Topological Origin of Three Fermion Generations from the Degeneracy Locus of the Observer's Density Matrix

with Chirality and Normal Neutrino Hierarchy as Structural Predictions

Jin Woo Lee, M.D.

Independent Researcher, Seoul, Republic of Korea

Member, Korean Medical Association; Member, Korean Physical Society

iiorange@gmail.com

March 2026 (revised April 2026)

Abstract

Within the Relational-Informational Model (RIM), gauge fields emerge from the Uhlmann curvature of the observer's reduced density matrix ρ_{Obs}. I show that the number of fermion generations is determined by the topology of the *degeneracy locus*—the set of spacetime points where eigenvalues of ρ_{Obs} coincide.

By the von Neumann–Wigner non-crossing theorem, pairwise eigenvalue degeneracy in a Hermitian family requires three independent real conditions (codimension 3). In four-dimensional spacetime each such condition therefore defines a one-dimensional submanifold—a *degeneracy string*. For the SU(3) sector of RIM, the spectrum of ρ_{Obs} contains three eigenvalues $\{p_1, p_2, p_3\}$, admitting exactly $\binom{3}{2} = 3$ independent pairwise degeneracies; three topologically distinct degeneracy strings therefore exist, each carrying a distinct sector of the Uhlmann holonomy. I identify these three sectors with the three fermion generations.

An explicit qutrit toy model demonstrates that a closed loop in parameter space encircling all three degeneracy points produces a $\mathbb{Z}_3$ monodromy whose braid structure provides an information-geometric origin for the CKM and PMNS mixing matrices and CP violation. Two additional structural results extend the framework: the canonical orientation of the SU(2) degeneracy string provides a topological origin for $\mathrm{SU}(2)_L$ chirality; and the standard spectral ordering of eigenvalues, made canonical by the modular flow of ρ_{Obs}, is consistent with the normal neutrino mass hierarchy $m_1 < m_2 < m_3$.

These results are consistent with precision electroweak data ($N_\nu = 2.984 \pm 0.008$

from LEP), current LHC bounds on fourth-generation quarks, oscillation data mildly preferring the normal hierarchy, and hints of non-zero δ_{CP} from T2K and NOvA.

Keywords: Fermion generations; Degeneracy locus; Uhlmann curvature; von Neumann–Wigner theorem; Braid group; CKM matrix; PMNS matrix; CP violation; Chirality; Normal hierarchy; Modular flow; Quantum information geometry; Relational-Informational Model

---

Contents

1. Introduction

The Standard Model of particle physics contains exactly three generations of fermions. Each generation carries identical gauge quantum numbers under $\mathrm{U}(1) \times \mathrm{SU}(2) \times \mathrm{SU}(3)$ yet differs in mass by orders of magnitude. The number three is not derived from any known principle—it enters as an empirical input.

In the companion papers [1, 2], I established the Relational-Informational Model (RIM), in which the observer's reduced density matrix $\rho_{\mathrm{Obs}} = \mathrm{Tr}_{\mathrm{Sys}}|\Psi\rangle\langle\Psi|$, defined within a globally static Wheeler–DeWitt universe ($\hat{H}_{\mathrm{tot}}|\Psi\rangle = 0$), serves as the single fundamental object from which all interactions emerge. The real part of the mixed-state Quantum Geometric Tensor (msQGT)—the Quantum Fisher Information Matrix (QFIM)—defines the spacetime metric and yields the Einstein field equations via entanglement thermodynamics [1]. The imaginary part—the Uhlmann curvature [3]—produces Yang–Mills gauge fields, with the gauge group determined by the eigenvalue degeneracy structure of ρ_{Obs}: non-degenerate spectra yield U(1), two-fold degeneracy yields SU(2), and three-fold degeneracy yields SU(3) [2].

The degeneracy mechanism answers "which gauge group?" via the *size* of the degeneracy. The present paper addresses the complementary question: given that the gauge group is derived from eigenvalue degeneracies, what determines the number of fermion generations?

The answer lies not in the size of the degeneracy but in its *location* in spacetime. The key mathematical tool is the von Neumann–Wigner non-crossing theorem [4], which establishes a definite codimension for eigenvalue degeneracies in Hermitian families. Combined with the four-dimensionality of spacetime, this yields the number of fermion generations as a topological count—not a free parameter.

This represents a natural extension of the RIM programme: the same eigenvalue spectrum of ρ_{Obs} that generates the gauge group (via degeneracy size) also generates the generation structure (via degeneracy location) and the flavour mixing pattern (via braid topology). Two further structural consequences—the topological origin of chirality and the normal neutrino mass hierarchy—are developed in §8 and §9.

2. Framework: Two Roles of Eigenvalue Degeneracy

2.1. Review of RIM Gauge Structure

The observer's reduced density matrix, parametrised by spacetime coordinates x^μ ($\mu = 0, 1, 2, 3$), has the spectral decomposition

$$\rho_{\mathrm{Obs}}(x) = \sum_n p_n(x)\, |n(x)\rangle\langle n(x)|, \tag{1}$$

where $p_n(x) > 0$ and $\sum_n p_n = 1$. As established in [2], the Uhlmann connection $A_\mu^{(U)}$ on the bundle of purifications of ρ_{Obs} gives rise to the curvature

$$F_{\mu\nu}^{(U)} = \partial_\mu A_\nu^{(U)} - \partial_\nu A_\mu^{(U)} - i\big[A_\mu^{(U)}, A_\nu^{(U)}\big], \tag{2}$$

with gauge group determined by the eigenvalue degeneracy:

$$\begin{aligned} &\text{Non-degenerate:} \quad \mathrm{U}(1),\\ &\text{2-fold degeneracy:} \quad \mathrm{SU}(2),\\ &\text{3-fold degeneracy:} \quad \mathrm{SU}(3). \end{aligned}$$

2.2. Two Distinct Roles of Degeneracy

The eigenvalue spectrum of ρ_{Obs} plays two distinct roles:

(1) **Size of degeneracy** (how many eigenvalues coincide at a given point) determines the gauge group—the local algebraic structure of the Uhlmann connection [2].

(2) **Location of degeneracy** (where in spacetime eigenvalues coincide) determines the generation structure—the global topology of the degeneracy locus, the content of the present paper.

Even in the generic regime where three-fold degeneracy is only approximate, there exist codimension-3 loci where any two of the three eigenvalues coincide exactly. The topology of these loci determines the number of fermion generations.

3. The von Neumann–Wigner Theorem and Degeneracy Strings

3.1. The Non-Crossing Theorem

Proposition 3.1 (von Neumann–Wigner Codimension [4])**.** *For a generic smooth family of $N \times N$ Hermitian matrices parametrised by $x = (x^1, \ldots, x^d)$, the set of parameter values where two eigenvalues coincide, $p_a(x) = p_b(x)$, has codimension 3 in the parameter space.*

Proof sketch. In the neighbourhood of a degeneracy point, the relevant 2×2 block in the near-degenerate subspace reads

$$H_{\mathrm{eff}}(x) = \begin{pmatrix} \alpha(x) & \beta(x) + i\gamma(x) \\ \beta(x) - i\gamma(x) & \delta(x) \end{pmatrix}. \tag{3}$$

Eigenvalue coincidence requires precisely three independent real conditions: $\alpha(x) = \delta(x)$, $\beta(x) = 0$, $\gamma(x) = 0$. Hence codimension 3. □

3.2. Degeneracy Strings in Four-Dimensional Spacetime

Definition 3.2 (Degeneracy String)**.** In four-dimensional spacetime ($d = 4$), the *degeneracy string* $\mathcal{D}_{ab}$ is the one-dimensional submanifold

$$\mathcal{D}_{ab} = \{x \in \mathcal{M}^4 \,:\, p_a(x) = p_b(x)\}, \qquad \dim \mathcal{D}_{ab} = 4 - 3 = 1. \tag{4}$$

The four-dimensionality of spacetime plays an essential role here. In $d = 2$, pairwise degeneracies are generically absent (codimension exceeds dimension). In $d = 3$, they occur as isolated points with no topological linking structure. In $d = 4$, they form one-dimensional curves admitting non-trivial braid topology. In $d = 5$, they would be surfaces, yielding a continuum rather than a discrete generation count. The generation structure is therefore a special property of four-dimensional spacetime.

4. Three Generations from Three Degeneracy Strings

4.1. Counting in the SU(3) Sector

The SU(3) gauge symmetry arises from a sector of ρ_{Obs} with three near-degenerate eigenvalues $\{p_1, p_2, p_3\}$ [2]. The number of independent pairwise degeneracies is $\binom{3}{2} = 3$, corresponding to

$$\mathcal{D}_{12} = \{x : p_1(x) = p_2(x)\}, \quad \mathcal{D}_{23} = \{x : p_2(x) = p_3(x)\}, \quad \mathcal{D}_{13} = \{x : p_1(x) = p_3(x)\}. \tag{5}$$

Theorem 4.1 (Three Fermion Generations)**.** *In the RIM framework with four-dimensional spacetime and an* SU(3) *colour sector arising from a three-level subsystem of* ρ_{Obs}*, the number of fermion generations equals the number of topologically distinct pairwise degeneracy strings:*

$$N_{\mathrm{gen}} = \binom{3}{2} = 3. \tag{6}$$

Each string $\mathcal{D}_{ab}$ *carries a distinct sector of the Uhlmann holonomy, corresponding to one fermion generation.*

The self-consistency of this result is worth noting. The SU(3) gauge group is derived in [2] from a three-level subsystem of ρ_{Obs}. The present paper uses this same three-level structure to count $\binom{3}{2} = 3$ pairwise degeneracy strings, yielding three generations. The two steps address logically distinct questions—gauge group from degeneracy size, generation count from degeneracy location—and together show that the observed SU(3) gauge group and the observed generation number three both follow from the same three-level sector of ρ_{Obs}.

4.2. Why Not Four or More Generations?

If the colour gauge group were SU(4), the spectrum would contain four eigenvalues yielding $\binom{4}{2} = 6$ pairwise degeneracy strings. This possibility is excluded on two grounds. First, the observed gauge group is SU(3), not SU(4); within RIM this corresponds directly to a three-level (not four-level) subsystem of ρ_{Obs} [2]. Second, three degeneracy strings in four-dimensional spacetime form a stable braid configuration (§6); six strings generically reconnect under continuous deformation, losing the topological distinctness required for a discrete generation count.

4.3. The Role of SU(2)

The SU(2) sector arises from a two-fold degeneracy $\{q_1, q_2\}$: $\binom{2}{2} = 1$. The single SU(2) degeneracy string $\mathcal{D}_{12}^{(2)}$ produces the isospin doublet structure within each generation; it contributes no additional generation.

4.4. Complete Quantum Number Classification

Combining all three sectors of ρ_{Obs}:

$$\begin{aligned}
&\text{SU(3) sector:} \quad \binom{3}{2} = 3 \text{ strings} \longrightarrow 3 \text{ generations},\\
&\text{SU(2) sector:} \quad \binom{2}{2} = 1 \text{ string} \longrightarrow \text{doublet within each generation},\\
&\text{U(1) sector:} \quad \text{non-degenerate} \longrightarrow \text{charge quantisation.}
\end{aligned}$$

A fermion's identity is thus determined by three "addresses" in the spectral structure of ρ_{Obs}:

$$\text{Fermion} = \Big(\underbrace{\mathcal{D}_{ab}}_{\text{generation}}, \underbrace{\text{SU(2) position}}_{\text{isospin}}, \underbrace{\text{U(1) phase}}_{\text{charge}} \Big). \tag{7}$$

5. Explicit Toy Model: Qutrit Monodromy

5.1. Setup

Consider a three-level system with parameter-dependent eigenvalues

$$p_k(\theta) = \frac{1}{3} + \varepsilon \cos\left(\theta + \frac{2\pi(k-1)}{3} \right), \quad k = 1, 2, 3, \tag{8}$$

where $\theta \in [0, 2\pi)$ parametrises a closed loop and $0 < \varepsilon \ll 1/3$ ensures positivity. The normalisation $\sum_k p_k = 1$ is automatic.

5.2. Degeneracy Points on the Loop

Pairwise degeneracies occur at

$$\theta_{12} = \tfrac{4\pi}{3}, \qquad \theta_{23} = 0 \ (\equiv 2\pi), \qquad \theta_{13} = \tfrac{2\pi}{3}, \tag{9}$$

uniformly spaced at intervals of $2\pi/3$, manifesting the $\mathbb{Z}_3$ symmetry of the spectrum.

5.3. $\mathbb{Z}_3$ Monodromy

Starting from $\theta = 0^+$ with $p_1 > p_2 > p_3$ (for $\varepsilon > 0$), each passage through θ_{ab} exchanges the ordering of p_a and p_b. After a complete circuit $\theta \to \theta + 2\pi$, the eigenvalues undergo the cyclic permutation

$$(p_1, p_2, p_3) \longrightarrow (p_2, p_3, p_1). \tag{10}$$

This is the generator of $\mathbb{Z}_3 \subset S_3$, confirming that the monodromy group of the degeneracy locus is $\mathbb{Z}_3$.

5.4. Uhlmann Holonomy at Each Degeneracy Point

At each degeneracy point θ_{ab}, the two-dimensional degenerate subspace span$\{|a\rangle, |b\rangle\}$ admits an SU(2) Uhlmann connection. The total holonomy around the full loop is

$$U_{\text{hol}} = U_{13}(\theta_{13}) \cdot U_{23}(\theta_{23}) \cdot U_{12}(\theta_{12}), \tag{11}$$

where each U_{ab} is an SU(2) matrix acting on the $\{|a\rangle, |b\rangle\}$ subspace. The product of three SU(2) rotations in overlapping two-dimensional subspaces of a three-dimensional space generates an SU(3) transformation consistent with the $\mathbb{Z}_3$ eigenvalue permutation.

6. Braid Structure and the CKM Matrix

6.1. The Braid Group B_3

The three degeneracy strings $\mathcal{D}_{12}, \mathcal{D}_{23}, \mathcal{D}_{13}$ in four-dimensional spacetime are naturally viewed as three strands in a braid. Their mutual linking is described by the braid group B_3, generated by elementary exchanges σ_1, σ_2 satisfying

$$\sigma_1\sigma_2\sigma_1 = \sigma_2\sigma_1\sigma_2. \tag{12}$$

The abelianisation of B_3 is $\mathbb{Z}$, and its centre contains $(\sigma_1\sigma_2)^3$, which generates a natural $\mathbb{Z}_3$ subgroup corresponding to the cyclic permutation of the three strands—precisely the monodromy found in §5.

6.2. CKM Mixing as Braid Holonomy

Generation mixing arises when a fermion's worldline passes through or encircles a degeneracy string, acquiring Uhlmann holonomy that rotates it into a different generation sector. The CKM matrix is identified with a unitary representation of B_3 acting on the three-dimensional generation space:

$$V_{\text{CKM}} \sim R(\sigma_1^{n_1}\sigma_2^{n_2}\cdots), \tag{13}$$

where $R : B_3 \to \text{U}(3)$ is a unitary representation and n_i are winding numbers around each degeneracy string. A quantitative computation of the CKM angles and phase requires specifying the representation R and the winding numbers, which is deferred to future work.

6.3. CP Violation

The Jarlskog invariant J, which measures the magnitude of CP violation [12], is related to the linking number of the three degeneracy strings. A necessary condition for a CP-violating phase is that the relevant group be non-abelian. With only two degeneracy strings, $B_2 \cong \mathbb{Z}$ is abelian and cannot support a complex phase. With three strings, B_3 is non-abelian, making the CP-violating phase a structural consequence of three-strand braiding rather than an adjustable parameter.

7. Mass Hierarchy from Level Repulsion

The three fermion generations carry identical gauge quantum numbers but differ dramatically in mass. Within the RIM framework, the mass of a fermion in generation $\mathcal{D}_{ab}$ is related to the level repulsion at the corresponding degeneracy point. Near $\mathcal{D}_{ab}$, the eigenvalue gap behaves as

$$\Delta p_{ab}(x) = |p_a(x) - p_b(x)| \sim \sqrt{\alpha_{ab}^2 + \beta_{ab}^2 + \gamma_{ab}^2}, \tag{14}$$

where $\alpha_{ab}, \beta_{ab}, \gamma_{ab}$ are the three parameters of the von Neumann–Wigner decomposition (3). The slope of the level repulsion, $|\nabla \Delta p_{ab}|$ evaluated at $\mathcal{D}_{ab}$, determines the effective mass scale. Different strings generically have different slopes, providing a natural mechanism for mass hierarchy without fine-tuning. A quantitative derivation of the observed mass ratios requires specifying the detailed form of $\rho_{\mathrm{Obs}}(x)$, which remains an open problem.

8. Topological Origin of Chirality

I now show that the weak interaction couples exclusively to left-handed fermions—$\mathrm{SU}(2)_L$—as a consequence of the Uhlmann parallel transport law on the normal bundle of $\mathcal{D}_{12}^{(2)}$, the single SU(2) degeneracy string.

8.1. Uhlmann Holonomy and the Linking Number

Let $\gamma : [0,1] \to \mathcal{M}^4$ be a smooth closed fermion worldline, and let $\mathcal{D}$ be a degeneracy string. In the adiabatic limit, the holonomy phase is

$$\Phi_{\mathrm{hol}}[\gamma] = \exp\left(i \oint_\gamma A_\mu^{(U)} \dot{\gamma}^\mu \, d\tau \right). \tag{15}$$

In a tubular neighbourhood of $\mathcal{D} \subset \mathcal{M}^4$, the Uhlmann connection concentrates its curvature on $\mathcal{D}$, analogously to a magnetic vortex [8]. By Stokes' theorem,

$$\Phi_{\mathrm{hol}}[\gamma] = \exp\big(2\pi i \cdot \mathrm{lk}(\gamma, \mathcal{D})\big), \tag{16}$$

where $\mathrm{lk}(\gamma, \mathcal{D}) \in \mathbb{Z}$ is the linking number of the worldline with the degeneracy string [20].

Lemma 8.1. *A fermion worldline acquires a non-trivial* $\mathrm{SU}(2)$ *gauge phase if and only if* $\mathrm{lk}(\gamma, \mathcal{D}_{12}^{(2)}) \neq 0$.

8.2. Oriented Normal Bundle and the Sign of the Linking Number

At each point $x \in \mathcal{D}_{12}^{(2)}$, the global spacetime orientation form $\varepsilon_{\mu\nu\rho\sigma}$ and the unit tangent $\hat{t}^\mu(x)$ to $\mathcal{D}$ together induce a canonical orientation 2-form on the normal bundle:

$$\omega_{\nu\rho}(x) = \varepsilon_{\mu\nu\rho\sigma}\, \hat{t}^\mu(x)\, \hat{n}^\sigma(x), \tag{17}$$

where $\hat{n}^\sigma$ is the outward unit normal. This orientation is globally well-defined on any orientable spacetime.

8.3. Signed Linking Number and Handedness

The linking number $\mathrm{lk}(\gamma, \mathcal{D})$ is a signed integer: its sign is determined by whether the worldline is wound in the positive or negative sense relative to the canonical orientation $\omega_{\nu\rho}$. The resulting holonomy phases are:

$$\begin{aligned}
\mathrm{lk} > 0 : &\quad \Phi_{\mathrm{hol}} = e^{+2\pi i\,\mathrm{lk}} \neq 1 \quad \text{(non-trivial coupling)},\\
\mathrm{lk} < 0 : &\quad \Phi_{\mathrm{hol}} = e^{-2\pi i|\mathrm{lk}|} \quad \text{(conjugate phase)},\\
\mathrm{lk} = 0 : &\quad \Phi_{\mathrm{hol}} = 1 \quad \text{(no coupling)}.
\end{aligned}$$

Observation 8.2 (Topological Origin of $\mathrm{SU}(2)_L$). The canonical orientation $\omega_{\nu\rho}$ of $\mathcal{D}_{12}^{(2)}$ provides a topological distinction between two classes of fermion worldlines: those wound positively ($\mathrm{lk} > 0$) and those wound negatively ($\mathrm{lk} < 0$). I identify the positively-wound class with left-handed fermions and the negatively-wound class with right-handed fermions. Under this identification, $\mathrm{SU}(2)_L$ coupling emerges as a topological consequence of the signed linking structure:

- *Left-handed* ($\mathrm{lk} > 0$): acquire non-trivial holonomy; couple to $W^\pm$ and Z bosons.
- *Right-handed* ($\mathrm{lk} < 0$): acquire conjugate holonomy; decouple as Standard Model $\mathrm{SU}(2)$ singlets.
- The $\mathbb{Z}_2$ choice between $\mathrm{SU}(2)_L$ and $\mathrm{SU}(2)_R$ reflects the two possible global orientations of $\mathcal{M}^4$; a fixed arrow of time selects one.

The precise dynamical mechanism linking the spinor helicity of the Dirac equation to the winding sign $\mathrm{sgn}(\mathrm{lk})$ is the primary open problem of this section, deferred to future work.

9. Neutrino Sector and the Normal Mass Hierarchy

9.1. $\mathbb{Z}_3$ Monodromy and Mass Ordering

The positive $\mathbb{Z}_3$ monodromy is $(p_1, p_2, p_3) \to (p_2, p_3, p_1)$. The canonical direction of the monodromy assigns mass labels: the string traversed first in the positive cyclic ordering carries the lightest generation.

9.2. Modular Flow and the Canonical Ordering

The modular Hamiltonian $K = -\log \rho_{\text{Obs}}$ generates the modular flow (Tomita–Takesaki modular automorphism group [18, 19]):

$$\Delta^{is} : \mathcal{O} \mapsto \rho_{\text{Obs}}^{is}\, \mathcal{O}\, \rho_{\text{Obs}}^{-is}, \qquad s \in \mathbb{R}. \tag{18}$$

For the three-level subsystem, the modular flow acts as $\Delta^{is}|_{\mathcal{D}_{ab}} : |a\rangle\langle b| \mapsto e^{is(\log p_a - \log p_b)}|a\rangle\langle b|$. The labelling $p_1 > p_2 > p_3$ is the natural one in which eigenvalues are listed in decreasing order, standard in spectral theory. Once this labelling is adopted, the modular flow Δ^{is} for $s > 0$ cycles through the transitions $1 \to 2 \to 3 \to 1$ in the positive sense, defining a canonical direction for the $\mathbb{Z}_3$ monodromy consistent with the KMS condition.

9.3. Canonical Ordering and the Normal Hierarchy

The three neutrino mass eigenstates are identified with the three SU(3) degeneracy strings. Under the canonical spectral ordering, the level-repulsion slopes satisfy $m(\mathcal{D}_{13}) < m(\mathcal{D}_{12}) < m(\mathcal{D}_{23})$, yielding:

Conjecture 9.1 (Normal Neutrino Mass Hierarchy). *In the RIM framework, the standard spectral ordering $p_1 > p_2 > p_3$, made canonical by the modular flow of ρ_{Obs}, implies the normal neutrino mass hierarchy:*

$$m_1 < m_2 < m_3. \tag{19}$$

The inverted hierarchy $m_3 < m_1 < m_2$ would correspond to the inverse monodromy, topologically inequivalent to the canonical sector. This conjecture is falsifiable by DUNE, Hyper-Kamiokande, JUNO, and KM3NeT/ORCA.

9.4. Consistency with Current Data

Current oscillation data mildly prefer the normal hierarchy [21, 22]. The prediction will be tested at $\geq 5\sigma$ significance by DUNE [23]. The PMNS phase δ_{CP} is generically non-zero, consistent with hints from T2K and NOvA.

10. Falsifiable Predictions

1. **Generation number**: $N_{\text{gen}} = 3$ exactly. Falsified by any confirmed fourth-generation fermion at any mass scale.
2. **Normal neutrino mass hierarchy**: $m_1 < m_2 < m_3$ (Conjecture 9.1). Falsified by confirmation of the inverted hierarchy at $\geq 5\sigma$ by DUNE, Hyper-

Kamiokande, JUNO, or KM3NeT/ORCA.

3. **Non-zero PMNS CP violation**: $\delta_{CP} \neq 0, \pi$, as a consequence of three-strand braid non-abelianness.
4. **Chiral weak coupling**: SU(2) couples exclusively to left-handed fermions (Observation 8.2). Falsified by any confirmed $\mathrm{SU}(2)_R$ gauge coupling at any energy scale.

Precision electroweak measurements at LEP: $N_\nu = 2.984 \pm 0.008$ [13], consistent with exactly three light generations. LHC searches place stringent bounds on sequential fourth-generation quarks [14]. All existing data are consistent with the RIM predictions.

11. Comparison to Prior Work

String theory. The number of generations equals $|\chi|/2$ of the compactification manifold, yielding three generations for specific choices in the vast landscape. The result depends sensitively on moduli stabilisation and is not topologically protected in the sense used here.

Grand unified theories. The generation number is an input parameter even in well-motivated GUT group structures; it must be imposed by hand.

Preon models. Generations arise from compositeness at a higher scale, replacing one empirical input with another.

The RIM approach. The generation number $\binom{3}{2} = 3$ is a topological invariant of the degeneracy locus of ρ_{Obs} in four-dimensional spacetime. It does not depend on compactification, does not require compositeness, and is not a free parameter. Two independently motivated ingredients suffice: (i) the SU(3) gauge group requires a three-level subsystem of ρ_{Obs}; (ii) the von Neumann–Wigner theorem in $d = 4$ converts each pairwise degeneracy (codimension 3) into a one-dimensional string. The combination $\binom{3}{2} = 3$ is then unavoidable.

12. Open Problems

1. **Quantitative CKM/PMNS parameters.** The braid-group interpretation identifies the structural origin of generation mixing but does not yet compute the numerical values of the mixing angles and phases from first principles. This requires specifying the representation R and the winding numbers for physical fermion trajectories.
2. **Mass ratios.** The level-repulsion mechanism qualitatively explains the mass hierarchy but not the specific ratios $m_t/m_c/m_u$. A quantitative derivation requires knowledge of the detailed form of $\rho_{\mathrm{Obs}}(x)$.
3. **Dynamical identification of chirality.** Observation 8.2 proposes that

physical fermion helicity corresponds to the winding sign sgn(lk) with respect to $\mathcal{D}_{12}^{(2)}$. Establishing this identification rigorously from the RIM action principle is the central open problem of the chirality sector.

4. **Arrow of time and spectral ordering.** Conjecture 9.1 is conditional on identifying the standard spectral ordering $p_1 > p_2 > p_3$ with the thermodynamic arrow of time. In the timeless Wheeler–DeWitt framework, demonstrating that the Page–Wootters mechanism [7] selects this ordering requires a separate derivation.
5. **Yukawa couplings.** A derivation of Yukawa coupling structure from the geometry of the degeneracy locus remains open.

13. Conclusion

I have shown that within the Relational-Informational Model, the number of fermion generations is not a free parameter but a topological invariant: $N_{\text{gen}} = \binom{3}{2} = 3$, determined by the number of pairwise degeneracy strings in the SU(3) sector of the observer's density matrix ρ_{Obs}, embedded in four-dimensional spacetime.

The argument rests on two independently motivated ingredients: (i) the SU(3) colour gauge group requires a three-level subsystem of ρ_{Obs}, as established in [2]; (ii) the von Neumann–Wigner theorem in a four-dimensional parameter space converts each pairwise degeneracy (codimension 3) into a one-dimensional string, of which there are exactly $\binom{3}{2} = 3$. The same eigenvalue spectrum of ρ_{Obs} that generates the gauge group (via degeneracy size) also generates the generation structure (via degeneracy location) and the flavour mixing pattern (via braid topology). CP violation is structurally unavoidable in a three-strand braid, as the braid group B_3 is non-abelian—unlike $B_2 \cong \mathbb{Z}$, which is abelian and cannot support a CP-violating phase.

Two further structural results extend the programme. The canonical orientation of $\mathcal{D}_{12}^{(2)}$, induced by the global spacetime orientation and the string's unit tangent, provides a topological partition of fermion worldlines into positively-wound and negatively-wound classes; I propose this as the topological origin of $\mathrm{SU}(2)_L$ chirality. The standard spectral ordering of eigenvalues, made canonical by the modular flow of ρ_{Obs}, implies the normal neutrino mass hierarchy $m_1 < m_2 < m_3$.

Combined with the results of [1, 2], the complete structure of the Standard Model fermion spectrum is encoded in ρ_{Obs}:

$$\rho_{\mathrm{Obs}} \longrightarrow \begin{cases} \text{Degeneracy size} & \to\ G_{\mathrm{gauge}} = \mathrm{U}(1) \times \mathrm{SU}(2) \times \mathrm{SU}(3), \\ \text{Degeneracy location} & \to\ N_{\mathrm{gen}} = 3, \\ \mathrm{SU}(2) \text{ signed linking} & \to\ \mathrm{SU}(2)_L \text{ chirality}, \\ \text{Spectral ordering} & \to\ m_1 < m_2 < m_3 \text{ (normal hierarchy)}, \\ \mathrm{SU}(2) \text{ position} & \to\ \text{isospin doublet}, \\ \text{Level repulsion slope} & \to\ \text{mass hierarchy (qualitative)}. \end{cases} \tag{20}$$

This represents a further step in the RIM programme: both the bosonic structure (gauge fields and spacetime geometry) and the multiplicity of fermionic families arise from a single quantum-informational object—the observer's reduced density matrix.

Acknowledgements

The author thanks Professor S. L. Braunstein for critical feedback on the companion papers that motivated the present investigation. No external funding was received.

Author Contributions

J.W.L. conceived the framework, performed all derivations, and wrote the manuscript.

Data Availability

Not applicable (theoretical study).

Competing Interests

The author declares no competing interests.

References

[1] J. W. Lee, *From information to geometry: A relational framework for emergent spacetime via quantum Fisher information*, Zenodo preprint (2026); submitted to Int. J. Theor. Phys.

[2] J. W. Lee, *Emergent gravity and gauge fields from the mixed-state quantum geometric tensor: The Relational-Informational Model*, Zenodo preprint (2026); submitted to Found. Phys.

[3] A. Uhlmann, Parallel transport and 'quantum holonomy' along density operators, *Rep. Math. Phys.* **24**, 229 (1986).

[4] J. von Neumann and E. P. Wigner, Über das Verhalten von Eigenwerten bei adiabatischen Prozessen, *Phys. Z.* **30**, 467 (1929).

[5] S. L. Braunstein and C. M. Caves, Statistical distance and the geometry of quantum states, *Phys. Rev. Lett.* **72**, 3439 (1994).

[6] T. Jacobson, Thermodynamics of spacetime: The Einstein equation of state, *Phys. Rev. Lett.* **75**, 1260 (1995).

[7] D. N. Page and W. K. Wootters, Evolution without evolution: Dynamics described by stationary observables, *Phys. Rev. D* **27**, 2885 (1983).

[8] M. V. Berry, Quantal phase factors accompanying adiabatic changes, *Proc. R. Soc. Lond. A* **392**, 45 (1984).

[9] O. Viyuela, A. Rivas, and M. A. Martin-Delgado, Uhlmann phase as a topological measure for one-dimensional fermion systems, *Phys. Rev. Lett.* **112**, 130401 (2014).

[10] E. Sjöqvist et al., Geometric phases for mixed states in interferometry, *Phys. Rev. Lett.* **85**, 2845 (2000).

[11] M. Kobayashi and T. Maskawa, CP-violation in the renormalizable theory of weak interaction, *Prog. Theor. Phys.* **49**, 652 (1973).

[12] C. Jarlskog, Commutator of the quark mass matrices in the standard electroweak model and a measure of maximal CP nonconservation, *Phys. Rev. Lett.* **55**, 1039 (1985).

[13] The ALEPH, DELPHI, L3, OPAL, and SLD Collaborations, Precision electroweak measurements on the Z resonance, *Phys. Rep.* **427**, 257 (2006).

[14] CMS Collaboration, Search for pair production of fourth-generation b' quarks, *Phys. Lett. B* **718**, 307 (2012).

[15] M. Van Raamsdonk, Building up spacetime with quantum entanglement, *Gen. Relativ. Gravit.* **42**, 2323 (2010).

[16] J. P. Provost and G. Vallée, Riemannian structure on manifolds of quantum states, *Commun. Math. Phys.* **76**, 289 (1980).

[17] J. D. Bekenstein, Black holes and entropy, *Phys. Rev. D* **7**, 2333 (1973).

[18] J. J. Bisognano and E. H. Wichmann, On the duality condition for a Hermitian scalar field, *J. Math. Phys.* **16**, 985 (1975).

[19] M. Tomita, On canonical forms of von Neumann algebras, *Fifth Functional Analysis Symposium* (Sendai, 1967); M. Takesaki, *Tomita's Theory of Modular Hilbert Algebras and Its Applications*, Springer LNM 128 (1970).

[20] J. Milnor, *Morse Theory*, Princeton University Press (1963), §5.

[21] NOvA Collaboration (M. A. Acero et al.), An improved measurement of neutrino oscillation parameters by the NOvA experiment, *Phys. Rev. D* **106**, 032004 (2022).

[22] T2K Collaboration (K. Abe et al.), Improved constraints on neutrino mixing from the T2K experiment with 3.13×10^{21} protons on target, *Phys. Rev. D* **103**, 112008 (2021).

[23] DUNE Collaboration (B. Abi et al.), Long-baseline neutrino oscillation physics potential of the DUNE experiment, *Eur. Phys. J. C* **80**, 978 (2020).