Functional Sparsity in State Spaces: A Pre-Dynamic Structural Principle

Ho Minh
Independent Researcher, Vietnam
Email: khongbiet06@gmail.com

Abstract

This paper formalizes a minimal structural principle governing constrained state spaces, termed the Geometric Sparsity Principle (GSP). We show that configurations satisfying functional constraints occupy a measure-small subset of the ambient state space, arising from multiplicative contraction induced by intersecting constraint sets.

The result is established under mild and broadly applicable conditions, without assumptions on dynamics, optimization procedures, or agent behavior. The underlying mechanism is geometric: layered constraints induce multiplicative contraction of admissible regions, leading to exponential sparsity in both discrete and continuous settings.

This perspective identifies functional sparsity as a pre-dynamic property of constraint-defined systems, independent of specific representations or procedures. As such, it provides a domain-agnostic foundation for understanding the rarity of viable configurations in high-dimensional spaces.

Implications for sampling, optimization, and learning are noted but not treated as intrinsic components of the principle.

Keywords: functional sparsity, constrained systems, high-dimensional geometry, measure concentration, probabilistic bounds, dependence structure

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1 — Introduction

In many complex systems, configurations satisfying functional constraints occupy only a small fraction of the ambient state space. This phenomenon appears across domains, including combinatorial optimization, control, biological systems, and engineered design. Despite its ubiquity, it is often attributed to domain-specific factors such as search limitations, imperfect control, or domain-specific procedural inefficiencies.

However, such explanations are incomplete: even under idealized exploration or optimal procedures, feasible configurations remain sparse. This suggests that the phenomenon is not merely procedural, but fundamentally structural.

In this work, we formalize this phenomenon as a pre-dynamic structural property of constrained state spaces. Given a state space Ω equipped with a reference measure μ, and a functional set $F \subseteq \Omega$ defined as the intersection of constraint sets, we show that under mild and broadly applicable conditions, $\mu(F)$ becomes exponentially small as the number of constraints increases.

The underlying mechanism is geometric: layered constraints progressively restrict admissible configurations, inducing a multiplicative contraction of the feasible region. This effect holds in both discrete and continuous settings and does not depend on specific dynamics, optimization procedures, or representations.

Related perspectives arise in constraint satisfaction, measure concentration, and statistical physics [1, 2, 3], typically in domain-specific or process-dependent forms.

This work makes two primary contributions:
(i) We identify functional sparsity as a minimal structural principle governing constrained systems, independent of domain-specific mechanisms.
(ii) We provide a unified quantitative framework for functional sparsity, where measure is a canonical instance of a broader notion of size across discrete and continuous regimes under bounded dependence.

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2 — Basic Definitions

To formalize this structural perspective, we begin with a representation-agnostic state-space framework. We consider a state space Ω, representing the set of all admissible configurations of a system.

A functional set is defined as a subset

$$F \subset \Omega$$

consisting of configurations that satisfy a given collection of constraints.

These constraints may arise from physical laws, design requirements, or task-specific conditions. We do not assume any particular structure beyond measurability or countability as appropriate to the setting.

We define:

- In discrete settings:

$$\mu(F) = \frac{|F|}{|\Omega|}$$

- In continuous settings:

$$\mu(F) = \frac{\mathrm{Vol}(F)}{\mathrm{Vol}(\Omega)}$$

whenever such quantities are well-defined.

This definition abstracts functionality purely as constraint satisfaction, independent of any dynamics or objective.

Remark 2.1 (Normalization).
Throughout the paper, we assume that the ambient space Ω is endowed with a normalized measure, i.e., $\mu(\Omega) = 1$.
This ensures that $\mu(\mathrm{F})$ can be interpreted as a relative size.
When absolute measures are used, all statements can be reformulated in normalized form without loss of generality.

We say that functionality is measure-small if

$$\mu(F) \ll 1$$

This notion captures the idea that functional configurations occupy only a small fraction of the ambient space.

Remark 2.2 (Robustness of sparsity).
The notion of measure-smallness is invariant under equivalent representations of the state space and does not depend on a particular parametrization of Ω.

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3 — Structural Setup

We model functional sets as arising from the intersection of multiple constraint layers.

Let

$$\mathcal{F} = \bigcap_{i=1}^{k} \mathcal{C}_i$$

where each $\mathcal{C}_i \subset \Omega$ represents a constraint-defined subset.

We assume that each constraint reduces the admissible region in a nontrivial way, i.e.,

$$\mu(\mathcal{C}_i) < 1$$

Under weak dependence conditions between constraints, the combined effect leads to a progressive contraction of admissible states.

Assumption 3.1 (Weak dependence).
We do not assume independence between constraints.
It suffices that the overlap between constraint sets does not systematically concentrate measure in a way that offsets their individual restricting effects.

This layered restriction induces a geometric structure in which the feasible region becomes increasingly concentrated.

This condition is satisfied in a wide class of systems where constraints do not exhibit systematic positive alignment; formal treatments may be developed in terms of correlation bounds or dependency graphs, but are beyond the scope of this work.

Remark 3.2 (Generality across regimes).
The above construction applies uniformly to both discrete and continuous state spaces, and is agnostic to the specific nature of the constraints, with cardinality and volume serving as parallel notions of measure.

This structural perspective motivates the main claim of the next section, where we formalize functional sparsity as a pre-dynamic principle.

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4 — Core Structural Principle

4.1 Informal Statement

Across a broad class of systems, configurations satisfying functional constraints occupy a vanishingly small subset of the ambient state space.
This phenomenon arises prior to any dynamics, optimization, or learning, and reflects a structural imbalance induced by constraint geometry.

Postulate (Functional Sparsity)

Functional configurations occupy a vanishingly small subset of the state space.

4.2 Formal Setting

Let (Ω, μ) be a measurable state space equipped with a normalized measure, i.e., $\mu(\Omega) = 1$. This includes both finite spaces with the uniform measure and compact subsets of R^n with normalized Lebesgue measure.

Let $F \subseteq \Omega$ denote the set of configurations satisfying a collection of functional constraints:

$$F = \bigcap_{i=1}^{k} C_i$$

where each $C_i \subseteq \Omega$ is measurable.

We express all quantities in ratio form:

$$\frac{\mu(\cdot)}{\mu(\Omega)}$$

No assumptions are made on dynamics, sampling, or optimization; the formulation is purely structural.

4.3 Structural Conditions

The following conditions are not imposed assumptions, but structural properties characterizing constraint-defined subsets in broad classes of systems.

Condition 4.3.1 (Nontrivial constraints).
There exist constants $p_i < 1$ such that

$$\frac{\mu(C_i)}{\mu(\Omega)} \leq p_i, \quad i = 1, \ldots, k$$

Condition 4.3.2 (Bounded dependence / overlap).
There exists a constant $\gamma \geq 1$, independent of k, capturing bounded overlap among constraints, such that

$$\frac{\mu\left(\bigcap_{i=1}^{k} C_i\right)}{\mu(\Omega)} \leq \gamma \prod_{i=1}^{k} \frac{\mu(C_i)}{\mu(\Omega)}$$

4.4 Structural Theorem

Theorem 4.4 (Geometric Sparsity Principle).
Under Conditions 4.3.1–4.3.2,

$$\frac{\mu(F)}{\mu(\Omega)} \leq \gamma \prod_{i=1}^{k} p_i$$

In particular, if $p_i \leq p < 1$ for all i, then

$$\frac{\mu(F)}{\mu(\Omega)} \leq \gamma p^k \longrightarrow 0 \quad \text{exponentially as } k \to \infty$$

4.5 Interpretation

The result is structural: sparsity arises from multiplicative contraction induced by constraints under bounded overlap.
No appeal is made to randomness, optimization, or dynamical failure.

4.6 Realizations of the Principle

The discrete and continuous cases are not separate results, but realizations of a single underlying structural principle.

Proposition 4.6.1 (Discrete realization).
Theorem 4.4 applies to finite state spaces equipped with the uniform probability measure.

In this setting, constraints act combinatorially, and the bound follows directly from multiplicative restriction of admissible configurations.

In particular, we obtain

$$\frac{|F|}{|\Omega|} \leq \gamma p^k$$

recovering exponential sparsity in combinatorial regimes.

Proposition 4.6.2 (Continuous realization).
Theorem 4.4 applies to compact subsets of R^n equipped with normalized Lebesgue measure.

In this setting, constraint geometry induces measure contraction, yielding exponential sparsity in high-dimensional spaces.

In particular,

$$\frac{\mu(F)}{\mu(\Omega)} \leq \gamma p^k$$

establishing exponential sparsity in continuous spaces.

4.7 Remarks

(i) Independence as a special case.
If the constraint sets are independent, then $\gamma = 1$, and

$$\frac{\mu(F)}{\mu(\Omega)} = \prod_{i=1}^{k} \frac{\mu(C_i)}{\mu(\Omega)}$$

(ii) Interpretation of bounded dependence.
The constant γ quantifies deviation from independence due to constraint overlap, while preserving multiplicative decay.

(iii) Structural asymmetry.
The overwhelming majority of configurations fail to satisfy functional constraints; admissible configurations form a measure-small subset.

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5. Structural Basis of Bounded Dependence

5.1 From Assumption to Structure

The bounded dependence condition in Theorem 4.4,

$$\mu\left(\bigcap_{i=1}^{k} C_i\right) \leq \gamma \prod_{i=1}^{k} \mu(C_i)$$

is not introduced as an arbitrary assumption.
It reflects an underlying structural property of how constraints interact within the state space.

We now identify general mechanisms under which such a bound arises.

All these mechanisms reflect a common principle:
constraint interactions remain sufficiently localized or weak,
thereby preventing uncontrolled accumulation of dependence.

5.2 Independence and Near-Independence

In the idealized case where the constraint sets $\{C_i\}$ are independent,

$$\mu\left(\bigcap_{i=1}^{k} C_i\right) = \prod_{i=1}^{k} \mu(C_i)$$

and the inequality holds with $\gamma = 1$.

More generally, if dependencies are weak or sufficiently diffuse, deviations from independence can be controlled, yielding a bounded inflation factor γ.
Thus, bounded dependence can be interpreted as a stability condition around independence.

5.3 Limited Overlap Structure

A sufficient structural condition for bounded dependence is limited overlap among constraints.

The key structural driver is that each constraint interacts with only finitely many others,

so dependency cannot accumulate beyond a controlled factor.

Suppose each constraint set C_i overlaps nontrivially with only a bounded number of other constraints, independent of k.
Then the cumulative effect of overlaps cannot grow arbitrarily with k, yielding the bound:

$$\mu\left(\bigcap_{i=1}^{k} C_i\right) \leq \gamma \prod_{i=1}^{k} \mu(C_i)$$

for some constant γ depending only on the maximal overlap degree.

This regime includes sparse interaction graphs and locally dependent constraint systems.

5.4 Correlation Decay and Weak Coupling

Bounded dependence also arises when correlations between constraints decay sufficiently fast.

If the dependence between C_i and C_j weakens with distance (in an appropriate metric or interaction graph), then long-range interactions become negligible, and the joint measure behaves approximately multiplicatively.

Such decay conditions are standard in statistical physics and probabilistic graphical models, where they ensure control over joint distributions.

5.5 Geometric Regularity in Continuous Spaces

In continuous settings, bounded dependence can emerge from geometric regularity.

If constraint sets are defined by smooth or low-curvature regions, and their intersections do not concentrate mass along highly degenerate manifolds, then the volume of intersections remains controlled relative to the product of individual volumes.

In high-dimensional spaces, this effect is often amplified by concentration phenomena, which further suppress large overlaps.

5.6 Stability Under Constraint Composition

The bounded dependence condition is stable under composition of constraint families.

If two collections of constraints each satisfy bounded dependence with constants γ_1 and γ_2, and their interactions remain controlled, then the combined system also satisfies bounded dependence with a constant depending only on γ_1, γ_2.

This stability reflects the robustness of the condition under layering and aggregation.

As a result, the joint measure behaves approximately multiplicatively, up to a bounded factor γ.

5.7 Interpretation

The constant γ quantifies the deviation from ideal multiplicative behavior.

- $\gamma = 1$: exact independence
- $\gamma > 1$: controlled overlap
- $\gamma \gg 1$: strong dependence, breakdown of sparsity guarantees

Thus, bounded dependence characterizes a regime in which constraint interactions are sufficiently limited to preserve exponential decay.

γ thus quantifies the effective structural complexity of constraint interactions.

5.8 Structural Insight

Bounded dependence is not a special property of particular models, but a structural feature of systems in which constraints are locally defined, interactions do not scale with system size, and global coherence is not enforced by dense coupling.

Under these conditions, multiplicative contraction dominates, leading to sparsity.

5.9 Bridge to Mechanism

The presence of bounded dependence ensures that each additional constraint contributes a multiplicative reduction in feasible volume.

This layered contraction is the structural basis for the emergence of directional effects in exploration and dynamics, which we analyze in the next section.

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6. Mechanism: Constraint Layering and Emergent Drift

6.1 Mechanism Overview

Under the Geometric Sparsity Principle, functional states occupy a measure-small subset of the state space. Section 5 showed that this sparsity arises structurally from bounded dependence: constraint interactions remain sufficiently localized to prevent uncontrolled accumulation.

We now describe the mechanism by which this structural property induces systematic directional effects.

The key observation is that functionality is defined by the intersection of multiple constraints. Each constraint reduces the admissible region, and their composition produces a layered restriction of the state space. This layered structure is not neutral: through multiplicative contraction, it induces an imbalance between entry into and retention within the functional set.

6.2 Constraint Layering as Multiplicative Contraction

Let

$$F = \bigcap_{i=1}^{k} C_i$$

denote the functional region defined by the constraint family $\{C_i\}_{i=1}^{k}$.

Under bounded dependence (Section 5), the joint measure contracts approximately multiplicatively:

$$\mu(F) \leq \gamma \prod_{i=1}^{k} \mu(C_i)$$

for a constant (γ) depending only on local structure.

Thus, even when each individual constraint is weak, their composition yields a strong reduction in feasible volume. This multiplicative contraction is the geometric core of the mechanism.

6.3 Asymmetry of Transitions

Consider an exploratory process over the state space.

From a functional state ($x \in F$), any perturbation risks violating at least one constraint.

From a non-functional state ($x \notin F$), entering (F) requires satisfying all constraints simultaneously.

This creates an inherent asymmetry:

$$\text{Exit probability} \gg \text{Entry probability}$$

Leaving functionality requires breaking a single constraint, whereas entering requires satisfying all of them. This asymmetry is structural and does not vanish under scaling.

6.4 Emergent Directionality (Drift Mechanism)

This asymmetry is a direct consequence of the multiplicative contraction induced by constraint layering.

As a result, transitions are statistically biased:

$$P(X_{t+1} \notin F \mid X_t \in F) > P(X_{t+1} \in F \mid X_t \notin F)$$

Under unbiased or weakly biased exploration dynamics, this yields:

$$E[\Delta 1_F] < 0$$

indicating a net drift away from the functional region.

This recovers the structural drift ($\Phi < 0$) introduced earlier.

Key point.
This drift does not arise from noise, poor strategies, or adversarial effects. It is a direct consequence of the geometric asymmetry induced by constraint layering.

6.5 High-Dimensional Amplification

In high-dimensional systems, the number of effective constraints typically scales with dimension. As dimension increases, the feasible region ($\mathcal{F}$) shrinks rapidly due to repeated multiplicative contraction.

Consequently, random entry into ($\mathcal{F}$) becomes negligible, while exit events dominate. This amplifies the asymmetry and strengthens the induced drift.

6.6 Structural Pipeline: From Geometry to Behavior

The mechanism can be summarized as:
Layered constraints → Multiplicative contraction → Transition asymmetry → Directional drift

Equivalently, at a more geometric level:
Constraint geometry → Measure concentration → Transition asymmetry → Emergent drift

This pipeline highlights a structurally grounded progression from constraint geometry to emergent statistical bias.

6.7 Implication

Functionality is fragile not because systems are poorly controlled, but because it resides in a geometrically constrained region of the state space.

It is easy to break but exponentially difficult to assemble.

This asymmetry is not algorithmic but structural, and therefore persists across models and implementations.

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7. Projected Consequence Classes

7.1 Structural Origin of Negative Expected Change

Under the mechanism in Section 6, transitions are inherently asymmetric: leaving the functional region requires breaking a single constraint, whereas entering requires satisfying all constraints simultaneously.

As a result, under unbiased or weakly biased exploration,

$$E[\Delta \mathbf{1}_F] < 0$$

indicating a negative expected drift away from the functional region.

This effect is structural: it follows from multiplicative contraction and does not depend on specific algorithms or noise models.

7.2 Persistent Bias in State Space

Because exit events are much more probable than entry events,
the system develops a persistent statistical bias toward non-functional states.

Over time, trajectories concentrate in partially satisfied constraint regions, while fully functional configurations become increasingly rare.

This induces:

- systematic deviation from functional states,
- long-term imbalance in state occupancy,
- effective repulsion from highly constrained regions.

7.3 Exploration Becomes Inefficient

The feasible region F occupies an exponentially small portion of the state space.

Consequently, random exploration:

- rarely enters F,
- is quickly exited once reached,
- and is rarely rediscovered.

Thus, exploration without structural guidance is inefficient and unstable.

7.4 Negative Long-Run Value

The asymmetry implies that, over time, the system loses functional structure faster than it gains it.

In expectation:

- gains (entering F) are rare and fragile,
- losses (leaving F) are frequent and easy.

This yields a structurally negative long-run expected value:

$$\mathrm{EV}_{\text{long-run}} < 0$$

Importantly, this is not due to poor strategy, but due to the geometry of the constraint system.

Proposition 7.4 (Structural negative drift).
Let $F = \bigcap_{i=1}^{k} C_i$ denote the functional region, and assume the bounded dependence condition (Section 5).
Consider an exploration process that is unbiased or only weakly biased with respect to the constraint structure.

Then there exists a constant $\delta > 0$, depending only on local structure, such that

$$E[\,\Delta \mathbf{1}_{\mathbf{F}} \mid X_t\,] \leq -\delta \quad \text{whenever } X_t \in F$$

In particular, the process exhibits a negative expected drift away from the functional region, and thus

$$\mathrm{EV}_{\text{long-run}} < 0$$

This follows from the negative expected drift established in Section 7.1.

This result formalizes that functionality is statistically unstable under unguided exploration. The asymmetry between entry and exit probabilities—induced by multiplicative contraction—creates a persistent tendency to leave F faster than it can be re-entered.

Importantly, the constant δ depends only on local interaction structure (e.g., overlap degree, mixing, geometry), not on the number of constraints k.

Remark.
The result does not rely on specific algorithmic choices or noise models.
It is a direct consequence of structural properties established in Sections 4–6.

Accordingly, these perspectives—drift, bias, and entropy—are not distinct mechanisms, but equivalent manifestations of the same underlying geometric constraint structure.

7.5 Entropic Perspective

The structural bias toward non-functional states also admits an entropic interpretation.

As trajectories concentrate away from the functional region, the distribution over states becomes increasingly dispersed across partially constrained configurations. This corresponds to an effective increase in entropy, reflecting the loss of coordinated constraint satisfaction.

Thus, the negative drift can be viewed equivalently as:

- a geometric effect (measure contraction),
- a dynamical effect (transition asymmetry),
- or an entropic effect (dispersion away from structured states).

7.6 Interpretation

Functionality is fragile not because systems are poorly designed, but because it resides in a geometrically constrained region of the state space.

It is easy to break, but exponentially difficult to assemble.

The resulting bias and negative drift are therefore inevitable consequences of structural geometry, not artifacts of dynamics.

7.7 Bridge Forward

To maintain or recover functionality, systems must counteract this structural bias.

This requires mechanisms that:

- guide exploration toward constraint satisfaction,
- stabilize trajectories within functional regions,
- or modify effective accessibility within the state space.

This figure summarizes the common structural architecture underlying drift, bias, asymmetry, and related downstream phenomena.

Taken together, these results support a unified structural interpretation.

Dynamic drift, statistical bias, long-run negative expected value, and entropy-related degradation may be understood as distinct projections of a shared underlying constraint geometry.

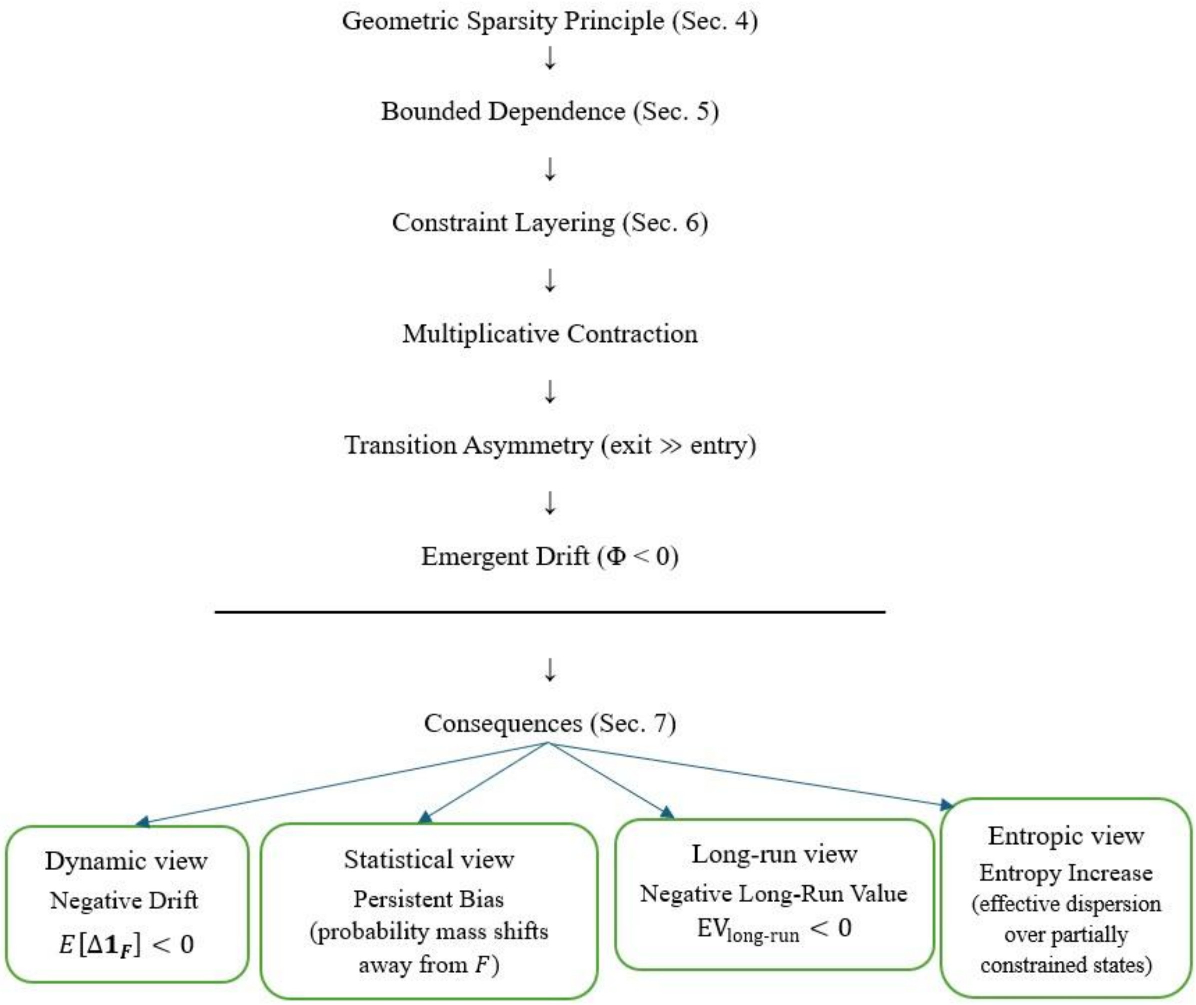

Figure X. ***Structural pipeline from geometric sparsity to equivalent consequence projections.*** *Geometric sparsity and bounded dependence induce constraint layering, producing multiplicative contraction, transition asymmetry, and emergent drift. These structural effects admit equivalent dynamic (drift), statistical (bias), long-run (negative expected value), and entropic (dispersion) projections.*

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8 — Discussion and Structural Implications

8.1 Unified Structural Interpretation

Different metrics do not necessarily represent competing explanations, but distinct observational lenses on the same underlying structural reality.

The present framework develops a layered account of constrained functionality.
Section 4 establishes that functional states are geometrically sparse.
Section 5 shows that bounded local dependence preserves this sparsity under realistic interaction structure.
Section 6 demonstrates that layered constraints induce multiplicative contraction and transition asymmetry.
Section 7 extends these implications into downstream domains, where asymmetry may

manifest dynamically as drift, statistically as bias, economically as long-run value erosion, or entropically as dispersion away from coordinated states.

Accordingly, drift, bias, long-run expected value, and entropy are best understood not here as separate mechanisms, but as distinct projections of a shared geometric constraint architecture.

This does not imply that these quantities are interchangeable; rather, they describe different observational layers of the same structural substrate.

8.2 Scope, Conditions, and Limits

The Geometric Sparsity Principle applies most naturally to systems where:

- a meaningful state space Ω can be defined,
- functionality corresponds to a constrained subset $F \subset \Omega$,
- multiple constraints must often be simultaneously satisfied, and
- structural transitions are sensitive to local violations.

This includes many biological, strategic, institutional, computational, and engineered systems, though applicability is not universal.

Importantly, sparsity depends on both representation and measure. Different formulations may alter the apparent size or accessibility of functional regions. However, across broad classes of constrained high-dimensional systems, functional subsets often remain disproportionately small.

The principle should therefore be interpreted neither as a universal law nor as an absolute guarantee, but as a recurring structural regularity that arises under commonly encountered conditions.

Its claim is fundamentally one of typicality:
functional order is often geometrically narrow relative to total possibility space.

8.3 Structural Architecture of Failure

A broader implication of this framework is that many recurring failures may not be fully explained by local errors, isolated inefficiencies, or individual shortcomings alone. In many constrained systems, structural geometry may systematically shape probabilistic flows before local behavior is fully executed.

Structural bias is not fate, but a systemic asymmetry within the space of possibilities. When functional regions are narrow, local exits are easy, and re-entry is combinatorially difficult, systems may exhibit persistent tendencies toward degradation even when agents are partially adaptive. Under this interpretation, local action still matters—but action often unfolds within architectures that may have already biased the outcomes.

Consequently, many forms of failure may be better understood not only behaviorally, but structurally.

Failure often has structure before it has dynamics.

This perspective does not eliminate agency, nor does it imply inevitability. Rather, it suggests that in many environments, observed failure may partially reflect deeper constraint geometry, transition asymmetry, and state-space architecture.

8.4 Position Within a Broader Framework

The Geometric Sparsity Principle is best understood as a foundational pre-dynamic layer rather than a complete theory of system behavior.

Its primary contribution is to isolate structural geometry as an upstream explanatory domain from which downstream phenomena—such as fragility, drift, bias, instability, and long-run erosion—may emerge under additional assumptions.

This shifts part of analysis from purely local performance toward architecture itself:
from what agents do alone toward the structural landscape in which those actions unfold.

More broadly, this framework suggests that functionality is often fragile not merely because systems are poorly controlled, but because coordinated functional order may itself occupy geometrically rare regions.

In this sense, the principle may serve as a structural basis for broader investigations into resilience, adaptation, institutional design, exploration efficiency, and recurring patterns of organized failure.

The broader implication is not that all systems fail for the same reason, but that many complex failures may share recurring structural architectures.

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9 — Conclusion

This work isolates a minimal pre-dynamic structural property of constrained systems:
functional states often occupy a geometrically sparse subset of the ambient state space.

This sparsity arises prior to dynamics, optimization, or learning, and follows directly from constraint geometry. Because functionality requires coordinated satisfaction of multiple conditions, while non-functional configurations dominate unconstrained possibility space, viable states may be structurally rare even before any local process unfolds.

Under bounded dependence and layered constraint structure, this geometric sparsity may propagate downstream into transition asymmetry, directional drift, persistent bias, long-run value erosion, and entropy-related dispersion. These phenomena need not be interpreted solely as failures of local behavior, but may often reflect deeper architectural asymmetries within the space of possibilities.

The Geometric Sparsity Principle is not proposed as a universal law, but as a recurring structural regularity across broad classes of constrained systems.

Beyond explanation, this framework may also help guide the design of systems that preserve functionality by reshaping constraint architecture rather than merely correcting local behavior. *Functionality is not the default; it may be structurally exceptional.*

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