Experimental Investigation of β^+ and β^- Energy Spectra

Analysis of Observed Deviations from Theoretical Models

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to

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Synopsis

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Report on: Understanding the discrepancies in Beta Spectroscopy

In this experiment, the energy spectra of β^- and β^+ particles were studied using a magnetic spectrometer and a Geiger–Müller (GM) counter. The GM tube was first set to find its stable operating region. The measured counts were converted into energy values and analyzed using non linear fitting and derivative methods to determine the peak energies.

For the β^- spectrum, the peak energy was found to be approximately 1039.7 keV, while for the β^+ spectrum, the peak occurred at approximately 212.82 keV. This indicates that the range of energy in which both of these were measured are a bit different

The observed spectra differ significantly from theoretical predictions. This discrepancy arises because the measurements probe limited portions of the beta spectrum, where effects such as Coulomb interactions, scattering, and detector responses dominate. As a result, the curves are distorted, leading to deviations from the expected theoretical behavior.

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INTRODUCTION:

Beta decay is a nuclear process in which an unstable nucleus emits an electron (β^-) or a positron (β^+), along with a neutrino. Unlike alpha decay, beta decay produces a continuous energy spectrum because the decay energy is shared between the emitted particle and the neutrino. Studying the shape of this spectrum provides insight into nuclear structure and decay dynamics.

In this experiment, the energy spectra of β^+ and β^- particles were measured using a magnetic spectrometer and a Geiger–Müller (GM) counter. The spectrometer selects particles of specific energy through a magnetic field, allowing the count rate to be studied as a function of energy.

The primary objective of this work is not only to obtain the beta spectra but also to investigate the discrepancy between the experimentally observed spectra and the theoretical predictions. In particular, the experiment explores why the measured β^- and β^+ spectra appear inverted and signififcantly distorted compared to the expected theoretical behavior.

THEORY:

Beta decay produces a continuous energy spectrum because the decay energy is shared between the emitted particle (electron or positron) and the neutrino. As a result, the kinetic energy of the emitted particle can vary from zero up to a maximum value known as the endpoint energy.

The general form of the beta spectrum is given by:

$$N(E) \propto p^2(E_{max} - E)^2$$

where (p) is the momentum of the emitted particle and (E_{max}) is the endpoint energy. This expression predicts a spectrum that rises from low energy, reaches a peak, and then falls to zero at (E_{max}).

An important correction to this spectrum comes from the Coulomb interaction between the emitted particle and the nucleus, described by the Fermi function. For β^- decay, the electron is attracted to the positively charged nucleus, which increases the probability of low-energy electrons. For β^+ decay, the positron is repelled, which suppresses low-energy emission. This leads to a difference in the shapes of the β^+ and β^- spectra.

In this experiment, a magnetic spectrometer is used to relate the momentum of the particles to the magnetic field through the relation:

$$p = eBr$$

where (e) is the charge of the particle, (B) is the magnetic field, and (r) is the radius of curvature. By varying the magnetic field, particles of different momenta (and hence energies) can be selected and counted.

The detection of particles is carried out using a Geiger–Müller (GM) counter, which records the number of incoming particles but does not measure their energy directly. Therefore, the energy information is obtained indirectly through the magnetic field selection.

It is important to note that the accessible range of magnetic field in the experiment limits the range of energies that can be measured. As a result, only a portion of the beta spectrum is observed, which plays a crucial role in interpreting the results.

GM TUBE CHARACTERISATION:

Before performing the beta spectroscopy measurements, the Geiger–Müller (GM) counter was characterized by studying the variation of count rate with applied voltage. This was done to identify the plateau region where the detector operates reliably.

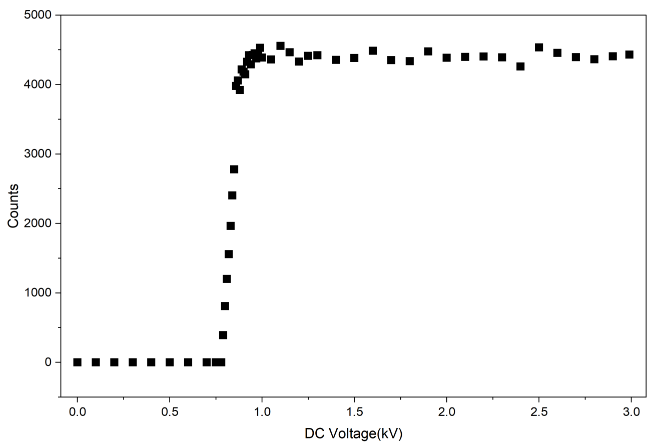

FIGURE 1: GM tube characteristic curve (Counts vs Voltage)

The plot shows a sharp increase in counts at lower voltages, followed by a plateau region where the count rate remains nearly constant with increasing voltage. This plateau region corresponds to stable operation of the GM counter and was chosen for all subsequent measurements.

Comparison with RadLAB Simulation

To better understand the detector response, a similar measurement was performed using the RadLAB simulation. The simulated result was compared with the experimentally obtained curve.

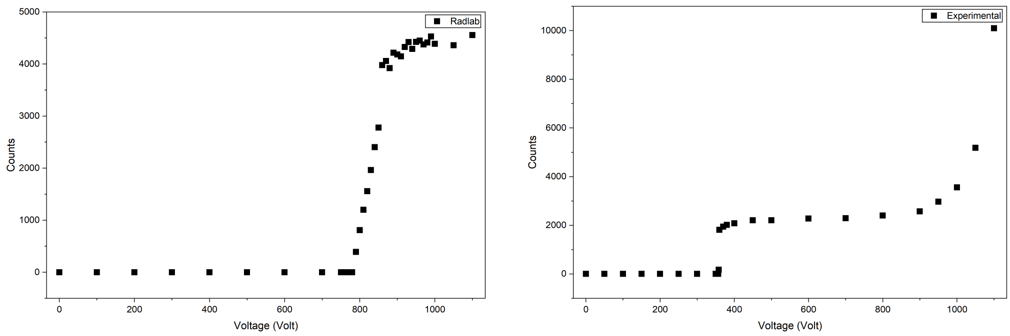

FIGURE 2: Comparison of GM tube response: RadLAB simulation (left) and experimental data (right)

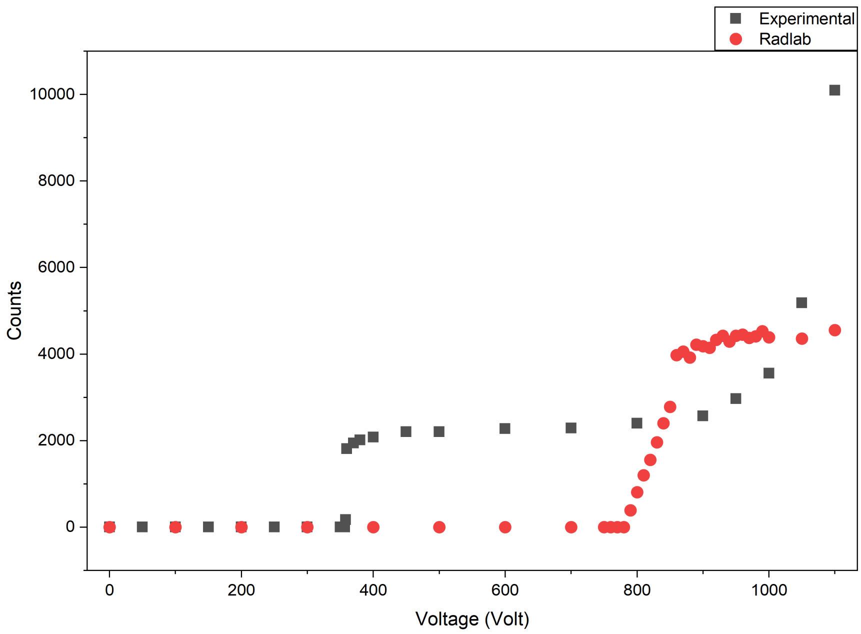

FIGURE 3: Combination of GM tube response obtained from RadLAB simulation and experimental measurement

Both the simulated and experimental graphs show the characteristic behavior of a GM tube, including the initial rise and the plateau region. However, slight differences are observed in the slope and extent of the plateau, which can be attributed to experimental uncertainties, detector imperfections, and environmental factors. Overall, the comparison validates the general behavior of the detector used in the experiment.

RESULTS AND ANALYSIS:

The measured magnetic field values were converted into corresponding particle energies, and the count rates were plotted as a function of energy. For each measurement, two readings were taken and averaged, and the uncertainty in counts was estimated using Poisson statistics.

0.1 β^- Spectrum

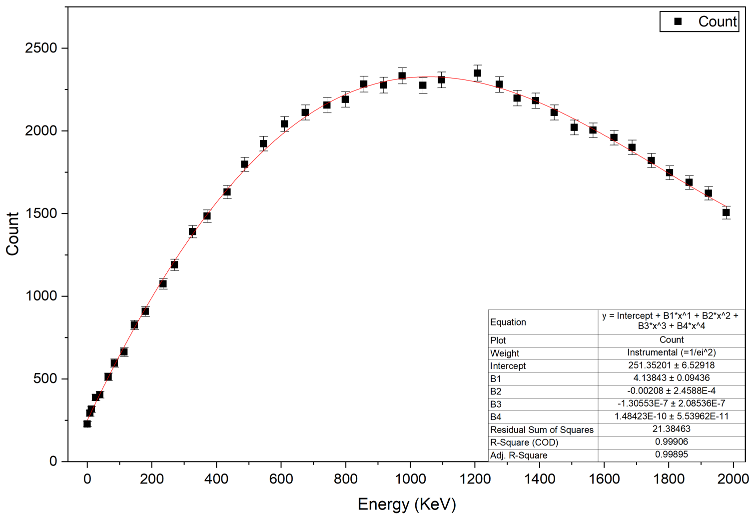

FIGURE 4: β^- spectrum with polynomial fit

The β^- spectrum shows a smooth variation of count rate with energy. A fourth-order polynomial fit was applied to obtain a continuous representation of the data. The peak of the spectrum was determined by taking the derivative of the fitted curve.

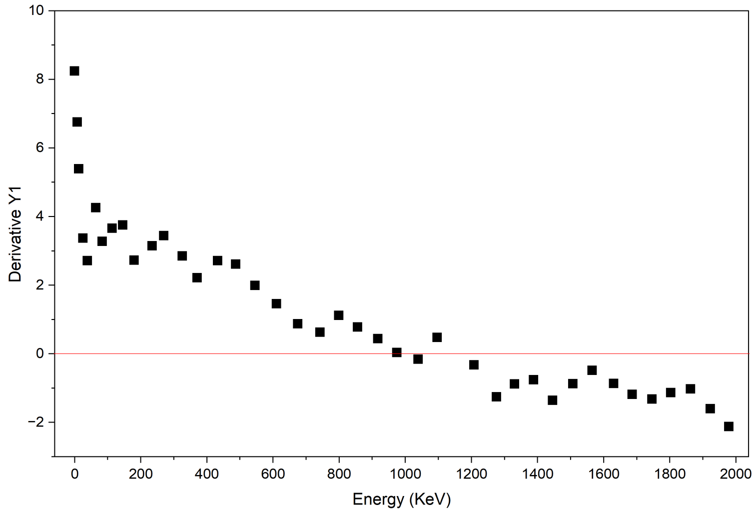

FIGURE 5: Derivative of the fitted β^- spectrum

From the zero of the derivative, the peak energy was found to be:

$$E_{\text{peak}}^{\beta^-} \approx 1039.72\,\text{keV}$$

0.2 β^+ Spectrum

A similar analysis was performed for the β^+ spectrum. Due to increased fluctuations in the data, a signal processing fit was applied to obtain a clearer representation of the spectrum. It was done by the Savitzky-Golay method. The derivative of the smoothed curve was then used to determine the peak position.

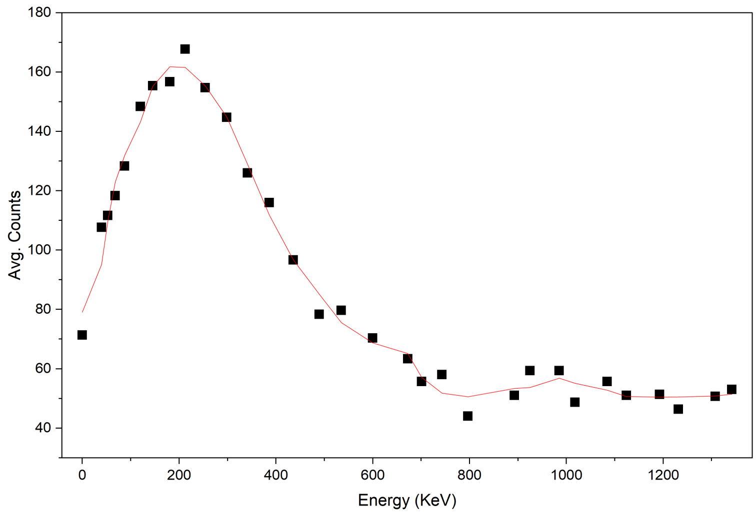

FIGURE 6: β^+ spectrum after Savitzky–Golay smoothing (to reduce fluctuations)

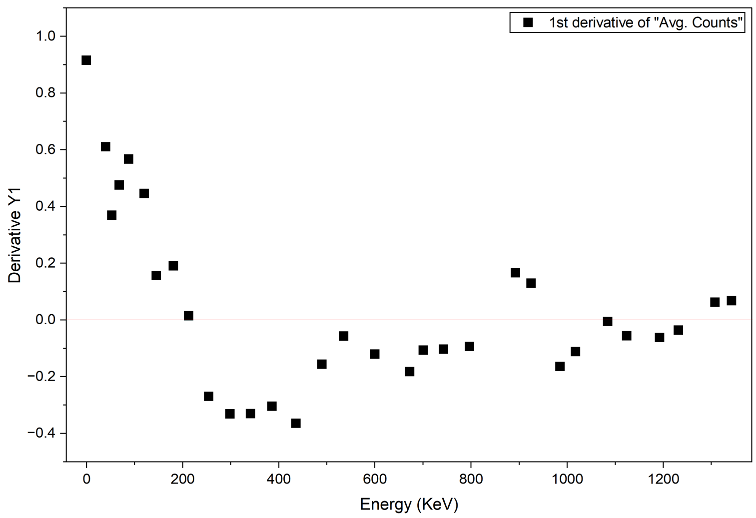

FIGURE 7: Derivative of the fitted β^+ spectrum

The peak energy for the β^+ spectrum was found to be:

$$E_{\text{peak}}^{\beta^+} \approx 212.82\,\text{keV}$$

0.3 Endpoint Energy Estimation

The endpoint energy was estimated using the empirical relation:

$$E_{max} \approx 3E_{\text{peak}}$$

Using this relation, the endpoint energies were found to be:

$$E_{max}^{\beta^-} \approx 3119.16\,\text{keV}$$

$$E_{max}^{\beta^+} \approx 638.46\,\text{keV}$$

These values represent effective endpoint energies within the experimentally accessible energy range.

Although the energy axis extends to higher values, the spectrum does not approach zero within the measured range, indicating that the true endpoint energy is not fully captured by the experiment.

DISCUSSION:

The experimentally obtained β^- and β^+ spectra show clear deviations from the theoretical expectations. In theory, the beta spectrum extends up to an endpoint energy in the 2-3 MeV range and follows a characteristic shape. However, in the present experiment, the observed spectra differ both in shape and in the accessible energy range.

The β^- spectrum peaks at around 1039.72 keV while the β^+ spectrum peaks at around 212.82 keV. This is actually opposite to that predicted by the theory. The relative positions of the peaks differ from theoretical expectations, indicating that the experimentally observed spectra do not follow the ideal beta decay distribution.

The most important factor contributing to this discrepancy is the limited energy range of the spectrometer. The magnetic field values used in the experiment correspond to only a restricted range of particle energies. As a result, the measurements probe only a portion of the beta spectrum rather than the full distribution. In particular, for the Beta spectra, the measured energies lie in the keV range, which is quite below the true endpoint energy of beta decay.

In this low-energy regime, the Coulomb interaction between the emitted particle and the nucleus plays a dominant role. For β^- decay, the emitted electron is attracted to the positively charged nucleus, which enhances the probability of detecting low-energy electrons. For β^+ decay, the emitted positron is repelled, which suppresses low-energy emission. Since the experiment primarily samples this low-energy region, these Coulomb effects significantly distort the observed spectral shape.

Another important factor is scattering. As beta particles travel through the apparatus, they can lose energy due to interactions with matter. This causes higher-energy particles to be detected at lower apparent energies, leading to a redistribution of counts and a distortion of the spectrum.

The response of the GM counter also contributes to the observed differences. The detector records only the number of particles and does not measure their energy directly. Its efficiency and dead-time effects can influence the measured count rates, particularly at higher count values.

For the β^+ spectrum, additional effects such as positron annihilation may also play a role. The emitted positron can annihilate with an electron, producing gamma rays, which may affect the detected counts and further modify the observed spectrum.

It is also important to note that although the energy axis extends to higher values, the spectrum does not approach zero within the measured range. This indicates that the true endpoint energy is not fully captured by the experiment. Therefore, the estimated endpoint energies obtained from the peak positions represent effective values within the accessible range rather than the true physical endpoints.

Taken together, these factors explain why the experimentally observed β^+ and β^- spectra appear different from the theoretical predictions and, even appear inverted. The experiment highlights the importance of considering both physical effects and experimental limitations when interpreting measured data.

CONCLUSION:

In this experiment, the energy spectra of β^- and β^+ particles were measured using a magnetic spectrometer and a GM counter. The data was analyzed by converting magnetic field values into energies and applying smoothing and fitting techniques to determine the peak positions of the spectra.

The peak energies were found to be approximately 1039.72 keV for the β^- spectrum and 212.82 keV for the β^+ spectrum. Using an empirical relation, the corresponding endpoint energies were estimated within the experimentally accessible range.

The observed spectra showed noticeable deviations from theoretical expectations. This is primarily due to the limited energy range of the experiment, as well as the influence of Coulomb interactions, scattering effects, and detector limitations. As a result, the measured spectra do not represent the complete beta distribution.

Overall, the experiment demonstrates how practical limitations and physical effects influence the observed beta spectra, highlighting the importance of careful interpretation of experimental data.

In addition to this, the experiment highlights the importance of understanding both physical processes and experimental constraints when interpreting nuclear measurements.