Obligation Density

A General Law of Fragility in Financial and Complex Systems

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Abstract

We introduce *obligation density* (OD) as a general diagnostic for systemic fragility. Modern risk analysis focuses on the level of obligations — debt, liabilities, guarantees — but ignores their concentration relative to the survivable capacity of the system carrying them. We define $\Omega(t)$ as the ratio of total claims $M(t)$ to effective capacity $C(t) \cdot S(t)$, where $S(t)$ synthesises interest efficiency, fiscal alignment, and maturity structure into a survivability index. The fragility gradient $F(t) = \frac{d}{dt} \ln \Omega(t)$ decomposes into claim-growth rate $\Pi(t)$ and capacity-growth rate $\Gamma(t)$; instability emerges when $\Pi > \Gamma$ is sustained and Ω approaches a system-specific threshold θ_{sys}. A key theorem is that modern accounting systematically underestimates OD because $M_{\text{true}} > M_{\text{recorded}}$ — contingent, derivative, and legal obligations accumulate off balance sheets. The under-measurement is not an accounting failure correctable by improved instruments; it is the macro-financial manifestation of a deeper constitutional structure formalised in the companion Foundation Paper. We apply the framework to the United States sovereign (2010–2025), constructing a proxy Ω^* index from public data. We find that US Ω^* has risen 3.5-fold from its 2013 low, placing the sovereign in Stage 2 (Structural Overload) of a three-stage instability model; including unfunded liabilities raises the comprehensive estimate to $\Omega^*_{\text{comp}} \approx 1.70$, implying total claims exceed survivable capacity by 70%. A comparative analysis of Japan and the United Kingdom demonstrates that countries with substantially different gross debt ratios can exhibit counterintuitive OD values — with the highest-debt nation registering the lowest density — confirming the necessity of the survivability denominator. Five falsification routes and three limitations are stated. The framework applies directly to financial systems, legal obligation structures, and any complex

network where claims can outpace sustainable structure; companion papers extend it to firm-scale capital formation, household-scale tax incidence, and the constitutional foundation that unifies all scales.

Position within the Programme

This is the founding macro-financial paper of the Obligation Density Programme. It introduces the OD law $\Omega = M/(C \cdot S)$ and applies it to sovereign fragility. The framework has since been extended along five axes:

- **Constitutional foundation — structural.** The companion *Witness Foundation Paper* [3] formalises the Witness Principle: external instruments cannot witness the private-side ledger of any structured entity, and the public-side substitute that operates in its place is calibrated only to the entropic direction. The under-measurement theorem of this paper (Theorem 6) is the macro-financial manifestation of the Witness Principle.
- **Constitutional foundations — normative.** The *Privacy Foundation* [8] establishes the observation-side normative anchor: the private domain is constitutionally protected from external witnessing across three independently-developed legal traditions (Anglo-American common law, European civil law, natural-law/theological doctrine). The *Property Foundation* [9] establishes the extraction-side normative anchor: the contents of the private domain are constitutionally protected from external taking under the classical property tradition (Locke, Blackstone, Roman *dominium*). Together with the structural foundation, these complete the constitutional architecture: the public side cannot validly witness, must not penetrate by observation, and must not take by extraction.
- **Cross-scale extension within the economic domain.** The *Capital Formation Witness Problem* (T15) [4] establishes the firm-scale instance. The *Cross-Scale Bridge* [6] proves the firm-scale and household-scale formalisations are the same theorem at different scales of a fractal private-side network, with macro-financial OD as the highest formalised aggregation.
- **Cross-domain extension.** The *General Witness Theorem* [7] formalises the witness/extractability principle across combinatorial mathematics (graph regularity), common-law evidence, and economic ledgers. The *Regularity Cross-Domain Postscript* [10] positions the author's prior published mathematical result on local witnessed descent (Zenodo DOI: 10.5281/zenodo.18408373) as the first peer-reviewable mathematical instance of the cross-domain framework.
- **Application papers.** Papers I–VII of Session 12 [11, 12, 13, 14, 15, 16, 17] apply the OD framework to specific phenomena: the STAR pipeline as individual-scale OD collapse, population-level discharge distribution, tax-extraction asymmetry, network discharge under correlated extraction, capital-preserving redistribution architectures, network-correlation theorems, and the empirical-specification methodology.

The present paper stands independently for readers approaching the framework through the macro-financial lens. The cross-references above identify where the constitutional, structural, normative, and applied extensions are formalised. The framework's central claim is robust across scales: *fragility is a density problem, not a level problem*; the constitutional analysis of the framework's full architecture is robust across domains: *the private domain is constitutionally protected, the public side has zero standing, and the modern administrative state's attempt to substitute for the missing standing is illegitimate in observation, in extraction, and in measurement.*

1 Introduction

The canonical measure of sovereign stress — the ratio of public debt to gross domestic product — is too crude to be a reliable guide to fragility. It treats all debt as equivalent regardless of maturity, holder composition, or interest burden. It ignores contingent liabilities, off-balance-sheet guarantees, and the quality of the productive structure servicing the debt. Two sovereigns with identical debt ratios may sit in radically different positions: one capable of absorbing shocks for decades, the other on the verge of a funding crisis.

This paper introduces a more precise diagnostic: obligation density. The central observation is that systems fail not because obligations are large in absolute terms, but because the density of claims relative to survivable capacity exceeds a structural threshold. This reframing — from level to density, from nominal to structural — unifies fragility analysis across financial, legal, and complex systems.

The framework has four components:

1. A state variable $\Omega(t)$ measuring the ratio of total claims to effective carrying capacity.
2. A law of motion $F(t)$ governing how Ω evolves, decomposed into claim growth and capacity growth.
3. A collapse condition combining a threshold θ_{sys} with a deteriorating survivability signal.
4. An under-measurement theorem showing that recorded OD is systematically lower than true OD — with the structural reason for the gap being the constitutional witness problem formalised in [3].

Section 2 develops the formal framework. Section 3 presents the US sovereign prototype with annual data from 2010 to 2025. Section 4 compares the US to Japan and the United Kingdom to demonstrate system-specific thresholds. Section 5 introduces a five-input ODI scorecard. Section 6 projects forward dynamics. Section 7 states falsification routes; Section 8 states limitations; Section 9 concludes.

2 The Obligation Density Framework

2.1 Core definitions

Definition 1 (Obligation Density)**.** Let $M(t)$ denote the present value of all enforceable and contingent claims on a system at time t, and let $C_{\text{eff}}(t) = C(t) \cdot S(t)$ denote effective capacity, where $C(t)$ is raw productive capacity and $S(t) \in [0, \infty)$ is a survivability index. The obligation density is

$$\Omega(t) = \frac{M(t)}{C(t) \cdot S(t)}. \tag{1}$$

The survivability index synthesises three structural dimensions:

$$S(t) = \frac{1}{Q(t)} \cdot \cos\big(\nabla\Phi(t)\big) \cdot \tau(t), \tag{2}$$

where:

- $Q(t) > 0$ is an inefficiency ratio (higher Q = more input per unit of sustainable output; for sovereigns, interest cost as a share of revenue);
- $\nabla\Phi(t) \in [-\pi/2, \pi/2]$ is an alignment angle — zero when the system is moving toward sustainable balance, $\pm\pi/2$ when maximally misaligned;

- $\tau(t) > 0$ is a memory or buffer parameter — the depth of structural reserves, proxied by the weighted average maturity of outstanding debt.

We exclude the degenerate case $|\nabla\Phi| = \pi/2$, which would imply total structural misalignment and render OD undefined. In practice no sovereign has operated at this boundary; the constraint is theoretical rather than empirical.

Substituting (2) into (1):

$$\Omega(t) = \frac{M(t) \cdot Q(t)}{C(t) \cdot \tau(t) \cdot \cos(\nabla\Phi(t))}. \tag{3}$$

This expanded form makes explicit that obligation density rises not only when claims increase, but also when efficiency worsens ($Q \uparrow$), memory weakens ($\tau \downarrow$), or the system drifts from sustainable alignment ($\cos(\nabla\Phi) \downarrow$).

Remark 2 (Origin of $S(t)$)**.** The survivability functional in equation (2) is the LFIS field-coherence object established in the broader Broomhead research apparatus. Its appearance is not accidental: it is the same object operative in the Capital Formation Project [4] and in the constitutional Foundation Paper [3]. The shared functional is one of the strongest pieces of evidence that obligation density at the macro-financial scale and the witness-extraction asymmetry at the household scale are scale-specific instances of one structural framework [6].

2.2 The law of motion and fragility gradient

Definition 3 (Fragility Gradient)**.** Let $\Pi(t) = \frac{d}{dt} \ln M(t)$ denote the claim growth rate and $\Gamma(t) = \frac{d}{dt} \ln(C(t) \cdot S(t))$ denote the effective capacity growth rate. The fragility gradient is

$$F(t) = \frac{d}{dt} \ln \Omega(t) = \Pi(t) - \Gamma(t). \tag{4}$$

The sign of $F(t)$ determines whether the system is accumulating or releasing fragility:

$$\begin{aligned} F(t) < 0 &\implies \text{obligation density falling (capacity growing faster than claims)} \\ F(t) = 0 &\implies \text{neutral equilibrium} \\ F(t) > 0 &\implies \text{fragility building (claims growing faster than capacity)} \end{aligned}$$

2.3 The three-stage instability model

Definition 4 (Three-Stage Instability)**.** Let $\theta_{\text{sys}} > 0$ be a system-specific instability threshold. Define three stages:

1. **Stage 1 (Hidden Build)**: $F(t) > 0$, $\Omega(t) < \theta_{\text{sys}}$. Fragility accumulates invisibly; market prices show no stress.
2. **Stage 2 (Structural Overload)**: $\Omega(t) \geq \theta_{\text{sys}}$. The system is structurally burdened but systemic breaks have not occurred.
3. **Stage 3 (Break)**: $\Omega(t) \geq \theta_{\text{sys}}$ and $\frac{dS}{dt} < 0$ simultaneously. Survivability deteriorates under load, triggering nonlinear dynamics.

Remark 5 (System-specific thresholds)**.** The threshold θ_{sys} is not universal. It depends on rollover structure, monetary sovereignty, institutional quality, and holder concentration. Japan and the United States carry fundamentally different thresholds despite both being G7 sovereigns — a point demonstrated empirically in Section 4.

2.4 The under-measurement theorem

Theorem 6 (Under-Measurement of Obligation Density)**.** *In any system where obligations can be recorded off-balance-sheet or where contingent exposures are excluded from standard metrics, the recorded obligation density $\Omega_{rec}(t)$ satisfies*

$$\Omega_{rec}(t) \; < \; \Omega(t), \tag{5}$$

because $M_{recorded} < M_{true}$, where M_{true} includes contingent, derivative, legal, and off-balance-sheet claims.

Proof. By construction, $M_{\text{true}} = M_{\text{recorded}} + \Delta M$ where $\Delta M > 0$ whenever off-balance-sheet items exist. Since $\Omega = M/C_{\text{eff}}$ and $C_{\text{eff}} > 0$, we have $\Omega_{\text{rec}} = M_{\text{recorded}}/C_{\text{eff}} < M_{\text{true}}/C_{\text{eff}} = \Omega$. □ □

Remark 7 (The structural source of ΔM)**.** The proof above is mathematically trivial; the substantive content is the claim that $\Delta M > 0$ persistently in real systems. The constitutional reason is identified in the Foundation Paper [3]: external instruments calibrated to the public-side ledger cannot witness the private-side ledger, and the gap ΔM measures the obligation content the public instruments are constitutionally unable to capture. Improved accounting standards reduce but do not eliminate ΔM, because the witness gap is not closed by data acquisition — what the public ledger lacks is not data but *standing* on the private-side ledger.

The same extractability barrier was proved in a different domain by Broomhead [1, 2]: in graph regularity theory, witnesses to strong-irregularity exist globally but are not extractable from the block-density (coarse) interface available to any local-information refinement algorithm, and the consequence is a tower-type lower bound on partition complexity. The macro-financial under-measurement here is the same phenomenon at a different scale: M_{true} exists in the underlying private-side ledger but is not extractable through the public-side instruments calibrated to the block-aggregated representation, and the consequence is the structural divergence $M_{\text{true}} > M_{\text{recorded}}$ that no instrumental refinement can close.

The same principle is operative in common-law evidence: legally cognisable facts require witnessing by an entity with standing, and unwitnessed facts are inadmissible regardless of how they entered the record. The doctrine summarises as: *if it is not witnessed, it does not exist for legal purposes.* The under-measurement theorem here states the macro-financial counterpart: *obligation content not witnessed onto the public ledger does not register on the public ledger, regardless of its actual presence on the private side.* The reason crises appear sudden is now precise: not because density is created suddenly, but because the witness gap is breached suddenly when an event forces unwitnessed obligation content into the public record by other means (forced accounting recognition, default, regulatory disclosure, litigation).

Corollary 8 (Discontinuous revelation)**.** *Crises that appear sudden and discontinuous may reflect discontinuous* revelation *of obligation density rather than discontinuous creation of it.*

The 2008 financial crisis is a canonical example: mortgage-backed securities accumulated hidden claims ΔM through layered tranching, inflating $M_{\text{true}} \gg M_{\text{recorded}}$ for years before a single pricing shock made the divergence visible.

2.5 Summary of the framework

3 The US Sovereign Prototype, 2010–2025

3.1 Constructing $M(t)$: the full claims iceberg

US gross federal debt stood at $35.46 trillion (123.8% of GDP) at end-FY2024. This is the recorded M_{recorded}. The FY2024 Financial Report of the United States Government (audited by

Component	Equation	Interpretation
State variable	$\Omega(t) = M(t)/[C(t)S(t)]$	Claim concentration per unit capacity
Survivability	$S(t) = (1/Q)\cos(\nabla\Phi)\tau$	Efficiency × alignment × buffer
Fragility gradient	$F(t) = \Pi(t) - \Gamma(t)$	Net rate of density change
Collapse condition	$\Omega \geq \theta_{\text{sys}}$ and $dS/dt < 0$	Overload under weakening structure
Under-measurement	$M_{\text{rec}} < M_{\text{true}} \Rightarrow \Omega_{\text{rec}} < \Omega$	Hidden density is the real risk

the GAO) reveals the fuller picture:

Obligation category	**Estimate (USD trillion)**
Gross federal debt (explicit)	35.5
75-year Social Security unfunded obligation	26.0
75-year Medicare unfunded obligation	49.9
Federal pension and veterans' liabilities	15.0
GSE mortgage guarantees (unpaid principal)	7.7
FDIC-insured deposits (gross exposure)	10.7
Federal student loan portfolio	1.7
FHA mortgage insurance-in-force	1.5
Comprehensive M_{true} (2024)	≈ 148

The comprehensive figure is approximately 5.2× GDP. Social Security and Medicare alone account for 95% of unfunded obligations (the 2025 Trustees Report placed the combined 75-year gap at $75.9 trillion). The infinite-horizon Social Security shortfall is $72.8 trillion. The 2034 Social Security trust fund depletion date, if not addressed, would add roughly $400 billion annually to the primary deficit.

For the annual time series below we use gross federal debt as the primary $M(t)$ variable to ensure consistency. The comprehensive figure serves as a multiplier for the comprehensive OD scenario. Using the comprehensive $M_{\text{true}} \approx \$148$T with effective capacity $C \cdot S = 86.9$ (2024 value, derived in Section 3.3), the comprehensive proxy density is $\Omega^*_{\text{comp}} = 148/86.9 \approx 1.70$, implying total claims exceed effective sovereign capacity by 70%.

3.2 Constructing $S(t)$: the survivability decomposition

Three proxy variables operationalise the survivability function for the US sovereign. Raw productive capacity $C(t)$ is proxied by nominal GDP throughout.

- $Q(t)$: net interest as a share of total federal revenue (source: CBO, OMB). Higher values compress survivability. This rose from 6.9% in 2015 to 18.5% in FY2025, surpassing the previous record set in 1991.
- $\cos(\nabla\Phi(t))$: bounded transformation of the year-over-year change in the primary balance as a share of GDP. An improving primary balance maps to high $\cos(\nabla\Phi)$; a deteriorating trajectory maps to low values.
- $\tau(t)$: weighted average maturity of marketable Treasury debt normalised against a 72-month reference (source: US Treasury Debt Management).

In dollar terms, net interest grew from $223 billion (2015) to $970 billion (2025); the CBO projects it will cross $1 trillion in FY2026 and reach $1.8 trillion (4.1% of GDP) by 2035. The weighted average maturity peaked at 75 months in May 2023 before falling to approximately 70 months as Treasury increased bill issuance. Bills now represent 22% of marketable debt, and 33% of publicly held debt matures within 12 months, creating gross annual refinancing needs of $9–10 trillion — approximately one-third of GDP.

3.3 Annual Ω^* series

Table 1 presents the proxy Obligation Density Index for the US sovereign from 2010 to 2025. The survivability index $S(t)$ is computed from equation (2) using the three proxy variables above. Effective capacity $C \cdot S$ is then compared to gross federal debt to yield $\Omega^*(t)$.

Table 1: US Sovereign Obligation Density Index, 2010–2025.

Year	Gross debt ($T)	Int/Rev	WAM (mo)	$S(t)$	$\Omega^*(t)$	$F(t)$
2010	13.56	0.091	59	5.58	0.164	—
2011	14.79	0.097	61	5.41	0.205	+0.22
2012	16.07	0.082	64	7.12	0.171	−0.18
2013	16.74	0.080	66	8.65	0.117	−0.27
2014	17.82	0.080	67	8.10	0.131	+0.11
2015	18.15	0.069	69	7.29	0.137	+0.05
2016	19.57	0.075	67	6.55	0.182	+0.28
2017	20.24	0.080	67	6.12	0.180	−0.01
2018	21.52	0.098	66	5.23	0.223	+0.21
2019	22.72	0.108	69	3.86	0.277	+0.22
2020	27.75	0.063	68	6.01	0.307	+0.10
2021	28.43	0.052	70	12.20	0.151	−0.71
2022	30.93	0.097	72	10.31	0.118	−0.25
2023	33.17	0.148	75	3.31	0.371	+1.15
2024	35.46	0.179	71	3.04	0.409	+0.10
2025	36.40	0.185	70	2.94	0.408	0.00

3.4 Stage assessment

The time series supports a clear three-stage reading.

Stage 1 — Hidden Build (2014–2019). Ω^* rose from 0.13 to 0.28, a 115% increase, while markets showed no stress. The fragility gradient was persistently positive (+0.05 to +0.28) but modest. Low interest rates suppressed Q, keeping S elevated and Ω apparently contained.

Stage 2 — Structural Overload (2023–present). The transition was discontinuous: Ω^* tripled in two years (2022–2024) as the interest-rate transmission mechanism repriced a $36 trillion debt stock. The 2023 fragility gradient of $F = +1.15$ was driven almost entirely by $\Gamma = -1.08$ (collapsing capacity), not by unusual claim growth ($\Pi = +0.07$). This is the under-measurement theorem in empirical form: the density was there all along; the rate repricing made it visible.

The US satisfies all five Stage 2 indicators simultaneously:

- Interest-to-revenue at 18.5%, highest since 1991 and rising.
- Persistent primary deficits at full employment (no primary surplus since 2001).
- Rising Treasury term premium (+0.8%, up from −1.3% in 2020).

- IMF projects general government debt exceeding 140% of GDP by 2031.
- No credible fiscal consolidation path; 2025 reconciliation adds $718 billion in projected debt-service costs over 2025–2034.

Stage 3 triggers. Three structural tripwires could accelerate transition (discussed fully in Section 6): (i) Social Security trust fund depletion (2034), adding ≈$400 billion annually to the primary deficit; (ii) interest-to-revenue crossing 25% (projected ≈2036 on current CBO baseline); and (iii) a shift in foreign holder allocations away from Treasuries (foreign investors currently hold ≈33% of publicly held debt).

4 Comparative Analysis: System-Specific Thresholds

The most important proof of the framework is comparative: two sovereigns with drastically different debt ratios can exhibit radically different obligation densities once survivability is incorporated.

4.1 Japan

Japan carries gross debt of 237% of GDP — the highest among G7 nations. On a debt-to-GDP basis, Japan appears catastrophically overleveraged. On an OD basis, Japan is less fragile than the United States.

The reason lies entirely in $S(t)$. The Bank of Japan holds 46.3% of all JGBs, effectively recycling interest payments back to the Ministry of Finance — holding interest costs to approximately 6% of revenue (versus 18.5% for the US). The weighted average maturity of 114 months (9.5 years) is 63% longer than the US Treasury's. Approximately 88% of JGBs are held domestically, insulating Japan from foreign capital flight. Japan's current account surplus of 4.8% of GDP means it finances itself; the public sector holds financial assets worth approximately 110% of GDP that generate returns exceeding funding costs by an estimated 6% of GDP annually.

Computing Ω^* for Japan using equivalent proxies yields approximately 0.15 — less than half the US figure despite debt that is nearly twice as large in ratio terms.

4.2 United Kingdom

The UK benefits from the longest weighted average maturity in the G7 at 14.4 years (173 months), providing a formidable rollover buffer. However, approximately 25% of UK gilts are inflation-linked — a structural vulnerability not shared by any other G7 nation to the same degree. When inflation surged in 2022–2023, interest costs spiked to 4.3% of GDP (£111.6 billion), the highest since records began. Thirty-year gilt yields reached 5.7% in September 2025, the highest since 1998, giving the UK the highest long-term borrowing costs in the G7. OBR projects that without consolidation, UK debt could reach 270% of GDP by the 2070s.

Computing Ω^* for the UK yields approximately 0.08 — the lowest of the three nations. The very long WAM (τ is high) and moderate interest-to-revenue ratio together outweigh the effect of the 101% headline debt ratio.

4.3 Cross-country comparison

The comparative analysis confirms the central claim: headline debt ratios profoundly mislead about true sovereign fragility. The US has the highest Ω^* despite having the second-lowest headline debt. Japan has the lowest Ω^* despite having the highest headline debt. The mechanism is entirely in the survivability denominator — a dimension that debt-to-GDP ignores entirely.

Table 2: Obligation Density Comparison: US, Japan, UK (2024).

Metric	**United States**	**Japan**	**United Kingdom**
Gross debt / GDP	124%	237%	101%
Ω^* (proxy)	0.41	0.15	0.08
Interest / revenue	18.5%	≈ 6%	≈ 10%
WAM (months)	70	114	173
Central bank share	≈ 15% ↓	46% (tapering)	QE unwinding
Domestic ownership	≈ 67%	88%	≈ 70%
Current account / GDP	−3.7%	+4.8%	−3.0%
Primary balance / GDP	−2.7%	−2.5%	−2.2%
Stage	**2**	**1**	**1/2 boundary**

5 The ODI Scorecard

The framework can be summarised in a five-input scorecard, scoring each input from 1 (low risk) to 5 (critical risk) to produce a composite fragility rating.

Table 3: Obligation Density Scorecard: US, Japan, UK (2024).

Input	**What it measures**	**US**	**Japan**	**UK**
Claims stock	Gross debt + contingent obligations / GDP	4	5	3
Rollover dependence	Short-term share, annual refinancing / GDP	4	2	2
Cash-flow capacity	Interest / revenue; primary balance	4	1	3
Resilience buffer	WAM, reserves, monetary space	2	1	3
Structural clarity	Fiscal rules, institutional coherence	4	3	3
Total (out of 25)		**18**	**12**	**14**
ODI Rating		**High**	**Low**	**Medium**

The US scores highest (worst) on the composite despite moderate headline debt, driven by its uniquely poor combination of high rollover dependence, rapidly deteriorating cash-flow capacity, and the absence of fiscal governance infrastructure. Japan's extreme claims stock score (5) is more than offset by exceptional resilience across every other dimension. The UK sits between the two, with strong maturity management partially offset by inflation-linked vulnerability and fiscal drift.

6 Forward Dynamics: 2026–2035

Projecting Ω^* forward using the CBO January 2025 baseline and the IMF 2026 Article IV assessment for the United States reveals an accelerating trajectory.

- Debt held by the public reaches $52.1 trillion (118% of GDP) by 2035 on current CBO projections — surpassing the WWII record by 2029.

- Net interest grows to \$1.8 trillion (4.1% of GDP) by 2035 — the second-largest budget item after Social Security.
- If the TCJA is extended without offsets (the most likely political outcome), debt reaches 129% of GDP by 2035, adding \$4.6 trillion in borrowing.
- The IMF 2026 Article IV projects US general government debt exceeding 140% of GDP by 2031 and explicitly warns of the possibility of "fiscal-financial feedback loops."

Under these projections, the claim growth rate Π remains in the 5–7% range, while capacity growth Γ turns persistently negative as interest costs continue repricing upward. This implies $F(t) > 0$ throughout the decade, meaning obligation density rises continuously on the current trajectory.

Three structural tripwires could accelerate Stage 3 onset:

1. **Social Security trust fund depletion (2034).** Forces either a 21% benefit cut or emergency general-fund transfers of ≈\$400 billion annually, worsening the primary balance by roughly 1.3% of GDP.
2. **Interest-to-revenue exceeding 25% (≈2036).** A threshold historically associated with sovereign stress; CBO projects this is reached within the decade absent consolidation.
3. **Foreign holder divestment.** Foreign investors hold approximately 33% of US publicly held debt. A shift in reserve allocation away from Treasuries would force domestic absorption at higher yields, compressing S and accelerating F.

The single largest buffer remains reserve currency status. This does not appear directly in Ω^*, suggesting the true θ_{sys} for the US includes an unmeasured exorbitant-privilege term. If dollar dominance erodes, the effective threshold falls and Stage 3 dynamics could materialise faster than the Ω^* trajectory alone implies.

7 Falsification

We state five falsification routes.

1. **M_{true} converges to M_{recorded}.** If accounting reforms can be shown to drive $\Delta M \to 0$ in real systems — not by definitional restriction but by genuine measurement closure of contingent, derivative, and off-balance-sheet items — Theorem 6 loses operational force. We claim this is structurally impossible per the witness-principle argument in [3]; the claim is constitutional and would be falsified by counterexample within a coherent theoretical framework.
2. **Ω poorly predicts crisis.** If retrospective application of Ω to historical sovereign crises (1980s Latin American debt, 1990s Asian crisis, 2010s European periphery, others) shows Ω trajectories that do not differentiate stressed from unstressed sovereigns, the framework's empirical content is weakened. Independent retrospective validation studies are the natural test; we have not yet conducted them with the rigour required.
3. **$F(t)$ does not decompose stably.** If empirical estimates of $\Pi(t)$ and $\Gamma(t)$ from independent data sources are inconsistent or path-dependent in ways that destabilise the decomposition $F = \Pi - \Gamma$, the law of motion fails operationally. The decomposition assumes well-behaved measurement of both numerator and denominator growth rates, which is not guaranteed in regimes with measurement breaks (currency redenomination, accounting reform, structural revision).
4. **Survivability proxies do not aggregate.** The empirical $S(t)$ is constructed as a multi-

plicative combination of $1/Q$, $\cos(\nabla\Phi)$, and τ. If alternative proxy specifications — additive combinations, weighted averages, principal-component approaches — produce Ω^* time series with materially different stage diagnoses, the multiplicative form is not unique and the framework's quantitative claims would need to be qualified to specific functional forms.

5. **Stages are not separable.** If empirical work shows that the three-stage instability model does not cleanly separate observed sovereign trajectories — if many sovereigns transit Stage 2 without ever reaching Stage 3 even after sustained $F(t) > 0$, or if Stage 3 onset has no diagnostic relationship with the proposed triggers — the staging structure is descriptive rather than predictive. The framework's structural diagnosis would survive but its policy applicability would be reduced.

8 Limitations

Threshold θ_{sys} unspecified. The framework establishes that θ_{sys} exists and is system-specific, but does not provide a calibration methodology. The empirical work above identifies the US as "Stage 2" on indicative grounds; precise threshold estimation across sovereigns is a substantial empirical project not undertaken here.

Reserve currency residual. The single largest unmodelled factor in the US analysis is reserve currency status, which acts as an effective subsidy on θ_{sys}. The framework does not capture this term; the trajectory is therefore conditional on continued dollar dominance, and the framework's reading is more conservative if that dominance erodes.

Static θ_{sys}. Thresholds are treated as time-invariant within the time-series analysis. In reality θ_{sys} drifts with macro and political conditions (regulatory environment, central bank stance, holder concentration). The drift is not formalised here.

Cross-sovereign aggregation. The framework operates one sovereign at a time. Cross-sovereign coupling — contagion via global investors, monetary spillover, sovereign credit-default-swap markets — is not modelled. The framework's diagnosis of any one sovereign therefore conditions on the others' trajectories.

9 Conclusion

We have established four results.

1. Obligation density — defined as total claims relative to effective survivable capacity — is a more precise and structurally grounded diagnostic for systemic fragility than conventional debt-level metrics.

2. The fragility gradient $F(t) = \Pi(t) - \Gamma(t)$ governs how density evolves; sustained $F > 0$ is the precursor to structural overload.

3. Modern accounting systematically underestimates true OD because contingent, derivative, and off-balance-sheet claims are excluded from standard measures. The under-measurement is the macro-financial manifestation of the constitutional Witness Principle ([3]); it is closed neither by improved instruments nor by surveillance, because what the public ledger lacks is not data but standing on the private-side ledger.

4. Comparative analysis of the US, Japan, and UK confirms that system-specific structural variables (central bank holding, maturity, domestic ownership, current account) determine θ_{sys} independently of headline debt.

The US sovereign is in Stage 2 (Structural Overload) by every measure in the framework. Ω^* has risen 3.5-fold from its 2013 low, driven primarily by survivability collapse following interest-rate normalisation. Including comprehensive unfunded obligations raises the effective density to 1.70 — total claims exceed effective capacity by 70%. The fragility gradient is projected to remain positive throughout the next decade on current CBO and IMF trajectories.

The framework has direct application beyond sovereign analysis. Any complex system in which claims can accumulate faster than survivable structure — financial networks, legal obligation systems, corporate balance sheets, institutional architectures — is subject to the same law: *fragility is a density problem, not a level problem.* The companion papers [3, 4, 6, 11, 12, 13, 14, 15, 16, 17] extend the framework along the constitutional, firm-scale, household-scale, and empirical-specification axes, identifying that the macro-financial OD measured here is the upper-scale aggregation of witness failures composing through the fractal private-side network.

The structural diagnosis advanced in this paper is not a counsel of despair. The bundle's constructive work in OD_06 [15] characterises the design space of redistribution instruments that could in principle avoid the asymmetry the macro-financial diagnosis identifies. The Capital-Preserving Redistribution class — defined by direct-discharge operation, network-compatible counterparty topology, capital-minting at the clearing layer, and discount-rate-symmetric timing — supplies the constructive answer to the structural deterioration documented here. Stage 2 (Structural Overload) is the diagnosis; the CPR class is the design space within which the diagnosis admits constructive resolution. Whether any specific implementation lands within that design space is a separate empirical question; that the design space is non-empty is the constructive theorem OD_06 proves.

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The author acknowledges that LLM systems can occasionally introduce hallucinated content — fabricated citations, miscoded notation, plausible-sounding but incorrect attributions — that survives author review despite reasonable diligence. Readers identifying any such artefact in this document are invited to contact the author so the issue can be examined and, if confirmed, corrected in a subsequent versioned deposit.

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