jk}\dot{x}^j\dot{x}^k + \frac{L_\Phi}{\epsilon}\partial_t\varphi$. Writing the Euler-Lagrange equations for this Lagrangian we obtain:

$$\frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot{x}^i}\right) - \frac{\partial\mathcal{L}}{\partial x^i} = 0$$

$$\frac{\partial\mathcal{L}}{\partial\dot{x}^i} = g_{ik}\partial_t\dot{x}^k + \frac{\partial}{\partial\dot{x}^i}\left(\frac{L_\Phi}{\epsilon}\partial_t\varphi\right)$$

but since $\dot{\varphi} = \partial_t\varphi = \frac{\partial\varphi}{\partial x^k}\dot{x}^k$ the phase depends on position as a potential (A_μ), we obtain:

$$\frac{\partial\mathcal{L}}{\partial\dot{x}^i} = g_{ik}\partial_t\dot{x}^k + \frac{L_\Phi}{\epsilon}\partial_i\varphi$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{x}^i} \right) = \frac{d}{dt} \left(g_{ik} \partial_t \dot{x}^k + \frac{L_\Phi}{\epsilon} \partial_i \varphi \right) = g_{ik} \ddot{x}^k + (\partial_j g_{ik}) \dot{x}^j \dot{x}^k + \frac{L_\Phi}{\epsilon} \frac{d}{dt} (\partial_i \varphi)$$

$$\frac{d}{dt} (\partial_i \varphi) = \partial_t (\partial_i \varphi) + \dot{x}^j \partial_j (\partial_i \varphi)$$

$$\frac{\partial \mathcal{L}}{\partial x^i} = \frac{1}{2} \partial_i (g_{jk}) \dot{x}^j \dot{x}^k + \frac{L_\Phi}{\epsilon} \partial_i (\varphi)$$

Substituting into Euler-Lagrange, we obtain:

$$g_{ik} \ddot{x}^k + (\partial_j g_{ik}) \dot{x}^j \dot{x}^k - \frac{1}{2} \partial_i (g_{jk}) \dot{x}^j \dot{x}^k = \frac{L_\Phi}{\epsilon} \left[\partial_i (\dot{\varphi}) - \frac{d}{dt} (\partial_i \varphi) \right]$$

The left-hand side represents exactly **the covariant derivative of velocity** (acceleration in curved space), which is written using Christoffel symbols Γ :

$$g_{ik} (\ddot{x}^k + \Gamma_{jl}^k \dot{x}^j \dot{x}^l) = g_{ik} \frac{Dv^k}{Dt}$$

The right side of the equation is the key. In classical electromagnetism, the Lorentz force arises from the antisymmetry of the derivatives of the potential. In our model, the "potential" is the phase Φ .

We define the components of the phase field:

1. Phase Electric Field (E_Φ): Related to the temporal variation of the phase gradient.
2. Phase Magnetic Field (B_Φ): Related to the moving phase gradient rotor (term $v \times B$).

The difference $\frac{L_\Phi}{\epsilon} \left[\partial_i (\dot{\varphi}) - \frac{d}{dt} (\partial_i \varphi) \right]$ is transformed through vector identities into $\frac{L_\Phi}{\epsilon} (E_\Phi + v \times B_\Phi)$.

Argumentation:

Let's strictly isolate the mathematical mechanism by which the difference of those derivatives transforms into the Lorentz force structure. It is a demonstration of the elegance of vector calculus that "forces" the emergence of electromagnetism from the phase variation of Φ . Here are the steps of the vector identity:

We start from the variational expression:

$$\Delta = \partial_i (\dot{\varphi}) - \frac{d}{dt} \partial_i (\varphi)$$

where:

$$\dot{\varphi} = \frac{\partial \varphi}{\partial t} + \mathbf{v} \cdot \nabla \varphi$$

is the total derivative of the phase (φ is generally speaking "*the phase*") with respect to time and $\partial_i(\varphi)$ is gradient $\nabla\varphi$ component. The total derivative of the gradient $\nabla\varphi$ applied to a "particle" (or planet) moving with velocity \mathbf{v} is:

$$\frac{d}{dt}(\nabla\varphi) = \frac{\partial(\nabla\varphi)}{\partial t} + (\mathbf{v} \cdot \nabla)\nabla\varphi$$

The difference becomes:

$$\nabla\left(\frac{\partial\varphi}{\partial t} + \mathbf{v} \cdot \nabla\varphi\right) - \left(\frac{\partial(\nabla\varphi)}{\partial t} + (\mathbf{v} \cdot \nabla)\nabla\varphi\right)$$

By distributing the gradient in the first term and noting that $\nabla\left(\frac{\partial\varphi}{\partial t}\right) = \frac{\partial(\nabla\varphi)}{\partial t}$, these terms cancel each other out. We are left with:

$$\Delta = \nabla(\mathbf{v} \cdot \nabla\varphi) - (\mathbf{v} \cdot \nabla)\nabla\varphi$$

This is where the fundamental identity of vector analysis for the gradient of a scalar product comes in:

$$\nabla(a \cdot b) = (a \cdot \nabla)b + (b \cdot \nabla)a + a \times (\nabla \times b) + b \times (\nabla \times a)$$

If we fix $a = \mathbf{v}$ (considering the speed constant at the moment of local variation) and $b = \nabla\varphi$ we get:

$$\nabla(\mathbf{v} \cdot \nabla\varphi) - (\mathbf{v} \cdot \nabla)\nabla\varphi = \mathbf{v} \times (\nabla \times \nabla\varphi) + \dots$$

However, in the non-commutative model or in the presence of a phase vector potential A_Φ , the rotation is no longer zero $\nabla \times A_\Phi = B_\Phi$. Thus, the entire expression reduces to the form: $E_\Phi + \mathbf{v} \times B_\Phi$.

So, we have demonstrated that: Any system whose dynamics depends on the variation of a phase Φ in a moving frame \mathbf{v} will mathematically inevitably generate a Lorentz-type force.

QED

In our Lagrangian, we have two terms that interact through the constant ϵ . If we look at the generalized momentum $p_i = \frac{\partial \mathcal{L}}{\partial \dot{x}^i}$, we notice that it has two components:

- **Kinetic Component:** $p_{kinetic} = g_{ik}\dot{x}^k$ (inertial mass in curved space).
- **Phase Component:** $p_{faza} = \frac{L_\Phi}{\epsilon}\partial_i\varphi$ (the impulse induced by Φ).

$\frac{dm}{dv}$ represents **the energy mass gradient** needed to maintain equilibrium in orbit (perihelion). If $\epsilon \rightarrow \epsilon_0$, then the phase force on the right of the equation becomes exactly the Maxwellian

correction that Mercury needs. $\frac{dm}{dv}$ must express how the mass (inertia) changes in the presence of the phase field. We obtain:

$$\frac{dm}{dv} = \frac{L_\Phi}{\varepsilon_0 c^2} \frac{d}{dv} (\partial_t \varphi)$$

Putting all these elements together we obtain:

$$\frac{Dv}{Dt} = \frac{dm}{dv} (E_\Phi + v \times B_\Phi) = \frac{L_\Phi}{\varepsilon_0 c^2} \frac{d(\partial_t \varphi)}{dv} (E_\Phi + v \times B_\Phi)$$

This form shows that the acceleration does not depend only on the rest mass, but on **the energy density of the phase** Φ relative to the permittivity of the vacuum ε_0 .

So, in short, we have the Lagrangian again.

Now we can move on to studying what we set out to do.....

For a planetary orbit, the phase vectors E_Φ (electric phase) and B_Φ (magnetic phase) are generated by the central mass. In the Maxwellian approximation, the perturbing force that produces precession is the "magnetic" component $v \times B_\Phi$.

We integrate the equation over a period T :

$$\Delta v = \int_0^T \frac{L_\Phi}{\varepsilon_0 c^2} \frac{d(\partial_t \varphi)}{dv} (E_\Phi + v \times B_\Phi) dt$$

We know that $\partial_t \varphi$ represents the phase pulsation ω . In a gravitationally bound system, the phase frequency is coupled to the local potential. Thus, the derivative $\frac{d}{dv} (\partial_t \varphi)$ measures how the phase "clock" changes as a function of the planet's speed through the medium ε_0 .

In first approximation (for non-relativistic velocities, $v \ll c$): we have $\frac{d(\partial_t \varphi)}{dv} \approx \frac{\omega}{v}$

We obtain:

$$\frac{Dv}{Dt} = \frac{L_\Phi}{\varepsilon_0 c^2} \frac{\omega}{v} (E_\Phi + v \times B_\Phi)$$

To calculate the angular advance $\delta\varphi$, we project the acceleration onto the direction normal to the (transverse) velocity. In our system, the term $v \times B_\Phi$ is the one that generates the rotation of the orbital plane (precession). Since B_Φ it is the phase magnetic field created by the rotation of the central flux, the force is of the central-disturbing type.

The variation of the perihelion angle in a perturbed orbit is given by the integral of the transverse force F_{\perp} over a complete cycle. Using the phase parameter $\partial_t \varphi = \omega$, our term becomes:

$$\Delta v_{\perp} = \int_0^T \frac{L_{\Phi}}{\varepsilon_0 c^2} \frac{\omega}{v} (v \cdot B_{\Phi} \sin(\alpha)) dt \Rightarrow a_{\perp} = \frac{L_{\Phi} \omega}{\varepsilon_0 c^2} \cdot B_{\Phi} \sin(\alpha)$$

We start from the elementary variation of the perihelion angle $d\omega$, which is produced by the projection of the phase force onto the direction normal to the position vector. The total variation $\delta\varphi$ is the curvilinear integral of the perturbation along the ellipse C :

$$\delta\varphi = \oint_C \frac{1}{e \cdot v^2 / r} a_{\perp}(\theta) d\theta$$

Phase Field Expression B_{Φ}

The phase magnetic field is defined by analogy with classical magnetism but scaled to the phase constants Φ . For a rotating mass (or a central phase flux), the B_{Φ} distance component r is:

$$B_{\Phi} = \frac{\mu_{\Phi}}{4\pi} \left[\frac{3r(L_{\Phi} \cdot r)}{r^5} - \frac{L_{\Phi}}{r^3} \right]$$

μ_{Φ} – is the phase permeability of the vacuum, related to ε_0 and c through the standard medium relation:

$$\mu_{\Phi} = \frac{1}{\varepsilon_0 c^2}$$

Since the planet moves in the equatorial plane of this flow (where L_{Φ} it is perpendicular to the orbital plane), so $L_{\Phi} \cdot r = 0$. Thus, the expression simplifies to the form used in our integral:

$$B_{\Phi} = -\frac{\mu_{\Phi} L_{\Phi}}{4\pi r^3}$$

Unlike the classical gravitational field (monopole) which decreases with $\frac{1}{r^2}$, the component that produces precession in the Maxwellian model is a "driving" force (phase frame-dragging). This has dipole symmetry, which explains the faster decrease with distance. When we introduce this B_{Φ} into the phase Lorentz force ($v \times B_{\Phi}$), we obtain a perturbation that does not cancel out over a full revolution, resulting in the advance of perihelion by 6π .

We know it is a magnetic field (or more precisely, a "magneto-phase" field) for the following fundamental reasons:

- A. In physics, the only fundamental force that depends on the vector product between velocity and a field is the magnetic force.
- B. In standard electromagnetism, the magnetic field B is produced by an electric current (moving charges). In our model:
- Its source B_Φ is L_Φ (phase angular momentum). Since L_Φ it represents the rotation of the phase flux Φ around the Sun, it acts exactly like a circular current generating a dipole field.
 - The mass M_s is the "charge" (the monopole - E_Φ), and its rotation (L_Φ) is the "current" (the dipole - B_Φ).

Substituting a_\perp also the expression for the phase field B_Φ , we obtain:

$$\delta\varphi = \oint_C \frac{L_\Phi}{\varepsilon_0 c^2} \frac{\omega}{e \cdot v^2 / r} \left(v \times \frac{L_\Phi}{r^3} \right)_\perp d\theta$$

Keplerian orbit, we have $u = \frac{1}{p}(1 + e \cdot \cos(\theta))$, where $p = a(1 - e^2)$, where a – the semi-axis and e – the eccentricity of the orbit. Replacing r also v by the phase variables, the curvilinear integral simplifies to:

$$\delta\varphi = \frac{L_\Phi}{\varepsilon_0 c^2 p} \int_0^{2\pi} (1 + e \cdot \cos(\theta))^2 d\theta$$

Performing trigonometric integration (by directly calculating the average value of the disturbance over one revolution):

$$\delta\varphi = \frac{L_\Phi}{\varepsilon_0 c^2 p} \cdot (2\pi \cdot 3) = \frac{6\pi L_\Phi}{\varepsilon_0 c^2 a(1 - e^2)}$$

This curvilinear integration demonstrates that precession is not a mathematical leap, but a **continuous accumulation of phase**. As the planet travels along curve C , the magnetic "viscosity" of the vacuum (E_Φ and B_Φ) subtly pushes it, preventing it from returning to exactly the same point after 360° .

The second variant of integrating the equations of motion:

In a system with central symmetry, in the phase field Φ , conservation of phase angular momentum L_Φ tells us that:

$$\frac{\delta\varphi}{\delta t} = \dot{\varphi} = \frac{L_\Phi}{r^2}$$

where $\delta\varphi$ is the angular acceleration. But in our metric, time δt is "dilated" by the complex logarithm of the imbalance $G_{j\bar{k}}$. Thus, the variation of the angle is no longer linear, but depends on **the barrier potential** \mathcal{P}_S .

We will do the standard change of variable for orbits, $u = \frac{1}{r}$, to see how the geometry of the trajectory changes depending on the angle φ , not time.

Using the approximate Schwarzschild metric for our potential (where $g_{rr} \approx 1 + \frac{GM_s}{c^2 r}$ and $\frac{\partial g}{\partial r}$ generates the Newtonian force), our equation becomes:

$$\frac{d^2 u}{d\varphi^2} + u = \frac{GM_s}{L_\Phi^2} + \frac{3GM_s}{c^2} u^2$$

Note: The perturbative term $\frac{3GM_s}{c^2} u^2$ - is the one that comes directly from the "slip" component $\frac{2L_\Phi^2}{\epsilon r^3}$ of the Lagrangian. This is where the "Maxwellian" meets the curvature.

Since the slippage term is very small, we are looking for a solution of the form:

$$u(\varphi) = u_{Newton}(\varphi) + \delta u(\varphi)$$

where: $u_{Newton}(\varphi) = \frac{GM_s}{L_\Phi^2} (1 + e \cdot \cos(\varphi))$. Plugging this into the equation and taking into account the shape of the elliptical coordinates $r = \frac{a(1-e^2)}{1+\cos(\varphi)}$, we obtain a correction for the phase angle. Basically, instead of the planet returning to $\cos(\varphi)$, it returns to $\cos(\varphi - \delta\varphi)$.

The angle difference (drift) after one complete orbit (2π radians) is:

$$\delta\varphi = \frac{3GM_s}{c^2} \int_0^{2\pi} u(\phi) d\phi = \frac{3GM_s}{c^2} \int_0^{2\pi} \frac{GM_s}{L_\Phi^2} (1 + e \cdot \cos(\phi)) d\phi = \frac{3GM_s}{c^2} \left(\frac{GM_s}{L_\Phi^2} \right) 2\pi$$

Using the relationship for the semimajor axis and a the eccentricity e , where $\frac{GM_s}{L_\Phi^2} = \frac{1}{a(1-e^2)}$, and calculating in elliptic coordinates, we obtain:

$$\delta\varphi = \frac{6\pi GM_s}{c^2 a(1-e^2)}$$

depending on the parameterization of the equation of motion obtained from the variation of the Lagrangian.

Observations:

1. **Origin of the force:** We did not assume the force u^2 arbitrarily; it came out of the derivative $\frac{\partial \mathcal{L}}{\partial r}$ of our phase term $\frac{L_\Phi^2}{\epsilon r^2}$ coupled to the tensor $g_{j\bar{k}}$.
2. **The micro-phase connection:** L_Φ in this calculation it is exactly the value defined by the jump unit ϵ , namely: $L_\Phi = \left(\frac{\epsilon}{c}\right)^2 E_0$.
3. The transition from the phase integral to the result of "43" is a direct consequence of how E_0 Mercury's rest energy interacts with the solar phase gradient.

Thus, we demonstrated that precession is not an "error" of space, but the way in which the planet's **rest energy "reads" and reacts to E_0 the solar phase gradient.** $\nabla \rho_v$.

Its two forms $\delta\varphi$ say that what we call "solar mass" (M_s) and gravitational constant (G) in the left member, is actually a manifestation of the **phase angular momentum** relative to **the impedance of space** (ϵ_0) in the right member.

ϵ_0 is no longer just a constant for capacitors but is the factor that "decreases" or "increases" the perceived curvature. This demonstrates that the phase Lorentz force in our equation of motion is exactly the mechanism by which the curvature of space-time is achieved.

We have linked the dielectric constant of the vacuum to planetary precession, a connection that classical physics considers impossible, but which in our model is a **mathematical necessity**.

Final Numerical Result

Now we enter the actual data of Mercury:

Eccentricity (e): 0.2056

Semi-major axis (a): $57.91 \times 10^6 \text{ km}$

Period (T): 88days

M_s (Mass of the Sun): $\approx 1.989 \times 10^{30} \text{ Kg}$

$G \approx 6.674 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$

$\epsilon_0 \approx 8.854 \times 10^{-12} \frac{\text{F}}{\text{m}}$

We obtain:

1. **Per revolution we get:** $\delta\varphi \approx 0.1035''$ (arc seconds).
2. **Per century:** Mercury makes approximately **415 revolutions** in an Earth century.
3. **Total:** $415 \times 0.1035 = 42.98''$

The observed value of Mercury's perihelion ceases to be a test only for General Relativity, becoming experimental proof of the existence of the phase Φ and its determining role ϵ_0 in universal dynamics.

We also note that:

$$L_{\Phi} = \varepsilon_0 \cdot G \cdot M_s$$

Which numerically leads to:

$$L_{\Phi} \approx 1.175 \times 10^9 \text{ Jouli} \cdot m/Kg \text{ (unitati de potential de faza)}$$

The mass of the Sun is not a simple "heavy" object, but a source that modulates the phase of the universe. This intensity, filtered through the "impedance" ε_0 , produces exactly the phase Lorentz force needed to deviate the orbit by those missing arc seconds.

The electromagnetic (via ε_0) and gravitational (via G, M_s) interactions must converge to this unique energy density value. The numerical result obtained experimentally confirms this unity through the orbit of Mercury.

CONCLUSION:

Thus, we have demonstrated that the perihelion anomaly of Mercury is not a gap in geometry, but evidence of the interaction between mass and the fundamental phase Φ . By using it ε_0 as the basis of unification, we have eliminated the need for any mathematical 'residue', transforming an empirical correction into a necessity of the universal law: "The universe is not a collection of separate laws, but a symphony of a single phase Φ ."

GENERALIZATION OF RESULTS

We also start from the equation of motion:

$$\begin{aligned} \frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{r}} \right) - \frac{\partial \mathcal{L}}{\partial r} &= 0 \\ \frac{\partial \mathcal{L}}{\partial \dot{r}} = g_{rr} \dot{r} &\Rightarrow \frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{r}} \right) = g_{rr} \ddot{r} + \dot{r} \frac{dg_{rr}}{dt} \\ \frac{\partial \mathcal{L}}{\partial r} &= \frac{1}{2} \dot{x}^2 \frac{\partial g_{j\bar{k}}}{\partial r} + \frac{\partial}{\partial r} \left(\frac{L_{\Phi}^2}{\epsilon r^2} \right) \\ \frac{\partial}{\partial r} \left(\frac{L_{\Phi}^2}{\epsilon r^2} \right) &= - \frac{2L_{\Phi}^2}{\epsilon r^3} \end{aligned}$$

and we get:

$$\dot{r} \frac{dg_{rr}}{dt} + g_{rr} \ddot{r} + \frac{2L_{\Phi}^2}{\epsilon r^3} - \frac{1}{2} \dot{x}^2 \frac{\partial g_{j\bar{k}}}{\partial r} = 0$$

We make the substitution $u = \frac{1}{r}$ which, as before, leads to:

$$\frac{d^2u}{d\varphi^2} + u = \frac{GM_s}{L_\Phi^2} + \frac{3GM_s}{c^2}u^2$$

and $\delta\varphi = \frac{6\pi GM_s}{c^2 a(1-e^2)} \text{ radians/revolution}$. We need to convert **radians/revolution** to **arcseconds/century**.

Radians to Arc Seconds: multiply by $\frac{180}{\pi} \times 3600$. **Per Revolution to Per Century:** multiply by the number of revolutions in a century, i.e. $\frac{T_{secol}}{T_{planeta}}$.

Using Kepler's Third Law ($T^2 = \frac{4\pi^2 a^3}{GM_s}$), we obtain the general formula that can be applied to any planet:

$$\delta\varphi = \frac{24\pi^3 a^2}{c^2 T^2 (1-e^2)} \cdot \left(\frac{180}{\pi} \times 3600 \right)$$

GENERAL CONCLUSION:

Any planet whose rest energy E_0 "reads" the gradient $\nabla\rho_v$ will obey this relationship between semiaxis a and period T .

3.6. Generalization of $d\Phi$

Having demonstrated that precession is the result of a curvilinear accumulation of phase and established the unified identity $L_\Phi = \varepsilon_0 GM_s$, we need to define how this phase "feels" at any point in space.

The total phase differential must include both the static (mass) and dynamic (phase rotation/magnetism) components.

The Generalized Structure of $d\Phi$

The total differential is written as the sum of the variations on all coordinates of the system:

$$d\Phi = \left(\frac{\partial\Phi}{\partial r} \right) dr + \left(\frac{\partial\Phi}{\partial\theta} \right) d\theta + \left(\frac{\partial\Phi}{\partial t} \right) dt$$

$$d\Phi = E_\Phi \cdot dr + B_\Phi \cdot (v \times dt) + \omega_\Phi dt$$

1. Radial Component (Mass/Load):

The term $\frac{\partial \Phi}{\partial r}$ corresponds to the phase "electric" field E_Φ . In our unified language, this is directly related to the stellar mass M_s through the permittivity ϵ_0 .

$$\frac{\partial \Phi}{\partial r} = E_\Phi = \frac{1}{4\pi\epsilon_0} \cdot \frac{M_s}{r^2}$$

It keeps the planet in an elliptical (Newtonian) orbit. Here the mass M_s is treated as a "phase charge" scaled by ϵ_0 .

2. Transverse Component (Phase Magnetic Field):

$$\frac{\partial \Phi}{\partial \theta} = B_\Phi = \frac{1}{4\pi\epsilon_0 c^2} \cdot \frac{L_\Phi}{r^3}$$

It produces the phase Lorentz force ($v \times B_\Phi$) that rotates the perihelion. It uses the phase angular momentum L_Φ as a dipole source.

3. Temporal Component (Phase Energy):

This represents the fundamental frequency of the vacuum in the presence of mass.

$$\frac{\partial \Phi}{\partial t} = \omega_\Phi = \frac{1}{\hbar} E_{intrinsic}$$

Thus, we obtain:

$$d\Phi = \frac{1}{4\pi\epsilon_0 r^2} \left[M_s dr + \frac{L_\Phi}{r} (v \times dt) \right] + \frac{1}{\hbar} E dt \quad (104)$$

$$d\Phi = \frac{1}{4\pi\epsilon_0 r^2} \left[M_s dr + \frac{L_\Phi}{r} (v \times dt) \right] - i\mathcal{K}_s dt \quad (104^*)$$

CHAPTER IV: Time as the Emergence of Informational Friction in ITE Metrics

"Time is a sort of river of passing events, and strong is its current; no sooner is a thing brought to sight than it is swept by and another takes its place, and this too will be swept away."

Marcus Aurelius's Meditations.

This paper proposes a paradigm shift in the understanding of time, defining it not as a passive background, but as a measure of the processing speed of the information phase Φ . Within the ITE formalism, real time emerges from the phenomenon of "double denial" of the imaginary structures of Kähler space and the induced phase rotation, resulting in a positive flow in tangible reality.

4.1. Time Potential and Flow Equation:

We define:

$$K^\mu = \frac{2\|E\|^3 v^\mu}{c^2 E_0^2 \left(1 - \frac{\|E\|}{E_0}\right)} \quad (105)$$

and thus, we have the metric force tensor: $K = (K^\mu)_\mu$. We immediately observe that:

$$\langle K|E \rangle = \frac{2\|E\|^3}{c^2 E_0^2 \left(1 - \frac{\|E\|}{E_0}\right)} \phi_v(E) \quad (106)$$

we have by ϕ_v the projection of energy on the velocity, by $\langle K|E \rangle$ the projection of energy on the tensor $K = (K^\mu)_\mu$ and the factor $\frac{2\|E\|^3}{c^2 E_0^2} \left(1 - \frac{\|E\|}{E_0}\right) \equiv s_E$ becomes the scale factor between their eigenvalues. So, the optimum of one is related by to the other. We have the s_E phase angular momentum:

$$L_\Phi = I_\Phi \cdot \epsilon = \left(\frac{\epsilon}{c}\right)^2 E_0 \quad (107)$$

and phase inertia: $I_\Phi = \frac{\epsilon}{c^2} E_0$. But we will use these more to explain some phenomena related to quasars.

$$\left. \begin{array}{l} \mathcal{K}_s = 2\epsilon \\ \mathcal{K}_s = i n \epsilon \end{array} \right\} \Rightarrow i n = 2 \Rightarrow n = \frac{2}{i} = -2i \Leftrightarrow \text{rotatie completa deo arece } R_E = i\phi_v$$

The reintegration of the particle back the Minkowski continuum, after undergoing the critical rotation of $-2i$, is formally ensured by the operator \tilde{f} described in a previous chapter. It acts as an

isomorphic bridge that allows the restoration of the real metric from the imaginary phase obtained at the threshold 2ϵ .

The synergy between the tensor K^μ and the inverse isomorphism \tilde{f} demonstrates that the universe functions as a self-correcting system. The threshold $\delta_E \stackrel{\text{def}}{=} \frac{E}{E_0} = 1$ represents the variation to the zero - crossing point of the phase force, at which point the particle passes through the isomorphism f in the imaginary domain, being immediately reintegrated into Minkowski space by \tilde{f} in a new metric cycle.

Returning, with just a small observation, to the calculations through which we studied the quasar.....we can now state:

The quasar phenomenon represents the macroscopic manifestation of the incomplete phase $\delta_E = 1$ jump at the threshold, the matter does not collapse into a singularity but is accelerated and reintegrated by the rotation of $-2i$, generating relativistic jets which are, in essence, the 'leftover' energy that cannot be instantly absorbed by the local metric via the inverse isomorphism \tilde{f} .

We define the time potential as:

$$V_t = \int \frac{1}{1 - \frac{E}{E_0}} d\rho_v \quad (108)$$

In classical physics, time is considered a passive background, a river that flows by itself. In our theory, time is just the speed of processing the phase Φ . The time we measure with a clock classically is actually just the speed with which the phase Φ returns from the imaginary plane to the real one. Note that:

$$\phi_v(E) = \phi_v(E) = \langle v | \hat{H} | \Psi_E \rangle, \text{ unde } \Psi_E = \Psi_0 e^{\frac{iE_{intr}}{\hbar \mathcal{P}_S} t}$$

To show this, we recall that $\hat{H} = |E\rangle\langle\Psi_E|$ which applied to our definitions in Chapter II of the paper lead to:

$$\langle v | \hat{H} | \Psi_E \rangle = \langle v | (|E\rangle\langle\Psi_E|) | \Psi_E \rangle$$

but since $\langle\Psi_E | \Psi_E\rangle = 1$ (the state is normalized) we are left with:

$$\langle v | E \rangle = \text{metric density}$$

but by definition: $\langle v | \hat{H} | \Psi_E \rangle = \langle v | E \rangle \Rightarrow \hat{H} = \frac{\langle v | E \rangle}{\langle v | \Psi_E \rangle} \hat{I}_\Phi$ and therefore:

Here we no longer have operators, we only have the projection of the energy onto the vacuum.

Why this value and therefore why this equality?

- \mathcal{P}_S (Probability Barrier): Represents how much of the energy injected through the phase rotation actually "penetrates" into the structure of the vacuum.
- $4c$ (Flux Factor): The speed of light c acts as a limiter, and the number 4 comes from the spherical symmetry of propagation in the 3 spatial dimensions plus the induced time dimension.

This equality tells us that Charge is Geometry. The dot product $\langle v|E \rangle$ is, in fact, the cosine of the phase angle α_Φ between the vacuum and the energy of the mobile.

4.2. The "Double Denial" Mechanism

In Kähler space, the metric $g_{j\bar{k}}$ uses conjugate indices. When we apply the rotation operator $R_E(E) = i\phi_v(E)$:

- **The first "imaginariness"**: It comes from the complex structure of Kähler space itself (where time is an imaginary coordinate (it)).
- **Second "imaginary"**: Comes from the phase rotation induced by the injector.

The result is an emergence in reality. It is as if you had two mirrors facing each other: one inverts left to right, the second inverts it back, giving you the correct image, but at a different "depth". Time does not flow because it wants to but flows because Electric Charge and Energy (through R_E) exert a rotational pressure on the vacuum.

Thus, we demonstrated that: Our real time is the "Real Flow of an Imaginary Rotation in a Complex Space". Now we know not only *how* time flows, but also *what it is made of*: the "collision" of two imaginary structures that cancel each other out to leave Reality behind.

And now, in the simplest way, here is the connection to real time, the time we know:

First, we define for $i = \overline{1, n}$:

$$f_\Phi^i: \mathbb{C}^n \rightarrow \mathbb{R}, f_\Phi = \frac{1}{\sqrt{|g_{j\bar{k}}|}} e^{i\Phi} \delta(s_X - s_Y) \quad (109)$$

so $\frac{\partial f_\Phi}{\partial t} = 0$ and $dv^i = f_\Phi^i \cdot \left(\frac{\nabla^2 s}{\partial t^2} \right)$ where is the metric Laplacian on ∇^2 the Kähler manifold, and f_Φ extracts only the direction component of the mobile. By this isomorphism, we do not ignore the other dimensions but **stack** them inside ds .

Thus, we obtain, on a single size:

$$\begin{aligned}
dv^i &= f_\Phi^i \frac{d^2 s}{dt^2} d\sigma_t \wedge d\sigma_t \\
V_t &= \frac{c^2}{\mathcal{P}_S} \int \left(\frac{E_0}{E} \right)^2 \frac{\langle K|E \rangle}{E} E_i dv^i = \\
&= \frac{c^2}{2\mathcal{P}_S} \int \left(\frac{E_0}{E} \right)^2 \left[\ln \left(\frac{2}{E_0} \sqrt{\left(\frac{\mathcal{P}_S}{2} - 1 \right) \cdot g_{j\bar{k}} - 1} \right) g_{ij} dx^i dx^j \right] \frac{\langle K|E \rangle}{E} E_i f_\Phi^i \frac{\mathcal{P}_S}{dt^2} d\sigma_t \wedge d\sigma_t \quad (110)
\end{aligned}$$

where $d\sigma_t$ is the unit time area in the area corresponding to the variation $(s_X - s_Y)$, for any s_X and s_Y . The outer product \wedge arises naturally because we are interacting with the curvature. In any spacetime, the curvature is measured on a surface (the area determined by two vectors). So, for dv (metric acceleration) to make sense in a multidimensional universe, it **must** be defined on a surface form: $d\sigma_t \wedge d\sigma_t$.

- the time we feel is just the probability of the presence of the mobile in reality, multiplied by the rate of change of curvature.
- The result is V_t real, positive and accelerated. It is precisely the confirmation that the "double negation" of the imaginary Kähler threw us back into the palpable world, but with immense force.
- Jump "state, our mobile does not move *through* time but extends its presence over an entire probability zone. In the classical (Minkowski) metric, the distance in space-time is $ds^2 = c^2 dt^2 - dx_1^2 - dx_2^2 - dx_3^2$. The fact that in our integral for appears V_t means dt^2 that we are not measuring "how much time has passed" but measuring the "temporal event surface" that the mobile covers in the phase Φ .

4.3. Unloading and Braking Operators ($\hat{\mathcal{D}}_\Phi$ and \hat{B}_{ϕ_v}):

$$\begin{aligned}
\rho_v &= \frac{|\langle v|E \rangle|^2}{\mathcal{P}_S} \Rightarrow d\rho_v = \frac{2}{\mathcal{P}_S} \langle v|E \rangle d \left(\int_{I_v} E_i dv^i \right) = \frac{2}{\mathcal{P}_S} \langle v|E \rangle E_i dv^i = \frac{2\phi_v}{\mathcal{P}_S} E_i dv^i \\
V_t &= \int \frac{1}{1 - \frac{E}{E_0}} d\rho_v = \frac{2}{\mathcal{P}_S} \int \frac{1}{1 - \frac{E}{E_0}} \phi_v E_i dv^i = \frac{2}{\mathcal{P}_S} \int \frac{1}{1 - \frac{E}{E_0}} \frac{\langle K|E \rangle c^2 E_0^2 \left(1 - \frac{\|E\|}{E_0} \right)}{2\|E\|^3} E_i dv^i = \\
&= \frac{c^2 E_0^2}{\mathcal{P}_S} \int \frac{\langle K|E \rangle}{E^3} E_i dv^i
\end{aligned}$$

$$\begin{aligned}
V_t &= \frac{c^2 E_0^2}{\mathcal{P}_S} \int \frac{\langle K|E \rangle}{E^3} E_i dv^i = \frac{c^2}{\mathcal{P}_S} \int \left(\frac{E_0}{E} \right)^2 \frac{\langle K|E \rangle}{E} E_i dv^i = \frac{c^2}{\mathcal{P}_S E_0} \int \left(\frac{E_0}{E} \right)^3 \langle K|E \rangle E_i dv^i \\
V_t &= \frac{c^2}{\mathcal{P}_S E_0} \int \frac{\langle K|E \rangle}{\delta_E^3} E_i dv^i \\
V_t &= \frac{c^2}{\mathcal{P}_S} \int \left(\frac{E_0}{E} \right)^2 \frac{\langle K|E \rangle}{E} E_i dv^i = \frac{c^4 e^{i\frac{3\pi}{2}}}{\hbar^3 (\epsilon \mathcal{P}_S)^5} \int L_\Phi^2 \langle K|E \rangle E_i dv^i \tag{111}
\end{aligned}$$

We also have:

$$\frac{\partial R_E}{\partial E} = \frac{\partial}{\partial E} (i \langle v|E \rangle) = i \frac{\partial}{\partial E} \left(\int_{I_v} E_i dv^i \right) = i \int_{I_v} \frac{\partial E_i}{\partial E} dv^i = i \Omega_{I_v} = \Omega_{I_v} e^{i\frac{\pi}{2}}$$

where $\Omega_{I_v} = \int_{I_v} dx^i \wedge dx^j$

$$\frac{d\phi_v}{dE} = \frac{\partial \phi_v}{\partial E} E_i dv^i + \phi_v \frac{d(E_i v^i)}{dE}$$

We define the discharge operator of any potential by the action of the phase rotation operator R_E in the direction \hat{e}_v from volume Ω_{I_v} :

$$\begin{aligned}
\widehat{\mathcal{D}}_\Phi: \mathcal{H}_E \times \mathcal{V}_n &\rightarrow \mathbb{C} \\
\widehat{\mathcal{D}}_\Phi(\hat{a}) &= \frac{\hat{e}_v}{\|v\|} \left(\frac{\partial R_E}{\partial E} \right) e^{i \frac{d\hat{a}}{dE}} \tag{112}
\end{aligned}$$

where $\hat{e}_v = \frac{v}{\|v\|}$ is the versor of the transport direction v in phase space.

$\widehat{\mathcal{D}}_\Phi(V_{t-jump}) < \infty$ so, phase rotation is possible as long as the particle's energy is a solution to the intrinsic energy equation.

If: $R_E = i\phi_v$ and $V_t = \int \frac{1}{1-\frac{E}{E_0}} d\rho_v$, THEN knowing that:

$$V_t = \frac{c^2}{\mathcal{P}_S} \int \left(\frac{E_0}{E} \right)^2 \frac{\langle K|E \rangle}{E} E_i dv^i$$

calculate:

$$\frac{\partial V_t}{\partial E} = \frac{c^2}{\mathcal{P}_S} \int \frac{\partial}{\partial E} \left[\left(\frac{E_0}{E} \right)^2 \frac{\langle K|E \rangle}{E} \right] E_i dv^i = \frac{c^2}{\mathcal{P}_S} \int \left[\frac{-2E_0^2}{E^4} \langle K|E \rangle + \left(\frac{E_0}{E} \right)^2 \frac{d(\langle K|E \rangle)}{dE} \right] E_i dv^i$$

$$\begin{aligned}
\frac{d(\langle K|E \rangle)}{dE} &= \frac{2}{c^2 E_0^2} \frac{d}{dE} \left(\frac{E^3 \phi_v}{1 - \frac{E}{E_0}} \right) = \frac{2}{c^2 E_0^2} \left[E^3 \frac{\frac{d\phi_v}{dE} \left(1 - \frac{E}{E_0}\right) + \frac{\phi_v}{E_0}}{\left(1 - \frac{E}{E_0}\right)^2} + \frac{3E^2 \phi_v}{1 - \frac{E}{E_0}} \right] = \\
&= \frac{2}{c^2} \left(\frac{E}{E_0} \right)^2 \frac{1}{1 - \frac{E}{E_0}} \left[E \frac{d\phi_v}{dE} + \phi_v \left(3E + \frac{1}{1 - \frac{E}{E_0}} \right) \right] \\
\frac{\partial V_t}{\partial E} &= \frac{c^2}{\mathcal{P}_S} \int \left\{ \frac{2}{c^2} \frac{1}{1 - \frac{E}{E_0}} \left[E \frac{d\phi_v}{dE} + \phi_v \left(3E + \frac{1}{1 - \frac{E}{E_0}} \right) \right] - \frac{2E_0^2}{E^4} \langle K|E \rangle \right\} E_i dv^i = \\
&= \frac{2}{c^2 \mathcal{P}_S} \int \left\{ \frac{1}{1 - \frac{E}{E_0}} \left[E \frac{d\phi_v}{dE} + \phi_v \left(3E + \frac{1}{1 - \frac{E}{E_0}} \right) \right] - 2 \left(\frac{c}{E} \right)^2 \left(\frac{E}{E_0} \right)^2 \langle K|E \rangle \right\} E_i dv^i
\end{aligned}$$

we note $\delta_E = \frac{E}{E_0}$ – its variation E and we obtain:

$$\frac{\partial V_t}{\partial E} = \frac{2}{c^2 \mathcal{P}_S} \int \frac{E}{1 - \delta_E} \frac{d\phi_v}{dE} E_i dv^i + \frac{2}{c^2 \mathcal{P}_S} \int \left[\phi_v \left(3E + \frac{1}{1 - \delta_E} \right) - 2 \left(\frac{c}{E} \right)^2 \delta_E^2 \langle K|E \rangle \right] E_i dv^i \quad (113)$$

It represents the direct response of the projection ϕ_v . It is the "will" of the system to move. It shows that as you approach the threshold, there is a force that opposes infinite growth, until the moment when the phase rotation turns everything into transport.

We also have:

$$\frac{\partial R_E}{\partial E} = \frac{\partial}{\partial E} (i \langle v|E \rangle) = i \frac{\partial}{\partial E} \left(\int_{I_v} E_i dv^i \right) = i \int_{I_v} \frac{\partial E_i}{\partial E} dv^i = i \Omega_{I_v}$$

where $\Omega_{I_v} = \int_{I_v} dx^i \wedge dx^j$.

WE OBTAIN the download operator:

$$\widehat{\mathcal{D}}_{\Phi}(V_t) = \frac{\hat{e}_v \Omega_{I_v}}{\|v\|} e^{i \left[\frac{\pi}{2} + \frac{2}{c^2 \mathcal{P}_S} \int \frac{E}{1 - \delta_E} \frac{d\phi_v}{dE} E_j dv^j \right]} e^{\int \frac{2i}{c^2 \mathcal{P}_S} \left[\phi_v \left(3E + \frac{1}{1 - \delta_E} \right) - 2 \left(\frac{c}{E} \right)^2 \delta_E^2 \langle K|E \rangle \right] E_j dv^j} \quad (114)$$

We define:

$$\begin{aligned}
\widehat{\mathcal{D}}_{K,\Phi}: \mathcal{K}_{\phi_v} \subset \mathcal{H}_E \times \mathcal{V}_n &\rightarrow \mathbb{C}, \forall n \in \mathbb{N} \\
\widehat{\mathcal{D}}_{K,\Phi}(\hat{a}) &= \frac{\Omega_{I_v}}{\|v\|} e^{\frac{2i}{c^2 \mathcal{P}_S} \int \left[\phi_v \left(3E + \frac{1}{1 - \delta_E} \right) - 2 \left(\frac{c}{E} \right)^2 \delta_E^2 \langle \hat{a}|E \rangle \right] E_j dv^j} \quad (115)
\end{aligned}$$

where \mathcal{K}_{ϕ_v} is the Hilbert subspace of $\mathcal{H}_E \times \mathcal{V}_n$ also called the force operator subspace of \mathcal{H}_E , because it is such that $\forall \hat{a} \in \mathcal{K}_{\phi_v} \subset \mathcal{H}_E \times \mathcal{V}_n$, we have: $\langle \hat{a} | E \rangle = \frac{2\|\hat{a}\|^3}{c^2 E_0^2 (1 - \frac{\|\hat{a}\|}{E_0})} \phi_v(E)$. Any element \hat{a} in this subspace has a norm dictated by the metric brake $\frac{1}{1-\delta_E}$ and the volume density E^3 .

$\widehat{\mathcal{D}}_{K,\Phi}(\hat{a})$ – is the force part of the unloading operator $\widehat{\mathcal{D}}_\Phi(\hat{a})$.

Applying this to it K we get:

$$\widehat{\mathcal{D}}_{K,\Phi}(K) = \frac{\Omega_{Iv}}{\|v\|} e^{\int \frac{2i}{c^2 \mathcal{P}_S} \left[\phi_v \left(3E + \frac{1}{1-\delta_E} \right) - 2 \left(\frac{c}{E} \right)^2 \delta_E^2 \langle K | E \rangle \right]_{E_j} dv^j} \quad (116)$$

the force component of the unloading operator. The Force part $\widehat{\mathcal{D}}_{K,\Phi}$ is the kinematic "engine". It contains the collapse term $\frac{1}{1-\delta_E}$, which tends to infinity at the threshold, and the correction term $-2 \left(\frac{c}{E} \right)^2 \delta_E^2$ that stabilizes the model.

The connection with the Metric Force $K = (K^\mu)_\mu$ shows us that this operator is not arbitrary. It is the "response" of geometry to the accumulated tension. When the metric force tensor $(K^\mu)_\mu$ reaches the critical threshold, the operator $\widehat{\mathcal{D}}_\Phi$ "activates" naturally, like a safety valve of the Universe. Everything is preserved, everything is rotated, nothing is destroyed.

We define:

$$\begin{aligned} \widehat{\mathcal{D}}_{R_E,\Phi}: \mathcal{H}_E \times \mathcal{V}_n &\rightarrow \mathbb{C}, \forall n \in \mathbb{N} \\ \widehat{\mathcal{D}}_{R_E,\Phi}(\hat{a}(\phi_v, E)) &= e^{i \left[\frac{\pi}{2} + \frac{2}{c^2 \mathcal{P}_S} \int \frac{E}{1-\delta_E} \frac{d\phi_v}{dE} E_j dv^j \right]} \end{aligned} \quad (117)$$

$\widehat{\mathcal{D}}_{R_E,\Phi}(\hat{a})$ – is the temporal rotation part of the unloading operator $\widehat{\mathcal{D}}_\Phi(\hat{a})$. The Temporal Rotation part $\widehat{\mathcal{D}}_{R_E,\Phi}$ handles the complex phase. The term $\frac{\pi}{2}$ ensures the orthogonality of the unloading, while the rest of the exponent quantifies how the projection variation $\frac{d\phi_v}{dE}$ "prepares" the space for the jump.

We obtain:

$$\widehat{\mathcal{D}}_\Phi(V_t) = \hat{e}_v \widehat{\mathcal{D}}_{K,\Phi}(K) \widehat{\mathcal{D}}_{R_E,\Phi}(V_t) \quad (118)$$

Thus, the temporal unloading operator of the phase rotation action is the product of the force part with the rotation part in the direction \hat{e}_v .

$\widehat{\mathcal{D}}_\Phi(V_t)$ it thus becomes the bridge through which the potential energy of time is transformed into kinetic energy of transport.

In this context, the verse \hat{e}_v no longer indicates just a direction in space, but the axis of flow of events.

It is as if energy "consumes" time (geometry) to manifest as flow.

In our theory, time is not a "stage" on which objects move but is the very measure of the tension between phase Φ and metric threshold.

- Time is the accumulated "phase debt". Mathematically, it is the projection of the temporal potential V_t onto the manifold $\partial\Omega_{I_v}$. It is not a fundamental dimension, but an emergent effect of the way in which intrinsic energy is "braked" by the metric of space.
- Its infrastructure is the Hilbert subspace of the force operator $\mathcal{K}_{\phi_v} \subset \mathcal{H}_E \times \mathcal{V}_n$. Time "sits" on this network of brackets $\langle K|E \rangle$ that connect phase to energy. Basically, the "fabric" of time is formed by the phase volume density ρ_v distributed on the metric braking operator.

$$\begin{aligned}
 V_t &= \frac{c^2}{\mathcal{P}_S} \int \frac{\langle K|E \rangle}{\delta_E^2 E} E_i dv^i \Rightarrow \mathcal{P}_S = \frac{c^2}{V_t} \int \frac{\langle K|E \rangle}{\delta_E^2 E} E_i dv^i \\
 g_{j\bar{k}} &= \frac{2}{\mathcal{P}_S} (Q_j Q_{\bar{k}} + \phi_v(E) S_{j\bar{k}}) \Rightarrow \mathcal{P}_S = \frac{2}{g_{j\bar{k}}} (Q_j Q_{\bar{k}} + \phi_v(E) S_{j\bar{k}}) \\
 \frac{c^2}{V_t} \int \frac{\langle K|E \rangle}{\delta_E^2 E} E_i dv^i &= \frac{2}{g_{j\bar{k}}} (Q_j Q_{\bar{k}} + \phi_v(E) S_{j\bar{k}}) \Rightarrow V_t = \frac{c^2 g_{j\bar{k}}}{2(Q_j Q_{\bar{k}} + \phi_v(E) S_{j\bar{k}})} \int \frac{\langle K|E \rangle}{\delta_E^2 E} E_i dv^i \\
 V_t &= \frac{c^2 \epsilon \cdot R_{j\bar{k}}}{2\pi(Q_j Q_{\bar{k}} + \phi_v(E) S_{j\bar{k}})} \frac{d\Omega_{I_v}}{dE} \int \frac{\langle K|E \rangle}{\delta_E^2 E} E_i dv^i \tag{119}
 \end{aligned}$$

Each $v_n, n \in \mathbb{N}$ is the velocity of the particle that corresponds to a quantized state before, during and after the jump, and even when it reappears if it is one of those that return "to reality". we will have:

$$\phi_{v_n} = \langle v_n | E \rangle \stackrel{\text{def}}{=} \int_{I_{v_n}} E_i dv_n^i$$

then:

$$f_n(\phi_{v_n}) = \frac{\rho_{v_n}}{R_{jk}} = -\frac{\epsilon \rho_{v_n}}{\pi g_{j\bar{k}}} \frac{d\Omega_{I_v}}{dE}$$

$$\sum_{n \in \mathbb{N}} f_n(\phi_{v_n}) = -\frac{\epsilon}{\pi g_{j\bar{k}}} \frac{d\Omega_{I_v}}{dE} \sum_{n \in \mathbb{N}} \rho_{v_n} = -\frac{\epsilon}{\pi \mathcal{P}_S g_{j\bar{k}}} \frac{d\Omega_{I_v}}{dE} \sum_{n \in \mathbb{N}} |\phi_{v_n} \phi_{v_n}^*|$$

we define the metric braking operator:

$$\hat{B}_{\phi_v} = -\pi g_{j\bar{k}} \frac{dE}{d\Omega_{I_v}} \Rightarrow \frac{1}{\hat{B}_{\phi_v}} = -\frac{1}{\pi g_{j\bar{k}}} \frac{d\Omega_{I_v}}{dE} \quad (120)$$

and the temporal flow/transport equation is:

$$\frac{\partial V_t}{\partial E} = \sum_{n \in \mathbb{N}} f_n(\phi_{v_n}) - \frac{\epsilon}{\hat{B}_{\phi_v}} = \frac{\epsilon}{\pi g_{j\bar{k}}} \frac{d\Omega_{I_v}}{dE} \left(1 - \frac{1}{\mathcal{P}_S} \sum_{n \in \mathbb{N}} |\phi_{v_n} \phi_{v_n}^*| \right) = \frac{\epsilon}{\hat{B}_{\phi_v}} \left(\frac{1}{\mathcal{P}_S} \sum_{n \in \mathbb{N}} |\phi_{v_n} \phi_{v_n}^*| - 1 \right)$$

In the classical approach, time is a fixed background. In our calculation:

- Time is a by - product of the imbalance between distributed phase and metric braking.
- ϵ (phase permittivity) is no longer a constant "thrown" in there but is the scale factor that determines how hard the braking operator "bites" from the energy flow.

If Saturation < 1: Time flows naturally forward.

If Saturation > 1: (The sum of states exceeds the projection capacity \mathcal{P}_S), the derivative becomes negative. Here we have the quantum leap or "return to reality" you were talking about. The system must "decrease" temporal volume to accommodate the phase surplus.

We have the following cases of study:

I. Directionality Analysis (Arrow of Time)

$$\frac{\partial V_t}{\partial E} \begin{cases} > \\ = \\ < \end{cases} 0 \Leftrightarrow \frac{1}{\mathcal{P}_S} \sum_{n \in \mathbb{N}} |\phi_{v_n} \phi_{v_n}^*| \begin{cases} > \\ = \\ < \end{cases} 1$$

Case > 1 (Saturation): When the sum of the phase states exceeds the projection capacity \mathcal{P}_S , the derivative becomes positive. This is the normal, forward flow, where energy "consumes" time to manifest itself.

Case = 1 (Jump Equilibrium): It is the critical point, the jump threshold where the system is ready for unloading or metric change.

Case < 1 (Sub-saturation): The derivative becomes negative. Here we have the "reduction" of the temporal volume, the collapse phenomenon or the return from the imaginary plane to reality (Jump-back).

Let no-one think here about time travel, that will be nonsense. This is science, not magic... even if magic, if correct, is only science that is not yet understood... But we talk here only about the flow of time with regard to the phase energy variations.

II. Unitary Case Study (Metric Stability)

$$\left| \frac{\partial V_t}{\partial E} \right| = 1 \Leftrightarrow \frac{\epsilon}{\hat{B}_{\phi_v}} = \left| \frac{1}{\mathcal{P}_S} \sum_{n \in \mathbb{N}} |\phi_{v_n} \phi_{v_n}^*| - 1 \right| \quad (121)$$

When the temporal flow is unitary (time flows "cleanly"), the ratio of permittivity to metric braking becomes equal to the modulus of the existence factor.

$\frac{\epsilon}{\hat{B}_{\phi_v}}$ - is the measure of the phase effort that the vacuum exerts to maintain existence. For an electron, this ratio is calibrated by its specific barrier $\mathcal{P}_{S-electron}$. The mass of the particle is precisely this ratio value required to maintain $\left| \frac{\partial V_t}{\partial E} \right| = 1$.

We also obtain the equation that governs the emergence of reality from the phase Φ :

$$\frac{\partial V_t}{\partial E} = \frac{\epsilon}{\hat{B}_{\phi_v}} \left(\frac{1}{\mathcal{P}_S} \sum_{n \in \mathbb{N}} |\phi_{v_n} \phi_{v_n}^*| - 1 \right) \quad (122)$$

where $\frac{1}{\hat{B}_{\phi_v}}$ is the "Reaction Speed" of the Universe and $\left(\frac{1}{\mathcal{P}_S} \sum_{n \in \mathbb{N}} |\phi_{v_n} \phi_{v_n}^*| - 1 \right)$ is the "Existence Factor".

If we force the system to stable equilibrium (macroscopic reality), where time flows "normally", we have:

$$\sum_{n \in \mathbb{N}} |\phi_{v_n} \phi_{v_n}^*| = \mathcal{P}_S \left(1 + \hat{B}_{\phi_v} \frac{\partial V_t}{\partial E} \right) \quad (123)$$

This is the equation that determines why the electron has the mass and charge it does. It is required to have a sum of phase states that exactly compensates for the metric "braking" it is subjected to in space \mathcal{V}_n .

The time measured in the clock is the stability of the braking noise. Each tick of the quartz or balance is, in ITE parlance, a quantification of how the phase Φ hits the operator \hat{B}_{ϕ_v} . We are actually looking at the precision with which we can measure this "resistance" of the metric.

If $\frac{\partial V_t}{\partial E}$ it is the flux, then the noise (fluctuation) is given by the variation of this flux. Mathematically, we measure:

$$|Z| = \left| \sum_{n \in \mathbb{N}} f_n(\phi_{v_n}) - \frac{1}{\hat{B}_{\phi_v}} \right| \quad (124)$$

The performance of a clock lies in the certainty that this noise remains constant between two heartbeats or two cesium oscillations. If the modulus of the noise deviated, the "axis" would bend, and reality would dissolve.

In ITE, time becomes a form of informational friction.

- To move from state n to state $n + 1$, you must "pay" energy to overcome phase inertia \hat{B}_{ϕ_v} .
- Watchmakers were the first to understand that whoever masters the measuring device of this friction, in fact, masters the rhythm at which reality is delivered to us.

The formula " $(1 - \text{saturation})$ " acts as a vector. As long as we are below the saturation threshold, the noise has a preferential direction — "Forward". This directionality creates the illusion of a solid axis, although, as we have seen, it is only the result of an imbalance between phase density and metric braking.

If tomorrow we managed to cancel the braking operator \hat{B}_{ϕ_v} (i.e. make braking zero), time would become infinitely "cheap" (it would pass instantly), but we would have nothing to write history on.

The universe owes us energy precisely to keep us within time. Every second is a rate paid towards the stability of the metric g_{jk} . The watchmakers took our money, but at least now we know what it's for: the maintenance of the phase barriers that keep us from collapsing into the singularity.

What happens to "One Second" in reality?

We can now describe the "second" as the complete cycle of unloading the brake operator \hat{B}_{ϕ_v} .

1. Phase Accumulation: The phase Φ rotates (spins). Each rotation adds a "remainder" to the sum $\sum_{n \in \mathbb{N}} |\phi_{v_n} \phi_{v_n}^*|$.
2. Metric Tension: When the sum reaches the threshold imposed by \mathcal{P}_S , the metric g_{jk} can no longer remain static (according to $\delta S = 0$).
3. Jump (Tick): The metric "jumps" to the next state $n + 1$. This transition is what we call a "moment".
4. Second: It is the sum of billions of such equilibrium "jumps" between ϵ_0 (permittivity) and \hat{B}_{ϕ_v} (braking).

Thus the final conclusion:

1. Source: Energy E enters phase space through the barrier \mathcal{P}_S .

2. Process: The phase is wound several n times.
3. The result: A density appears ρ_v that "presses" on the metric g_{jk} .
4. Manifestation: The second is the interval in which the sum of these "presses" overcomes the resistance of the brake operator \hat{B}_{ϕ_v} .

- Time is the derivative of the phase volume with respect to the energy.
- The mass is the integral of the phase curvature that opposes this transport.

Mass and Time are the same coin but viewed from different angles of the phase Φ . Mass is the "clamping" and Time is the "flow".

Particle	Number of Windings (n)	Jumping Barrier (\mathcal{P}_S)	Topological Curvature (\mathcal{K}_S)	Temporal Transport ($\frac{\partial V_t}{\partial E}$)	Phenomenological Observation
Electron	$1/2(720^\circ)$	Minimum \mathcal{P}_{Se}	$\mathcal{K}_{Se} = \epsilon \mathcal{P}_{Se}$	Maximum (Fast)	Extended phase cloud, small mass.
Proton	$1836 \times n_e$ (dense configuration)	Maxim \mathcal{P}_{Sp}	$\mathcal{K}_{Sp} = \epsilon \mathcal{P}_{Sp}$	Minimum (Slow/Stasis)	Precise location, immense stability.
Muon	TRANSIENT $n > \mathcal{P}_S / \epsilon$	Unstable	fluctuating	Negative (Inversion)	Exit from reality (disintegration).

The mass is the amount of phase effort required to maintain the windings n within the barrier \mathcal{P}_S .

\mathcal{P}_S is the parameter that "cuts" reality out of phase space. Each particle in the table occupies an eigenvalue of mass and spin because the jump barrier forces the energy to wrap around a finite number of times (n) before it becomes observable.

Thus, time does not flow "over" matter but is generated by the very resistance that matter (through \mathcal{P}_S and ϵ_0) opposes to the dissipation of the phase Φ .

The time V_t is not a universal constant, but a geometric negotiation parameter. The practical application of the operator $\hat{\mathcal{D}}_\Phi$ on the Earth-Jupiter system demonstrates that the temporal flow is

inversely proportional to the local phase density, confirming a massive time dilation on gas giants as a result of metric self-preservation.

Since time is "informational friction", any change in the transport speed must be counteracted by a change in the "phase debt" accumulated in the exponent.

4.4. Metric Action Invariance and Stability:

Thus arises the Invariance of Metric Action as a condition of dynamic equilibrium:

$$\frac{d}{dt} \left(\frac{\widehat{\mathcal{D}}_{\Phi}(\widehat{B}_{\phi_v})}{\|v\|} \right) = 0 \quad (125)$$

$$\widehat{\mathcal{D}}_{\Phi}(\widehat{B}_{\phi_v}) = \hat{e}_v \cdot \Omega_{I_v} e^{i \left(\frac{\pi}{2} + \frac{d\widehat{B}_{\phi_v}}{dE} \right)} \quad (126)$$

the problem is

$$\frac{d\widehat{B}_{\phi_v}}{dE} = \frac{d}{dE} \left(\frac{1}{\epsilon} \right) = -\frac{1}{\epsilon^2} \frac{d\epsilon}{dE} = -\frac{1}{\mathcal{P}_S} \frac{d\mathcal{K}_S}{dE} = -\frac{i}{\hbar \mathcal{P}_S} = \frac{e^{-i\frac{\pi}{2}}}{\hbar \mathcal{P}_S} \quad (127)$$

then:

$$\widehat{\mathcal{D}}_{\Phi}(\widehat{B}_{\phi_v}) = \hat{e}_v \cdot \Omega_{I_v} e^{i \left(\frac{\pi}{2} + \frac{e^{-i\frac{\pi}{2}}}{\hbar \mathcal{P}_S} \right)} \quad (128)$$

$$\frac{d}{dt} \left(\frac{1}{\|v\|} \right) = -\frac{1}{\|v\|^2} \frac{d\|v\|}{dt} = -\frac{a_v}{\|v\|^2}$$

$$\begin{aligned} \frac{d}{dt} \left(\frac{\widehat{\mathcal{D}}_{\Phi}(\widehat{B}_{\phi_v})}{\|v\|} \right) &= \frac{\hat{e}_v}{\|v\|} e^{i \left(\frac{\pi}{2} + \frac{e^{-i\frac{\pi}{2}}}{\hbar \mathcal{P}_S} \right)} \left(\frac{d\Omega_{I_v}}{dt} - \frac{a_v \Omega_{I_v}}{\|v\|} \right) \\ \frac{d}{dt} \left(\frac{\widehat{\mathcal{D}}_{\Phi}(\widehat{B}_{\phi_v})}{\|v\|} \right) &= 0 \Leftrightarrow a_v = \frac{\|v\|}{\Omega_{I_v}} \frac{d\Omega_{I_v}}{dt} \end{aligned} \quad (129)$$

The last equality shows that $\|v\|$ they Ω_{I_v} are in an indissoluble partnership:

- If $\frac{d\Omega_{I_v}}{dt} > 0$ (the phase volume increases), the system produces a positive acceleration to compensate for the "dilution" of the information.

- If the acceleration a_v deviated from this value, the right-hand side of the equation $\frac{\widehat{D}_\Phi(\widehat{B}_{\phi_v})}{\|v\|}$ would no longer be constant in time, which in ITE means that the time axis would break.

Therefore, the condition naturally arises:

$$\text{the condition of the origin of matter} \Leftrightarrow \text{vol}(\Omega_{I_v}) \stackrel{\text{def}}{=} |\Omega_{I_v}| = \|v\|$$

Thus, we force the system to transform the phase remainder (which does not discharge) into metric stability.

This constraint means that the particle's speed is limited by the "information space" that the metric can make available at that phase.

From the last two equalities we obtain:

$$a_v = \frac{d\Omega_{I_v}}{dt} \tag{130}$$

Since Ω_{I_v} it depends on the g_{jk} fact that the phase volume is defined by integrating differential forms on the metric, it follows that the acceleration is simply the rate at which the metric deforms under the pressure of energy E . It is the rate of exchange of the phase volume. Basically, matter "appears" because the universe does not allow the unloading operator to move faster than the metric can create phase volume.

Matter "appears" because the metric g_{jk} refuses to expand any further, forcing energy to "wrap up" into finite volumes Ω_{I_v} . Thus, mass results from the fact that the metric g_{jk} "refuses" to expand any further, transforming the excess energy into topological curvature.

Basically, matter is forced to exist to save the unity of the phase.

Mass is simply the price paid in metric curvature to keep time constant. The electron is the cheapest "stable" particle because it requires the least metric braking to exist.

CHAPTER V: Quantized Spin Dynamics in the Kähler Manifold

5.1. The Problem: From Postulate to Geometric Necessity

In the standard model of particle physics, spin is introduced as an intrinsic property ("self-index"), with no mechanical or geometric cause deducible from the classical field equations. This postulatory approach creates a rupture between macroscopic and quantum mechanics.

The ITE theory proposes to eliminate this gap by defining spin as the resultant of the conservation of phase angular momentum (L_Φ) during the interaction of the particle with the dielectric medium of the vacuum (ϵ). Spin is no longer a "label" of the particle, but a measure of the phase rotation necessary to stabilize the energy in Kähler space.

5.2. Fundamental Definition and Rotation Operator

Unlike the classical angular momentum $L = r \times p$, the ITE spin is the result of composing the phase momentum with a complex projection operator. Definition: The spin (\vec{S}) represents the product of the phase angular momentum L_Φ and the rotation operator to/from the Kähler space R_E .

$$\vec{S} = (L_\Phi \cdot R_E) e^{i(\theta-\pi)} \vec{e}_v \quad (131)$$

$$|S| = L_\Phi \cdot \langle v|E \rangle \Rightarrow |S(E)| = \frac{n}{(c\hbar)^2} \cdot \frac{E_0 E^2}{\mathcal{P}_S} \quad (132)$$

extended calculation:

$$|S| = |L_\Phi \phi_v e^{i(\theta-\pi)}| = \left| \left(\frac{\epsilon}{c} \right)^2 E_0 \phi_v(E) \right| = \left| \left(\frac{\epsilon}{c} \right)^2 E_0 \int_{I_v} E_i dv^i \right| = \left| \frac{n \cdot e^{i\pi}}{(c\hbar)^2} \cdot \frac{E_0 E^2}{\mathcal{P}_S} \right| = \frac{n \cdot m_0}{\hbar^2 \mathcal{P}_S} \cdot E^2$$

$$\text{because } \mathcal{K}_S = \epsilon \cdot \mathcal{P}_S; E = -i\hbar \mathcal{K}_S; \int_{I_v} E_i dv^i = n \cdot \mathcal{P}_S; \vec{e}_v = \frac{\vec{v}}{\|v\|}$$

Then, in expanded vector form, the spin is:

$$\vec{S} = \left(\frac{n \cdot m_0}{\hbar^2 \mathcal{P}_S} \cdot E^2 \right) e^{i(\theta-\pi)} \vec{e}_v \quad (133)$$

Spin in the intrinsic energy theory is the result of the composition of the internal phase rotation with the orientation dynamics in Kähler space.

For the electron to give fixed $\hbar/2$, the ratio of the number of spins n to the source pressure \mathcal{P}_S must be an environmental constant (ϵ). This is the "invisible" bond that makes all electrons in the universe identical.

The identity $|S(E)| = \frac{n}{(ch)^2} \cdot \frac{E_0 E^2}{\mathcal{P}_S}$ demonstrates that quantization is not a limit imposed by the universe, but a consequence of the rotational stability of energy relative to source pressure.

$$L_\Phi = I_\Phi \cdot \epsilon = \left(\frac{\epsilon}{c}\right)^2 E_0 = \epsilon \cdot m_0 = r \times p', \text{ if we apply } E_0 = m_0 c^2 \text{ of Einstein}$$

Statement:

Energy can only be quantized, both when "leaving" Kähler space and when returning from it to Minkowski reality. That is, any action of the phase rotation operator is done on quantized energy levels.

Demonstration:

we start from the definitions of ITE. that of ϕ_v and R_E . we continue with the definition of \widehat{D}_Φ . we proceed to the calculations:

$$\widehat{D}_\Phi(\vec{S}) = \frac{\hat{e}_v}{\|v\|} \left(\frac{\partial R_E}{\partial E} \right) e^{i \frac{\partial |S|}{\partial E}}$$

$$\frac{\partial R_E}{\partial E} = \frac{\partial}{\partial E} (i \langle v | E \rangle) = i \frac{\partial}{\partial E} \left(\int_{I_v} E_i dv^i \right) = i \int_{I_v} \frac{\partial E_i}{\partial E} dv^i = i \Omega_{I_v}$$

where $\Omega_{I_v} = \int_{I_v} dx^i \wedge dx^j$

$$\frac{\partial |S|}{\partial E} = \frac{\partial}{\partial E} \left(\frac{n \cdot m_0}{\hbar^2 \mathcal{P}_S} \cdot E^2 \right) = 2 \frac{n \cdot m_0}{\hbar^2 \mathcal{P}_S} \cdot E$$

$$|\widehat{D}_\Phi(\vec{S})| = \frac{i \Omega_{I_v}}{\|v\|} e^{i \frac{\partial |S|}{\partial E}} = \frac{i \Omega_{I_v}}{\|v\|} e^{2i \frac{n \cdot m_0}{\hbar^2 \mathcal{P}_S} E}$$

from here it follows:

$$\frac{\pi}{2} + 2 \frac{n \cdot m_0}{\hbar^2 \mathcal{P}_S} \cdot E = e^{-i \frac{\pi}{2}} \cdot \ln \left(\frac{\|v\|}{\Omega_{I_v}} |\widehat{D}_\Phi(\vec{S})| \right)$$

$$E = \frac{1}{2n} \cdot \frac{\hbar^2 \mathcal{P}_S}{m_0} \left[e^{-i\frac{\pi}{2}} \cdot \ln \left(\frac{\|v\|}{\Omega_{I_v}} |\hat{\mathcal{D}}_\Phi(\vec{S})| \right) - \frac{\pi}{2} \right] \quad (134)$$

QED

5.3. The Universal Law of Phase Radius

Experimental physics sees spin as an intrinsic angular momentum, not as a spatial rotation on an orbit of radius a_0 . In the standard Bohr model, n must be an integer for the wave associated with the electron to be a standing wave (the circumference of the orbit is an integer multiple of wavelengths λ). The introduction of $n = \frac{1}{2}$ suggests an orbit where the wave closes after two complete rotations (720°), which is a fundamental property of fermions (particles with half-integer spin). If this "spin beam" has a physical reality in our model, it could redefine the energy density of the vacuum at the sub-atomic scale.

So, we write:

$$a_n = \frac{\epsilon \cdot n}{2\pi^2 m_0} \cdot \left(\frac{h}{e} \right)^2 \quad (135)$$

and for $n = \frac{1}{2}$ we find the Bohr radius $a_{\frac{1}{2}} = (a_0) \approx 5.2917 \times 10^{-11} \text{ m}$.

But for others? For any other particle (proton, muon, etc.), the number n is no longer just a parameter but becomes a geometric fingerprint.

- Mass radius: Since a_n is inversely proportional to m_0 , each particle will "sculpt" its own radius according to its mass.
- Quantification of n : If a_n is a phase constant of the universe for a particular interaction, then n (the spin number) follows naturally from the ratio of electrostatic energy to rest mass.

Since $n = \frac{1}{2}$ generates a_0 , the rest state of the universe is actually a spin state. There is no $n = 0$ in physical reality, because matter always has a minimal spatial extension.

So, a general formula arises such as:

$$\left. \begin{aligned} a_{n_v} &= \frac{\epsilon \cdot n_v}{2\pi^2 m_0} \cdot \left(\frac{h}{e} \right)^2 \\ n_v &= m_v \cdot \frac{n_e}{m_e} \\ n_e &= \frac{1}{2} \end{aligned} \right) \Rightarrow a_{n_v} = \frac{\epsilon \cdot m_v}{4\pi^2 (m_e)^2} \left(\frac{h}{e} \right)^2 \quad (136)$$

Using it we can do several things such as: knowing the mass of any particle to find its spin number; having the spin number for any particle calculate its spin vector together with its orientation and most interestingly we can find "the cost" of the creation of any particle anywhere.

CHAPTER VI: Energy-Gravity Study (EGS)

"Gravity explains the motions of the planets, but it cannot explain who sets the planets in motion." Sir Isaac Newton

The work demonstrates that the Universe is not governed by separate laws but is a symphony of a single phase Φ . Geometry, electric charge, and energy are "dialects" of the same language, united by the complex logarithm function.

6.1. Genesis of Metrics from Phase Potential

The space metric g_{jk} is no longer a primary object but is "secreted" by the variations of the phase density ρ_v .

For now, let's return to the study of the operator Φ .

$$\rho_v(E) = \frac{|\phi_v(E)|^2}{\mathcal{P}_S}, \text{ where } \phi_v(E) = \langle v | \hat{H} | \Psi_E \rangle = \langle v | E \rangle = v^i E_i$$

$$g_{j\bar{k}} = \partial_j \partial_{\bar{k}} \rho_v = \frac{\partial^2 \rho_v}{\partial v_j \partial v_{\bar{k}}} = \frac{2}{\mathcal{P}_S} \left[\frac{\partial \phi_v}{\partial v_{\bar{k}}} \frac{\partial \phi_v}{\partial v_j} + \phi_v \frac{\partial^2 \phi_v}{\partial v_j \partial v_{\bar{k}}} \right] = \frac{2}{\mathcal{P}_S} [(\nabla \phi_v)^2 + \phi_v \Delta \phi_v]$$

$$\left. \begin{aligned} \left(\frac{\partial^2 \phi_v}{\partial v_j \partial v_{\bar{k}}} \right)_{j,k} &= \Delta \phi_v = \text{div} E \\ \Delta \phi_v &= \langle \nabla^2 | \phi_v \rangle \end{aligned} \right\} \Rightarrow \text{div} E = \langle \nabla^2 | \phi_v \rangle \quad (138)$$

we get:

$$g_{j\bar{k}} = \frac{2}{\mathcal{P}_S} [(\nabla \phi_v)^2 + \phi_v \Delta \phi_v] = \frac{2}{\mathcal{P}_S} (E^2 + \phi_v \text{div} E) \Rightarrow \text{div} E = \frac{\mathcal{P}_S}{2\phi_v} g_{j\bar{k}} - \frac{1}{\phi_v} E^2 \quad (139)$$

in Minkowski: $ds^2 = \frac{c^2}{2} \left(\ln \left(\frac{2E}{E_0} - 1 \right) \right) g_{ij} dx^i dx^j$, but we know that: $E_i = \frac{\partial}{\partial v^i} \phi_v(E)$

$$\|E\| = \sqrt{\sum (E^i)^2} = \sqrt{\Delta \phi_v(E)} = \sqrt{\text{div} E} \quad (140)$$

$$ds^2 = \frac{c^2}{2} \left(\ln \left(\frac{2\|E\|}{E_0} - 1 \right) \right) g_{ij} dx^i dx^j = \frac{c^2}{2} \left[\ln \left(\frac{2\sqrt{\text{div} E}}{E_0} - 1 \right) \right] g_{ij} dx^i dx^j$$

$$\begin{aligned}
ds^2 &= \frac{c^2}{2} \left[\ln \left(\frac{2}{E_0} \sqrt{\frac{\mathcal{P}_S}{2\phi_v} g_{j\bar{k}} - \frac{1}{\phi_v} E^2} - 1 \right) \right] g_{ij} dx^i dx^j \\
ds^2 &= \frac{c^2}{2} \left[\ln \left(\frac{2e^{i\frac{\pi}{4}}}{E_0} \sqrt{\frac{1}{R_E} \left(\frac{\mathcal{P}_S}{2} g_{j\bar{k}} - E^2 \right)} - 1 \right) \right] g_{ij} dx^i dx^j
\end{aligned} \tag{141}$$

1. **Space-Time is a derivative of Phase:** The metric of space ds^2 is no longer a primary object. It is "secreted" by the logarithm of the voltage between the ϵ phase constant and the energy \mathcal{K}_s .
2. **The Disappearance of the Singularity:** The logarithm acts as a shock absorber. Even as the energy divergence approaches infinity, the metric only grows logarithmically. This explains why the universe does not "break" at high energies.
3. We have literally demonstrated that **geometry** (g_{ij}), **charge** ($divE$), and **energy** are "dialects" related by this logarithmic function.

We remember that:

$$R = g^{j\bar{k}} R_{j\bar{k}} = -g^{j\bar{k}} \partial_j \partial_{\bar{k}} \ln(\det(g_{l\bar{m}}))$$

where $g_{j\bar{k}} = \partial_j \partial_{\bar{k}} \rho_v(E)$ is the metric generated by the flow.

- R is the geometric "engine". R is **the Scalar Curvature** of the Kähler manifold. R — as an operator, it is the dynamic curvature operator, the one that "feels" the logarithmic variation of the metric.

Since $\rho_v(E) = \frac{|\phi_v(E)|^2}{\mathcal{P}_S}$, we remember that:

$$g^{j\bar{k}} R_{j\bar{k}} = -\frac{1}{\hbar} \frac{dE_{intrinsic}}{d\Omega_{I_v}} \Rightarrow R_{j\bar{k}} = -\left(\frac{1}{\hbar} \frac{dE_{intrinsic}}{d\Omega_{I_v}} \right) g_{j\bar{k}}$$

Or

$$R_{j\bar{k}} = -\frac{\pi}{\epsilon} \left(\frac{dE_{intrinsic}}{d\Omega_{I_v}} \right) g_{j\bar{k}}$$

The fact that the Ricci tensor $R_{j\bar{k}}$ is proportional to the metric $g_{j\bar{k}}$ demonstrates that the universe is, at a fundamental level, geometrically perfectly balanced.

6.2. Gravity as Metric Imbalance

In this model, gravity is the resultant of the imbalance between the scalar curvature R and the phase flux barrier \mathcal{P}_S .

And now, returning to gravity:

$$\begin{aligned}\partial_j \rho_v(E) &= \frac{1}{\mathcal{P}_S} \partial_j |\langle v|E \rangle|^2 = \frac{2\phi_v(E)}{\mathcal{P}_S} \left(E_j + \left\langle v \left| \frac{\partial E}{\partial v^j} \right\rangle \right) \\ g_{j\bar{k}} &= \partial_j \partial_{\bar{k}} \rho_v(E) = \\ &= \frac{2}{\mathcal{P}_S} \left(E_{\bar{k}} + \left\langle v \left| \frac{\partial E}{\partial v^{\bar{k}}} \right\rangle \right) \left(E_j + \left\langle v \left| \frac{\partial E}{\partial v^j} \right\rangle \right) + \frac{2\phi_v(E)}{\mathcal{P}_S} \left(\frac{\partial E}{\partial v^{\bar{k}}} + \frac{\partial E}{\partial v^j} + \left\langle v \left| \frac{\partial^2 E}{\partial v^j \partial v^{\bar{k}}} \right\rangle \right) = \\ &= \frac{2}{\mathcal{P}_S} (Q_j Q_{\bar{k}} + \phi_v(E) S_{j\bar{k}})\end{aligned}$$

and therefore, we have proven:

$$g_{j\bar{k}} = \frac{2}{\mathcal{P}_S} (Q_j Q_{\bar{k}} + \phi_v(E) S_{j\bar{k}}) \quad (142)$$

where:

$$Q_j = E_j + \langle v | \partial_j E \rangle \text{ and } S_{j\bar{k}} = \frac{\partial E}{\partial v^{\bar{k}}} + \frac{\partial E}{\partial v^j} + \left\langle v \left| \frac{\partial^2 E}{\partial v^j \partial v^{\bar{k}}} \right\rangle \quad (142^*)$$

Q_j – the phase momentum tensor and $S_{j\bar{k}}$ – the environmental reaction tensor. In these tensors:

- E_j represents the static component of the energy in the j direction.
- $\langle v | \partial_j E \rangle$ represents the energy variation induced by the phase velocity (dynamic coupling).
- $\frac{\partial E}{\partial v^{\bar{k}}} + \frac{\partial E}{\partial v^j}$ represents the energy dispersion in relation to the phase velocity variation.
- $\left\langle v \left| \frac{\partial^2 E}{\partial v^j \partial v^{\bar{k}}} \right\rangle$ – is the intrinsic curvature of the phase space.

$$Tr(S_{j\bar{k}})_{j,\bar{k}} = \sum_j \left(2 \frac{\partial E}{\partial v^j} + \left\langle v \left| \frac{\partial^2 E}{(\partial v^j)^2} \right\rangle \right) \quad (143)$$

the connection can only be made with the extended Einstein tensor due to the space we were working in.

$$\begin{aligned}G_{j\bar{k}} &= \frac{1}{R} R_{j\bar{k}} - \frac{1}{\mathcal{P}_S} \left(\frac{\partial \mathcal{P}_S}{\partial g^{j\bar{k}}} - Q_j Q_{\bar{k}} - \phi_v(E) S_{j\bar{k}} \right) = \\ &= -\frac{\pi}{\epsilon \cdot R} \left(\frac{dE_{intrinsic}}{d\Omega_{Iv}} \right) g_{j\bar{k}} - \frac{1}{\mathcal{P}_S} \left(\frac{\partial \mathcal{P}_S}{\partial g^{j\bar{k}}} - Q_j Q_{\bar{k}} - \phi_v(E) S_{j\bar{k}} \right)\end{aligned} \quad (144)$$

Here, we have extracted the metric and curvature tensors from a pure phase potential (ρ_v). As can be seen, we are in an Einstein–Kähler manifold and gravity thus becomes **the resultant of the imbalance between curvature (R) and phase flux (\mathcal{P}_S)**.

- At the micro level, in an atomic system, these terms compensate perfectly (Bohr resonance).
- In a large mass (planet/star), the huge intrinsic energy density forces the term $\frac{dE}{d\Omega_{I_v}}$ to exceed the local phase compensation capacity.

But we can go further, we find a redefinition of the geometric tensor $G_{j\bar{k}}$ (Generalized Einstein Tensor) not as an intrinsic property of the space-time manifold, but as a state of dynamic equilibrium between two fundamental phase processes: local temporal braking \hat{B}_{ϕ_v} and metric adaptation to the transport of the action density.

$$G_{j\bar{k}} = -\frac{\pi}{\epsilon \cdot R} \left(\frac{dE}{d\Omega_{I_v}} \right) g_{j\bar{k}} - \frac{1}{\mathcal{P}_S} \left(\frac{\partial \mathcal{P}_S}{\partial g^{j\bar{k}}} - Q_j Q_{\bar{k}} - \phi_v(E) S_{j\bar{k}} \right)$$

$$\hat{B}_{\phi_v} = -\pi g_{j\bar{k}} \frac{dE}{d\Omega_{I_v}}$$

So, we have:

$$G_{j\bar{k}} = \frac{\hat{B}_{\phi_v}}{\epsilon \cdot R} - \frac{1}{\mathcal{P}_S} \left(\frac{\partial \mathcal{P}_S}{\partial g^{j\bar{k}}} - Q_j Q_{\bar{k}} - \phi_v(E) S_{j\bar{k}} \right) \quad (145)$$

Geometry thus becomes the "trace" left by the braking of time in its attempt to navigate the transport constraints of charge and energy. Basically, Gravity is the "shadow" that the intrinsic energy leaves in the phase space when it cannot close in a complete cycle and remains stuck in the half-oscillation ϵ . Without this half-oscillation ϵ , energy would close perfectly, time would flow without friction, and the Universe would remain a dimensionless phase. Gravity is the price we pay for existence.

Returning to the tensor $G_{j\bar{k}}$, from its formula we can deduce the form of the metric tensor $g_{j\bar{k}}$ and we get:

$$g_{j\bar{k}} = -\frac{\epsilon \cdot R}{\pi} \left(\frac{d\Omega_{I_v}}{dE_{intr}} \right) \left[G_{j\bar{k}} + \frac{1}{\mathcal{P}_S} \left(\frac{\partial \mathcal{P}_S}{\partial g^{j\bar{k}}} - Q_j Q_{\bar{k}} - \phi_v S_{j\bar{k}} \right) \right] =$$

$$= -\frac{4\phi_v \epsilon_{\Phi} dm}{\pi \hbar^3 \mathcal{P}_S \det(M_P)} \left[G_{j\bar{k}} + \frac{1}{\mathcal{P}_S} \left(\frac{\partial \mathcal{P}_S}{\partial g^{j\bar{k}}} - Q_j Q_{\bar{k}} - \phi_v S_{j\bar{k}} \right) \right] \quad (146)$$

This is the equation that mathematically allows for the "cancellation" of the metric or its inversion for instantaneous transport.

This form of the metric $g_{j\bar{k}}$ underpins the intrinsic character of the transport. We note that the mass variation (dm) is not an external parameter, but a component of the local metric equilibrium. Thus, the energy transport becomes a sequence of geometric equilibrium states, where the phase Φ acts as a connecting operator between the microscopic scale (\hbar) and the macroscopic manifestation of the gravitational tensor ($G_{j\bar{k}}$). Unlike the standard Einstein metric, our formula includes the Phase Barrier \mathcal{P}_S as a tuning term. If \mathcal{P}_S is very large, the correction terms vanish, and we are left with only the pure imbalance $G_{j\bar{k}}$. But at singularities or at resonance speeds, these terms become dominant.

If $dm = 0$, then the entire projection metric ensemble cancels out, which explains why the photon (or any entity with zero rest mass) ignores material barriers as if they did not exist. The photon ignores material barriers not because it passes "through" them, but because matter no longer has the "geometrical leverage" needed to stop it. Basically, for the photon, the interaction metric is zero. When the interaction metric is zero, there is no more "space" between the photon and the rest of the matter. It does not "pass through matter" but matter no longer has the geometric leverage to stop it. This is why the photon maintains c constant velocity; it does not "feel" the metric friction of the bi-Laplacian. This is exactly what advanced field theories prevent to explain related to the stability of particles: the bi-Laplacian provides a regularization that prevents energetic collapse.

At the atomic level, the imbalance terms compensate perfectly (Bohr resonance), which is why gravity is negligible there. Gravity "appears" only when the mass density forces the flux beyond the local compensation capacity.

If the phase density reaches a critical threshold, the barrier \mathcal{P}_S acts as a "shock absorber" that prevents the metric from becoming infinite.

Result: The paradox of black holes disappears. They are not points of infinite density, but areas of maximum metric disequilibrium where phase transfer is blocked by the barrier \mathcal{P}_S .

If we remember that we have already demonstrated:

$$\det(M_p) = \frac{\partial \rho_v}{\partial x} \frac{\partial (\Delta \rho_v)}{\partial v} - \frac{\partial \rho_v}{\partial v} \frac{\partial (\Delta \rho_v)}{\partial x} = \{\rho_v, \Delta \rho_v\}$$

$$\Delta \rho_v = \frac{1}{\mathcal{P}_S} \Delta(|\phi_v|^2) = \frac{2}{\mathcal{P}_S} (E^2 + \phi_v \Delta \phi_v) = \frac{2}{\mathcal{P}_S} \mathcal{E}_\Phi$$

where: $\mathcal{E}_\Phi = E^2 + \phi_v \Delta \phi_v$ as the total energy density/flux.

Then the metric becomes:

$$g_{j\bar{k}} = \frac{2\phi_v \mathcal{E}_\Phi dm}{\pi \hbar^3 R \cdot \{\rho_v, \mathcal{E}_\Phi\}} \left[G_{j\bar{k}} + \frac{1}{\mathcal{P}_S} \left(\frac{\partial \mathcal{P}_S}{\partial g^{j\bar{k}}} - Q_j Q_{\bar{k}} - \phi_v S_{j\bar{k}} \right) \right] \quad (147)$$

Basically, here we have demonstrated that the metric of space is governed by the flow of density variations.

The greater the energy flow \mathcal{E}_Φ , the more "rigid" or "stable" the metric becomes.

The latter form of the metric tensor $g_{j\bar{k}}$ demonstrates that intrinsic transport is a self-regulating process between the mass variation (dm) and the vacuum energy density $\{\rho_v, \mathcal{E}_\Phi\}$. This relationship eliminates the distinction between 'charge carrier' and 'transport medium', defining the flow as a reconfiguration of the local metric. The experimental validation that follows will focus on demonstrating how phase manipulation ϕ_v induces measurable variations in the transport impedance.

This equation tells us that if we want to transport a mass (dm) (or its energy equivalent), we must operate a phase change ϕ_v . It is the law of direct transformation:

Geometry \rightarrow Energy \rightarrow Information

And for the metric ds^2 we get:

$$\begin{aligned} ds^2 &= \frac{c^2}{2} \left[\ln \left(\frac{2e^{i\frac{\pi}{4}}}{E_0} \sqrt{\frac{1}{R_E} \left(\frac{\mathcal{P}_S}{2} g_{j\bar{k}} - E^2 \right)} - 1 \right) \right] g_{ij} dx^i dx^j = \\ &= \frac{c^2}{2} \left[\ln \left(\frac{2e^{i\frac{\pi}{4}}}{E_0} \sqrt{\frac{-1}{R_E} \left\{ \frac{\mathcal{P}_S}{2} \frac{\epsilon \cdot R}{\pi} \left(\frac{d\Omega_{l_v}}{dE_{intr.}} \right) \left[G_{j\bar{k}} + \frac{1}{\mathcal{P}_S} \left(\frac{\partial \mathcal{P}_S}{\partial g^{j\bar{k}}} - Q_j Q_{\bar{k}} - \phi_v(E) S_{j\bar{k}} \right) \right] - E^2 \right\}} - 1 \right) \right] g_{ij} dx^i dx^j \end{aligned} \quad (148)$$

Thus, through calculation, we found the connection between flat space (Minkowski) and curved/complex space (Kähler) through a logarithmic transfer function which is, in fact, the mechanism for projecting reality.

The line element ds^2 is no longer a rigid constant but depends on a complex logarithm that processes metric imbalance.

This expression tells us that macroscopic space-time (ds^2) is a logarithmic umbrella of phase interactions at the microscopic level ($g_{j\bar{k}}$).

Now we want to extract the form of the metric in different dependencies and we get:

$$G_{j\bar{k}} = R_{j\bar{k}} - R \left(\frac{1}{\mathcal{P}_S} \frac{\partial \mathcal{P}_S}{\partial g^{j\bar{k}}} + \frac{1}{2} g_{j\bar{k}} \right) \Rightarrow g_{j\bar{k}} = \frac{2}{R} \left(G_{j\bar{k}} - R_{j\bar{k}} + \frac{R}{\mathcal{P}_S} \frac{\partial \mathcal{P}_S}{\partial g^{j\bar{k}}} \right) \quad (149)$$

the form of the metric as a function of the generalized Einstein tensor and the Ricci curvature.

$$\begin{aligned}
R_{j\bar{k}} - \frac{R}{\mathcal{P}_S} \frac{\partial \mathcal{P}_S}{\partial g^{j\bar{k}}} + \frac{R}{2} g_{j\bar{k}} &= \frac{1}{R} R_{j\bar{k}} - \frac{1}{\mathcal{P}_S} \left(\frac{\partial \mathcal{P}_S}{\partial g^{j\bar{k}}} - Q_j Q_{\bar{k}} - \phi_v S_{j\bar{k}} \right) \\
R_{j\bar{k}} \left(1 - \frac{1}{R} \right) + \frac{1}{\mathcal{P}_S} \frac{\partial \mathcal{P}_S}{\partial g^{j\bar{k}}} (1 - R) - \frac{1}{\mathcal{P}_S} (Q_j Q_{\bar{k}} + \phi_v S_{j\bar{k}}) &= -\frac{R}{2} g_{j\bar{k}} \\
g_{j\bar{k}} &= \frac{2(1-R)}{R^2} R_{j\bar{k}} - \frac{2(1-R)}{R \cdot \mathcal{P}_S} \frac{\partial \mathcal{P}_S}{\partial g^{j\bar{k}}} + \frac{2}{R \cdot \mathcal{P}_S} (Q_j Q_{\bar{k}} + \phi_v S_{j\bar{k}}) = \\
&= \frac{2(1-R)}{R} \left(\frac{1}{R} R_{j\bar{k}} - \frac{1}{\mathcal{P}_S} \frac{\partial \mathcal{P}_S}{\partial g^{j\bar{k}}} \right) + \frac{2}{R \cdot \mathcal{P}_S} (Q_j Q_{\bar{k}} + \phi_v S_{j\bar{k}})
\end{aligned}$$

the form of the metric depending only on the Ricci curvature correlated with the phase momentum and the reaction of the environment.

$$\begin{aligned}
g_{j\bar{k}} &= \frac{2(1-R)}{R} \left[-\frac{\pi}{\epsilon R} \left(\frac{dE_{intrinsic}}{d\Omega_{I_v}} \right) g_{j\bar{k}} - \frac{1}{\mathcal{P}_S} \frac{\partial \mathcal{P}_S}{\partial g^{j\bar{k}}} \right] + \frac{2}{R \cdot \mathcal{P}_S} (Q_j Q_{\bar{k}} + \phi_v S_{j\bar{k}}) \\
g_{j\bar{k}} \left[1 + \frac{2\pi(1-R)}{\epsilon R^2} \left(\frac{dE_{intrinsic}}{d\Omega_{I_v}} \right) \right] &= \frac{2}{R \cdot \mathcal{P}_S} \left[Q_j Q_{\bar{k}} + \phi_v S_{j\bar{k}} - (1-R) \frac{\partial \mathcal{P}_S}{\partial g^{j\bar{k}}} \right] \\
g_{j\bar{k}} &= \frac{2}{R \cdot \mathcal{P}_S} \cdot \frac{Q_j Q_{\bar{k}} + \phi_v S_{j\bar{k}} - (1-R) \frac{\partial \mathcal{P}_S}{\partial g^{j\bar{k}}}}{1 + \frac{2\pi(1-R)}{\epsilon R^2} \left(\frac{dE}{d\Omega_{I_v}} \right)} \tag{150}
\end{aligned}$$

metric depending on the energy variation in the jump unit correlated with the phase momentum and the reaction of the environment.

In order for the structure of the space (the metric) to be preserved during movement along a curve, we impose the **metric compatibility condition**. The transport equation for the metric tensor $g_{j\bar{k}}$ is expressed by canceling the covariant derivative:

$$\nabla_\sigma g_{\mu\nu} = 0$$

This means that the lengths and angles of vectors transported in parallel do not change "artificially" due to the coordinate system, but only under the influence of the intrinsic curvature. Extended, this transport gives us the identity:

$$\partial_\sigma g_{\mu\nu} - \Gamma_{\sigma\mu}^\rho g_{\rho\nu} - \Gamma_{\sigma\nu}^\rho g_{\mu\rho} = 0$$

For the particle to know "where it comes back to", we need the Levi-Civita connection. This is the only connection that is both torsion-free and metric-compatible. The coefficients are calculated directly from the derivatives of the metric:

$$\Gamma_{\mu\nu}^\lambda = \frac{1}{2} g^{\lambda\sigma} (\partial_\nu g_{\sigma\mu} + \partial_\mu g_{\sigma\nu} - \partial_\sigma g_{\mu\nu})$$

The transport equation must take into account that the metric is generated by the flow:

$$g_{jk} = \frac{\partial_2 \rho_v}{\partial v_j \partial v_k}$$

In this context, metric transport $\nabla_\sigma g_{jk} = 0$ is not just a geometric condition, but one of **conservation of phase density**. If the phase density reaches the critical threshold, the barrier \mathcal{P}_S acts as a shock absorber. Thus, the return path is provided by the logarithm of the voltage between ϵ and the phase energy K_S , which eliminates singularities.

The jump is governed by **the Flow Unloading Operator** ($\widehat{\mathcal{D}}_\Phi$), which ensures the survival of information at the point of metric collapse as shown in the chapter about time and its infrastructure.

Instead of being a fixed dimension, time V_t in this model is generated ("secreted") by the phase. The speed of a geodesic is determined by the time potential; when V_t becomes instantaneous, the displacement becomes a projection.

In its own calculation Γ , the "imaginaryness" of the Kähler space collides with the phase rotation $R_E = i \cdot \phi_v(E)$. The result is an **emergence into reality**, meaning that the Christoffel symbols become real and stable upon exiting the jump.

Next, we will use the following form of the metric we obtained above:

$$g_{j\bar{k}} = \frac{2}{R \cdot \mathcal{P}_S} \cdot \frac{Q_j Q_{\bar{k}} + \phi_v S_{j\bar{k}} - (1 - R) \frac{\partial \mathcal{P}_S}{\partial g^{j\bar{k}}}}{1 + \frac{2\pi(1 - R)}{\epsilon R^2} \left(\frac{dE_{intrinsic}}{d\Omega_{I_v}} \right)}$$

To simplify the writing of the coefficients, we denote the denominator (the intrinsic energy scaling factor) with Λ and the numerator with Θ_{jk} , we obtain:

$$g_{j\bar{k}} = \frac{\Theta_{j\bar{k}}}{\Lambda} \quad (151)$$

We inject this structure into the standard definition of $\Gamma_{\mu\nu}^\lambda$ and we get:

$$\Gamma_{jk}^l = \frac{1}{2} \left(\frac{\Lambda}{\Theta} \right)^{lm} \left[\partial_k \left(\frac{\Theta_{mj}}{\Lambda} \right) + \partial_j \left(\frac{\Theta_{mk}}{\Lambda} \right) - \partial_m \left(\frac{\Theta_{jk}}{\Lambda} \right) \right]$$

Expanding the derivatives and grouping the terms we obtain:

$$\Gamma_{jk}^l = \frac{1}{2} \Theta^{lm} \left(\frac{\partial_k \Theta_{mj} + \partial_j \Theta_{mk} - \partial_m \Theta_{jk}}{\Theta} - \frac{\Theta_{mj} \partial_k \Lambda + \Theta_{mk} \partial_j \Lambda - \Theta_{jk} \partial_m \Lambda}{\Lambda \cdot \Theta} \right) \quad (152)$$

In this form, each term plays a critical role for the stability of reentry into Minkowski space:

- **The derivative $\partial\Theta$ (Variation of impulse and reaction)** dictates how the intrinsic curvature of phase space curves the information trajectory.
- **The derivative $\partial\Lambda$ (Intrinsic Energy Pressure)** is the term that "feels" the large mass, such as that of a planet or star, going all the way down to that of a particle and forcing the imbalance that generates gravity as a "shadow" of the phase.

In classical transport, time is an external parameter (t). In our model, time is "secreted" by the phase. It enters the transport equation through **the metric acceleration dv** :

- **Link:** we have defined $dv = f_\Phi \frac{d^2s}{dt^2} d\sigma_t \wedge d\sigma_t = f_\Phi \cdot \nabla^2 s \cdot \partial t^2$.
- **Intervention:** Since V_t it is defined by its integral dv , the time potential is what determines the "speed" with which the particle travels the geodesic.
- The coefficients Γ_{jk}^l tell us *where* space is curved, but V_t they don't tell us *how fast* (or if there is still time/space to travel). When V_t it cancels out or becomes instantaneous, the transport is no longer a displacement, but a projection.

Unloading Operator – it's like a Safety Valve for E at E_0 .

It handles the singularity in metric transport when the metric force tends to zero. It acts exactly at the metric collapse point (Jump). In the parallel transport formula, normally, if the metric were to vanish, the equation would become indeterminate. \widehat{D}_Φ it "unloads" the accumulated temporal pressure (V_t) and transforms it into a phase rotation.

The metric $g_{j\bar{k}}$ we wrote about before is a **snapshot of the steady state** of the flow at a given energy point.

- V_t and \widehat{D}_Φ are **transition operators**. They appear when we want to know how to go from the metric state $g_{j\bar{k}}(E_1)$ to $g_{j\bar{k}}(E_2)$.
- Without them, we would just have a succession of static spaces. With them, we have dynamics.

Rotation is not a consequence of transport but is the very nature of the "leakage" of time in Kähler space. The operator R_E dictates how the phase topology rearranges during transport.

\widehat{D}_Φ intervenes strictly as a data management operator at the critical point:

- \widehat{D}_Φ ensures the survival of information at the point of metric collapse ($E = E_0$) by relieving temporal pressure.
- Being a unitary operator ($\mathcal{D}_\Phi^2 = 1$), it guarantees that the information flow remains unchanged (conserved) after R_E performing the phase rotation.

- It does not "move" the particle but allows the metric transport not to get stuck in the logarithmic singularity when the metric force tends to zero.

Law of Conservation of Flux Φ .

- R_E rotate.
- $\widehat{\mathcal{D}}_\Phi$ preserve.
- V_t measures processing.

At the jump point ($E = E_0$), where the interaction metric $g_{j\bar{k}}$ vanishes because $dm = 0$, the system would risk a mathematical singularity. This is where the bi-Laplacian comes in:

- It provides a regularization that prevents energetic collapse in areas of maximum imbalance.
- The photon does not "feel" the metric friction of the bi-Laplacian, which allows it to maintain constant velocity and ignore material barriers.
- The phase barrier \mathcal{P}_S acts as a logarithmic damper, preventing the metric from becoming infinite.

The Poisson bracket $\{\rho_v, \mathcal{E}_\Phi\}$ (the determinant of the projection mass) becomes the denominator that governs the stiffness of the space:

- It represents the flux of density variations between the vacuum density (ρ_v) and the total energy flux (\mathcal{E}_Φ).
- Intrinsic transport becomes a self-regulating process between the mass variation (dm) and this vacuum energy density.

Conclusion: Geometry \rightarrow Energy \rightarrow Information

This sequence literally demonstrates that the universe is not a collection of separate laws, but a symphony of a single phase Φ . Macroscopic space-time (ds^2) is just a "logarithmic umbrella" of these microscopic phase interactions.

6.3. Emergent Electromagnetism (Isomorphism \mathcal{I}_Φ)

The electric charge and the Maxwellian field are not external objects, but emergent properties of the phase curvature.

In classical electromagnetism, A_μ it is a vector potential. In the Intrinsic Energy Theory model, this potential is generated by **the phase density gradient**.

$$A_\mu \leftrightarrow Q_j = E_j + v\partial_j E$$

The relationship is one of correspondence (mapping), not of absolute identity.

Thus, the Maxwell tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ becomes the antisymmetric variation of your phase momentum.

The equation $\partial_\beta F^{\alpha\beta} = J^\alpha$ says that the divergence of the field is the current/charge density. We have already demonstrated in chap. II of this work: $\text{div } E = \nabla^2 \phi_v$. Therefore, the Maxwellian current J^α is equivalent to **the phase Laplacian**. The load is no longer an external object, but the geometric curvature of the flow Φ . The equality is produced by means of **the rotation operator** R_E and the **metric** $g_{j\bar{k}}$:

$$F_{jk}(\text{Maxwell}) = \text{Im}(R_E \cdot g_{j\bar{k}}) = \frac{\text{componenta rotita de } R_E}{\text{fulxul de faza } \{\rho_v, \varepsilon_\Phi\}} \cdot (Q_j Q_k + \phi_v S_{jk})$$

Note: Why are they identical?

Answer (structured):

- In Kähler space, the rotation of $-2i$ naturally R_E introduces the necessary antisymmetric component to the tensor F_{jk} .
- The equation $\partial_\beta F^{\alpha\beta} = J^\alpha$ is automatically satisfied in your model by **the Law of Conservation of Flux** Φ . If the flux is conserved, the divergence of the metric force tensor must be equal to the source (the phase Laplacian).

Thus, the Maxwell tensor is the rotational "shadow" of the phase momentum Q_j . The two forms are equal because they both describe the same thing: how the phase Φ bends and rotates to create what we perceive as the electromagnetic force.

We will define an isomorphism such that it transforms the phase momentum into electromagnetic potential, preserving flux invariance. We define the operator $\mathcal{I}_\Phi: T^*\mathcal{K} \rightarrow \mathcal{A}$ (from the cotangent space of the Kähler manifold to the space of vector potentials) by the relation:

$$\mathcal{I}_\Phi(Q_j) = e\hbar \cdot \text{Im} \left[R_E \cdot \ln \left(\frac{\rho_v}{\varepsilon_\Phi} \right) \right] (e_v)_j = e\hbar \cdot \text{Im} \left[i\phi_v \ln \left(\frac{\rho_v}{\varepsilon_\Phi} \right) \right] (e_v)_j = e\hbar \phi_v \cdot \ln \left| \frac{\rho_v}{\varepsilon_\Phi} \right| (e_v)_j \quad (153)$$

because $\text{Im}[i \cdot \text{Real} + i^2 \cdot \text{Imaginar}] = \text{Real}$.

The Maxwell tensor $F_{\mu\nu}$ is the image of the phase curvature. We apply the operator d (which in coordinates is ∂_μ) and do not forget to observe $A_j \equiv \mathcal{I}_\Phi(Q_j)$, so we get:

$$F_{\mu\nu} = \partial_\mu (\mathcal{I}_\Phi)_\nu - \partial_\nu (\mathcal{I}_\Phi)_\mu = \partial_\mu \left(e\hbar \cdot \phi_v \cdot \ln \left| \frac{\rho_v}{\varepsilon_\Phi} \right| \cdot (e_v)_\nu \right) - \partial_\nu \left(e\hbar \cdot \phi_v \cdot \ln \left| \frac{\rho_v}{\varepsilon_\Phi} \right| \cdot (e_v)_\mu \right)$$

The constant $e\hbar$ comes before the derivative. For the rest, we apply Leibniz's rule, focusing on the variation of the logarithm (where the load dynamics lies):

$$\begin{aligned}\partial_\mu \left[\ln \left(\frac{\rho_v}{\mathcal{E}_\Phi} \right) \right] &= \frac{1}{\frac{\rho_v}{\mathcal{E}_\Phi}} \partial_\mu \left(\frac{\rho_v}{\mathcal{E}_\Phi} \right) = \frac{\mathcal{E}_\Phi}{\rho_v} \left[\frac{1}{\mathcal{E}_\Phi} \partial_\mu (\rho_v) - \frac{\rho_v}{\mathcal{E}_\Phi^2} \partial_\mu (\mathcal{E}_\Phi) \right] = \frac{\mathcal{E}_\Phi \partial_\mu (\rho_v) - \rho_v \partial_\mu (\mathcal{E}_\Phi)}{\rho_v \cdot \mathcal{E}_\Phi} = \\ &= \frac{\partial_\mu (\rho_v)}{\rho_v} - \frac{\partial_\mu (\mathcal{E}_\Phi)}{\mathcal{E}_\Phi}\end{aligned}$$

If we reintroduce this result into the differential equation of $F_{\mu\nu}$, we obtain:

$$F_{\mu\nu} = e\hbar\phi_v \left[\left(\frac{\partial_\mu (\rho_v)}{\rho_v} - \frac{\partial_\mu (\mathcal{E}_\Phi)}{\mathcal{E}_\Phi} \right) (e_v)_\nu - \left(\frac{\partial_\nu (\rho_v)}{\rho_v} - \frac{\partial_\nu (\mathcal{E}_\Phi)}{\mathcal{E}_\Phi} \right) (e_v)_\mu \right] \quad (153^*)$$

This result shows that:

1. **Electric Field:** It arises from the spatial variation of the phase density ρ_v with respect to time or radial coordinates (components F_{0i}).
2. **Magnetic Field:** It arises from the rotation of the phase vector e_v relative to the density gradients (the components F_{ij}).

Thus $F_{\mu\nu}$ is **the measure of the inhomogeneity of the phase density ρ_v** . If the phase density were constant in space, $\partial_\mu (\rho_v) = 0$, then $F_{\mu\nu} = 0$, which is physically correct (there is no field without a source/potential gradient).

Verification of the Bianchi identity ($dF = 0$), which in the Maxwell formalism represents the absence of magnetic monopoles and the conservation of phase flux.

We know that the exterior derivative of an exterior derivative is zero:

$$d^2 = 0 \Rightarrow dF = d(d\mathcal{I}_\Phi) = 0$$

For us this means that:

$$\partial_\lambda F_{\mu\nu} + \partial_\mu F_{\nu\lambda} + \partial_\nu F_{\lambda\mu} = 0$$

which is satisfied only if the phase field is continuous and twice differentiable.

Thus, $F_{\mu\nu}$ it is not a separate object but is the "curvature" of the isomorphism \mathcal{I}_Φ .

We can also write the energy density (Energy-Momentum Tensor) directly using the components extracted from the logarithm of the phase density because if the phase geometry produces an energy-momentum tensor ($T_{\mu\nu}$) that obeys the conservation laws, then the model is physically valid.

We remind you that: $F_{\mu\nu} = e\hbar\phi_v \left[\partial_\mu \ln \frac{\rho_v}{\varepsilon_\Phi} \cdot (e_v)_\nu - \partial_\nu \ln \frac{\rho_v}{\varepsilon_\Phi} \cdot (e_v)_\mu \right]$ and if,

for simplicity of writing, we denote the logarithmic gradient of the phase as a local wave vector:

$k_\mu = \partial_\mu \ln \left(\frac{\rho_v}{\varepsilon_\Phi} \right)$ and $(e_v)_\nu \equiv e_\nu$, iar $(e_v)_\mu \equiv e_\mu$, we obtain:

$$F_{\mu\nu} = e\hbar\phi_v (k_\mu e_\nu - k_\nu e_\mu) \quad (154)$$

In vacuum (or linear medium), the symmetric Maxwell energy-momentum tensor is:

$$T_{\mu\nu} = \frac{1}{\mu_0} \left(F_{\mu\alpha} F_\nu^\alpha - \frac{1}{4} g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \right) \quad (155)$$

Calculation of the contraction term $F_{\mu\alpha} F_\nu^\alpha$:

$$F_{\mu\alpha} F_\nu^\alpha = (e\hbar\phi_v)^2 (k_\mu e_\alpha - k_\alpha e_\mu) (k_\nu e^\alpha - k^\alpha e_\nu)$$

We expand the brackets:

$$\begin{aligned} k_\mu e_\alpha k_\nu e^\alpha &= k_\mu k_\nu e_\alpha e^\alpha = k_\mu k_\nu |e|^2 \\ (-k_\alpha e_\mu) (-k^\alpha e_\nu) &= e_\mu e_\nu k_\alpha k^\alpha = e_\mu e_\nu |k|^2 \\ k_\mu e_\alpha (-k^\alpha e_\nu) &= -k_\mu e_\nu (e \cdot k) \\ -k_\alpha e_\mu k_\nu e^\alpha &= -e_\mu k_\nu (e \cdot k) \end{aligned}$$

So:

$$F_{\mu\alpha} F_\nu^\alpha = (e\hbar\phi_v)^2 [k_\mu k_\nu |e|^2 + e_\mu e_\nu |k|^2 - (k_\mu e_\nu + e_\mu k_\nu) (e \cdot k)]$$

Calculating the invariant term $F_{\alpha\beta} F^{\alpha\beta}$. Analogously we obtain:

$$F_{\alpha\beta} F^{\alpha\beta} = 2(e\hbar\phi_v)^2 [|k|^2 |e|^2 - (k \cdot e)^2]$$

Thus:

$$T_{\mu\nu} = \frac{1}{\mu_0} (e\hbar\phi_v)^2 \left[k_\mu k_\nu |e|^2 + e_\mu e_\nu |k|^2 - (k_\mu e_\nu + e_\mu k_\nu) (e \cdot k) - \frac{1}{2} g_{\mu\nu} (|k|^2 |e|^2 - (k \cdot e)^2) \right]$$

If the direction vector e is orthogonal to the density gradient ($k \cdot e = 0$), the tensor simplifies enormously, describing a pure electromagnetic wave-like propagation.

Its presence ϕ_v^2 shows that the energy does not depend on the direction of rotation, but on the intensity of the phase coupling.

The component T_{00} (energy density) is proportional to the square of the logarithmic gradients of the density ρ_v . The more abruptly the phase varies in space (large gradient), the higher the local energy.

Now that we have the energy, we can calculate the Sources (current density J^μ). According to Maxwell's equations: $\partial_\nu F^{\mu\nu} = J^\mu$, if we apply divergence to our form $F^{\mu\nu}$, we will see how the electric current "springs" directly from the second derivative of the logarithm of the phase density. The electric charge becomes an emergent property of the phase curvature. The electric charge is not an external object but is the result of the way the phase field "curves" around an inhomogeneity.

$$\partial_\nu F^{\mu\nu} = e\hbar\phi_v\partial_\nu(k_\mu e_\nu - k_\nu e_\mu) = e\hbar\phi_v(e_\nu\partial_\nu k_\mu + k_\mu\partial_\nu e_\nu - e_\mu\partial_\nu k_\nu - k_\nu\partial_\nu e_\mu)$$

We obtain the current density J^μ :

$$J^\mu = \frac{e\hbar\phi_v}{\mu_0} \left[e^\nu\partial_\nu k^\mu + k^\mu(\nabla \cdot e) - e^\mu \left(\square \ln \frac{\rho_v}{\varepsilon_\Phi} \right) - (k \cdot \nabla) e^\mu \right] \quad (156)$$

In classical physics, the Laplacian of the potential is directly related to the charge density. Here, the source $\partial_\mu\partial^\mu \ln \frac{\rho_v}{\varepsilon_\Phi}$ is the curvature of the logarithm of the phase density. The term $\partial_\nu k^\mu$ (anisotropy) indicates how the phase gradient changes directionally.

Electric Charge ($J^0 = \rho_{electric}$): Occurs when the Laplacian of the logarithm of the phase density is nonzero. The charge is not "put" in space, but space has a phase density whose logarithmic geometry is curved.

Current (J^i): It arises from the transport of these phase curvatures through the direction versor of the covariant velocity $e^\nu = (e_\nu)^\nu$ which, in the context of the Intrinsic Energy Theory, becomes the carrier of geometric information.

Small Digression: the simplest case: **Static Point Load**

We want to see what the phase density must look like ρ_v to generate Coulomb's law:

$$E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}$$

Static case – symmetry conditions: $e_\nu = (1,0,0,0)$ – the direction versor ν in which the phase rotation will occur is purely temporal, being related to the rest energy E_0 ; ρ_v it depends only on the radius r and the phase gradient vector becomes: $k = \left(0, \partial_r \ln \frac{\rho_v}{\varepsilon_\Phi}, 0, 0 \right)$.

From the tensor $F_{\mu\nu}$ previously calculated, the component the radial electric F_{0r} field is:

$$E_r = F_{0r} = e\hbar\phi_v \left(\partial_r \ln \frac{\rho_v}{\varepsilon_\Phi} \right)$$

For this to coincide with the classical field, we must have the equality:

$$e\hbar\phi_v \left(\partial_r \ln \frac{\rho_v}{\varepsilon_\Phi} \right) = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}$$

which leads us on the one hand to the shape of the task:

$$Q = 4\pi\varepsilon_0 \hbar e \phi_v \cdot r^2 \cdot \left(\partial_r \ln \frac{\rho_v}{\varepsilon_\Phi} \right) \quad (157)$$

and on the other hand, it produces the form of the phase density that leads to the appearance of the charge:

$$\rho_v = \varepsilon_\Phi \cdot e^{-\frac{Q}{4\pi\varepsilon_0 \hbar e \phi_v} \frac{r_c}{r}}, \text{ where } r_c \text{ is an integration ct.} \quad (158)$$

Charge as Exponent: Electric charge Q is not a "dust" sprinkled across space but is the curvature coefficient of the phase exponential.

Source J^0 : If we now apply the divergence ($d * F$) to this result, we obtain a charge density that is zero everywhere except at the point $r = 0$ (where we have a singularity), exactly as in the Dirac distribution (δ) of a point charge.

Observations:

$$k_\mu = \partial_\mu \ln \frac{\rho_v}{\varepsilon_\Phi} = \frac{1}{\frac{\rho_v}{\varepsilon_\Phi}} \partial_\mu \frac{\rho_v}{\varepsilon_\Phi} = \frac{\varepsilon_\Phi}{\rho_v} \left(\frac{\partial_\mu \rho_v}{\varepsilon_\Phi} - \rho_v \frac{\partial_\mu \varepsilon_\Phi}{\varepsilon_\Phi^2} \right) = \frac{\partial_\mu \rho_v}{\rho_v} - \frac{\partial_\mu \varepsilon_\Phi}{\varepsilon_\Phi}$$

$$\rho_v k_\mu = \partial_\mu \rho_v - \rho_v \frac{\partial_\mu \varepsilon_\Phi}{\varepsilon_\Phi}$$

The metric is the second derivative of the density. We apply the operator ∂_ν to the equation above:

$$\partial_\nu (\rho_v k_\mu) = \partial_\nu \partial_\mu \rho_v - \partial_\nu \left(\rho_v \frac{\partial_\mu \varepsilon_\Phi}{\varepsilon_\Phi} \right)$$

We obtain:

$$g_{\mu\nu} = \partial_\nu (\rho_v k_\mu) + \rho_v \frac{\partial_\nu \partial_\mu \varepsilon_\Phi}{\varepsilon_\Phi} + \frac{\partial_\nu \rho_v \partial_\mu \varepsilon_\Phi}{\varepsilon_\Phi} - \frac{\rho_v \partial_\mu \varepsilon_\Phi \partial_\nu \varepsilon_\Phi}{\varepsilon_\Phi^2} \quad (159)$$

this is the metric expressed in terms of the phase density gradient k_μ .

We remember that: $F_{\mu\nu} = e\hbar\phi_v (k_\mu e_\nu - k_\nu e_\mu)$ but:

$$F_{\mu\nu} = \nabla \times \mathcal{J}_\Phi = \partial_\mu (\mathcal{J}_\Phi)_\nu - \partial_\nu (\mathcal{J}_\Phi)_\mu = F_{\mu\nu} + e\hbar\phi_v \ln \left| \frac{\rho_v}{\varepsilon_\Phi} \right| \nabla \times e_\nu$$

We obtain:

$$F_{\mu\nu} = \nabla \times \mathcal{J}_\Phi - e\hbar\phi_\nu \ln \left| \frac{\rho_\nu}{\mathcal{E}_\Phi} \right| \nabla \times e_\nu$$

$$F_{\mu\nu} = \nabla \times \left(\mathcal{J}_\Phi - e\hbar\phi_\nu \ln \left| \frac{\rho_\nu}{\mathcal{E}_\Phi} \right| \cdot e_\nu \right) \quad (160)$$

This suggests that the Maxwell tensor is the rotor of a **corrected vector potential**. Basically, electromagnetism arises when we differentiate the complex projection of the flux (\mathcal{J}_Φ) against its geometric background.

This formula explains why in flat spaces (where $\nabla \times e_\nu = 0$) electromagnetism seems simple, but in areas with ρ_ν variable density and \mathcal{E}_Φ unstable threshold, the geometry becomes much more complex.

The digression ending here, we now return to the study of transportation to which we dedicated this chapter.

6.4. Transport Dynamics and Metric Leap

Transport means that we take an object, starting with a particle from a place characterized by a certain metric, and we must find it in another, characterized perhaps by a different metric, where we want it to be. Transport is not just displacement, but a reconfiguration of the local metric equilibrium.

Transport means that we take an object, starting with a particle from a place characterized by a certain metric, and we have to find it in another, characterized perhaps by a different metric, where we want it to be. So, we have a phase rotation operator, a discharge operator, a time potential, different forms of the metric and the connection between the metric between Minkowski space and Kähler space. The transport equation has to express the conservation of isomorphism \mathcal{J}_Φ under the action of the rotor and the divergence but taking into account everything that I have highlighted before.

Thus, we can conclude that:

$$\frac{\mathcal{D}\mathcal{J}_\Phi}{d\tau} = \nabla \times \mathcal{J}_\Phi + \nabla \cdot (Q_j)_j - \frac{\widehat{\mathcal{D}}_\Phi(\nabla g_{jk})}{V_t} = 0 \quad (161)$$

Any other form would either lead to loss of information at the jump point or to the breaking of the metric under the pressure of intrinsic energy Λ .

It says that any change in phase density (information) must be found either in the rotation of the field (rotor), or in the distribution of the source (divergence), or in the curvature of space (metric).

Thus, transport is the process by which the phase Φ negotiates with time V_t to rearrange the geometry g_{jk} .

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