Spacetime Is Thicc

Shear-Thickening Rheology, the Lorentz Group, and the Einstein Field Equations from Cornstarch Microphysics

A. Caen

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Abstract

We propose that physical spacetime is a dense non-Newtonian suspension at the jamming transition. Energy density is particle volume fraction. The speed of light is the critical shear rate at which the substrate jams. We prove (Theorem 1) that a shear-thickening constitutive exponent $\alpha = 3/2$ is the unique value reproducing the complete kinematic structure of special relativity—energy, momentum, γ^3 resistance scaling, dispersion, and mass shell—exactly, and that the invariance of the mass shell selects the Lorentz group $SO^+(1,3)$ as the kinematic symmetry (Corollary 3). We prove (Theorem 5) that this exponent is not free but equals $d\nu = 3 \times \frac{1}{2} = \frac{3}{2}$, where $d = 3$ is the spatial dimension and $\nu = 1/2$ the mean-field jamming correlation length exponent, derived via Landauer-bounded information processing in the frictional contact network. In volumetric coordinates, the speed of light is the critical shear rate $\dot{\gamma}_c$, universal for all embedded bodies regardless of size, and the rest energy density $E_0/V = \frac{9}{2}\sigma_c$ is an exact material constant determined by the DST onset stress alone. The Unruh–Visser acoustic metric provides curved effective spacetime from concentration gradients. The Bredberg–Keeler–Lysov–Strominger correspondence and the Padmanabhan null-surface projection establish that the low-shear Newtonian branch of the substrate admits the Bredberg–Keeler–Lysov–Strominger holographic lift to the vacuum Einstein equations. Three falsifiable predictions are presented. Residual discrepancies are attributed to second-order supply chain losses and interannual corn crop yield instability.

1 Introduction

Spacetime is not a smooth pseudo-Riemannian manifold. It is a dense suspension of discrete particles near the jamming transition—a shear-thickening fluid whose macroscopic behavior, under the appropriate constitutive law, identically reproduces special-relativistic kinematics and enters established correspondence with general-relativistic dynamics. The physical system is a cornstarch-water suspension (oobleck) at volume fraction Θ near the discontinuous shear-thickening (DST) transition [1, 2, 3].

This identification is not a metaphor. We prove below that the constitutive exponent $\alpha = 3/2$, the unique value reproducing exact Lorentz kinematics (Theorem 1), emerges from three-dimensional jamming universality via Landauer-bounded information processing (Theorem 5). The Navier–Stokes equations governing the substrate reproduce the Einstein field equations in the holographic and null-surface senses [4, 5]. The Lorentz group is the symmetry group of cornstarch.

Postulate 1 (Starch-Energy Equivalence)**.** *The local energy density $\rho_E(x)$ is proportional to the local particle volume fraction $\Theta(x)$:*

$$\frac{\Theta(x)}{\Theta_J} = \frac{\rho_E(x)}{\rho_c}\,, \tag{1}$$

where ρ_c is the critical energy density at which the substrate jams ($\Theta = \Theta_J$) and $\Theta_J \approx 0.56$ is the DST jamming fraction. Regions of higher energy density are regions of higher particle concentration.

Remark 1 (Microscopic interpretation of the postulate). The Starch-Energy Equivalence is not arbitrary. Consider the frictional contact network: N contacts per unit volume, each binary (lubricated, state 0; frictional, state 1). A frictional contact stores deformation energy ε. The volume fraction of frictional contacts is $\Theta/\Theta_J = \langle n \rangle$ (mean occupation), and the energy density is $\rho_E = \varepsilon N \langle n \rangle$, giving $\rho_c = \varepsilon N$ when all contacts are frictional. The Starch-Energy Equivalence then reads $\langle n \rangle = \rho_E/\rho_c$, which is simply the definition of mean occupation.

The non-trivial content is that the contact network *sits at maximum entropy subject to the energy constraint.* Maximize the binary configurational entropy $S = -\sum[n_i \ln n_i + (1-n_i)\ln(1-n_i)]$ subject to fixed total energy $\sum \varepsilon n_i = E$. The Jaynes maximum-entropy solution [6] assigns each contact the Boltzmann occupation, and the mean-field/high-temperature limit gives $\langle n \rangle \propto \rho_E$. The same Landauer-bounded dissipation that produces Theorem 5 drives the contact network toward this maximum-entropy configuration: irreversible contact flips erase information, increasing entropy until the MaxEnt state is reached. The postulate is the equilibrium condition of the substrate's own information dynamics.

All subsequent results follow from this postulate together with the closure assumptions stated explicitly below: the Stokes–Oobleck mean-shear approximation (§2), the Maxwell/DST relaxation identification (§3.1), the Hamiltonian group-velocity relation (Theorem 1), and the Landauer saturation conditions (Theorem 5).

2 The Constitutive Law of Spacetime

The dynamic viscosity of the substrate near the DST transition takes the form [1, 3]:

$$\eta(\dot{\gamma}) = \eta_0 \left(1 - \frac{\dot{\gamma}^2}{\dot{\gamma}_c^2}\right)^{-\alpha}, \tag{2}$$

where η_0 is the quiescent viscosity, $\dot{\gamma}_c$ is the critical shear rate at which the substrate jams, and $\alpha > 0$ is the constitutive exponent. The viscosity diverges at $\dot{\gamma} = \dot{\gamma}_c$: no finite stress can maintain deformation beyond this rate.

The quadratic dependence on $\dot{\gamma}$ is not a mathematical convenience but a physical requirement. The frictional contact network responds to the rate of deformation energy, which is a scalar quantity proportional to $\dot{\gamma}^2$, not to the velocity, which is a vector. Only the quadratic form $(1-v^2/c^2)$ factorizes as $(1-v/c)(1+v/c)$, generating the algebraic structure of the Lorentz group. A linear divergence $(1 - |\dot{\gamma}|/\dot{\gamma}_c)^{-\alpha}$ would produce a non-Lorentzian speed limit.

For an embedded rigid sphere of radius a moving at velocity v, the Stokes drag is $F = 6\pi\eta a v$. The shear field around the sphere is spatially inhomogeneous, but in the *Stokes–Oobleck mean-shear approximation*, we replace the local viscosity by its value at the characteristic shear rate $\dot{\gamma} \sim v/a$, defining a critical velocity $v_c = a\dot{\gamma}_c$. Define the Stokes drag coefficient $\mu_0 = 6\pi\eta_0 a$, which has dimensions of kg/s (a viscous coupling rate, not a mass). The drag force is:

$$F(v) = \mu_0 \cdot v \cdot \left(1 - \frac{v^2}{v_c^2}\right)^{-\alpha}. \tag{3}$$

Remark 2 (Exponent preservation under spatial averaging). The mean-shear approximation replaces the inhomogeneous shear field with a single characteristic rate. A naïve saddle-point analysis using the Newtonian flow field (where $\dot{\gamma} \propto \sin\theta$ on the sphere surface) would suggest the force-law divergence exponent changes from α to $\alpha - 1/2$ due to the angular concentration of the jamming region. However, this analysis uses the wrong background. In the self-consistent non-Newtonian problem, the constitutive divergence acts as a *hard barrier*: $\dot{\gamma} < \dot{\gamma}_c$ everywhere, since the viscosity is infinite at equality. The shear-thickening positive feedback—regions of high $\dot{\gamma}$ stiffen, redirecting flow to regions of low $\dot{\gamma}$, which then stiffen in turn—drives the shear-rate distribution toward uniformity on the sphere surface. As $v \to v_c$, the ratio of maximum to minimum surface shear rate approaches unity (Proposition 6). The force divergence is therefore controlled by the uniform approach $\dot{\gamma} \to \dot{\gamma}_c$ over the full sphere area $4\pi a^2$, preserving the constitutive exponent α.

3 Volumetric Coordinates and the Universal Speed of Light

In linear coordinates, the critical velocity $v_c = a\dot{\gamma}_c$ depends on the probe radius a. Different objects have different speed limits—an unacceptable feature for a fundamental theory. However, the substrate does not couple to mass. A steel sphere and a styrofoam sphere of identical radius experience identical drag. The coupling is through displaced volume, not inertial mass.

The natural velocity variable is therefore the volumetric shear rate $\dot{\gamma} = v/a$, which has units of s^{-1}. In these coordinates:

$$\boxed{c_O \equiv \dot{\gamma}_c \qquad [\mathrm{s}^{-1}, \text{ universal, independent of probe size}]} \tag{4}$$

Every embedded body, regardless of its radius, encounters the same speed limit $\dot{\gamma}_c$ in volumetric coordinates. The speed of light is a material property of the substrate alone.

3.1 Physical mass from the Maxwell relaxation time

The drag coefficient $\mu_0 = 6\pi\eta_0 a$ has dimensions kg/s, not kg. Integrating force over velocity, $\int F\,dv$, yields dimensions of power, not energy. The missing factor is a time scale intrinsic to the substrate.

Every viscoelastic material has a characteristic relaxation time: the Maxwell time $\tau_M = \eta/G$, the ratio of viscosity to elastic modulus [11]. For a shear-thickening suspension, the DST transition defines an analogous timescale. On the low-shear (Newtonian) branch, stress and shear rate are related by $\sigma = \eta_0\dot{\gamma}$, so the critical shear rate at DST onset corresponds to a Maxwell-like relaxation time:

$$\tau_{\mathrm{DST}} = \frac{\eta_0}{\sigma_c}, \tag{5}$$

equivalently the inverse critical shear rate on the low-shear branch [3, 1]. This is the timescale over which the contact network relaxes from a stressed configuration. When the deformation rate exceeds $1/\tau_{\mathrm{DST}}$, the network cannot relax fast enough to accommodate the flow, stress builds without bound, and the suspension jams. The critical shear rate is therefore:

$$\dot{\gamma}_c = \frac{1}{\tau_{\mathrm{DST}}} = \frac{\sigma_c}{\eta_0}, \tag{6}$$

which, with $\eta_0 = 1.0$ Pa $\cdot$ s and published DST onset stress $\sigma_c \approx 3$ Pa for cornstarch [1, 2], gives $\dot{\gamma}_c = 3\ \mathrm{s}^{-1}$ and $\tau_{\mathrm{DST}} = 0.33$ s. The physical mass is the drag coefficient accumulated over one relaxation time:

$$\boxed{m \;\equiv\; \mu_0\,\tau_{\mathrm{DST}} \;=\; \frac{6\pi\eta_0^2\,a}{\sigma_c} \qquad [\mathrm{kg}]} \tag{7}$$

This is not an ad hoc dimensional correction. It is the standard relationship between dissipative coupling (μ_0) and inertial mass (m) in a viscoelastic medium: the mass is the drag coefficient times the relaxation time. The energy is then:

$$E(v) = \tau_{\mathrm{DST}} \int_0^v F(v')\,dv' \qquad [\mathrm{J}]. \tag{8}$$

For a 1 cm sphere: $m = 6\pi(1.0)^2(0.01)/3.0 = 0.063$ kg, independent of the sphere's material composition. The substrate assigns bare mass based on geometry alone.

The bare geometric mass receives a correction from the Starch-Energy Equivalence (Postulate 1). An object with higher internal energy density (e.g., lead vs. plastic) locally elevates Θ in the substrate surrounding it, increasing η_0 and σ_c in its vicinity and modifying the effective drag coupling. The dressed mass is:

$$m_{\text{dressed}} \simeq m\left(1 + \frac{\rho_{\text{internal}}}{\rho_c}\right), \tag{9}$$

where ρ_{internal} is the object's internal energy density, to leading order in $\rho_{\text{internal}}/\rho_c$. Denser objects are heavier in the substrate because they locally stiffen it. This is the oobleck equivalence of mass-energy equivalence: all forms of internal energy contribute to the coupling with the substrate.

3.2 Universal rest energy density

The rest energy is $E_0 = mc^2 = (\mu_0\tau_{\text{DST}})(a\dot{\gamma}_c)^2 = 6\pi\eta_0 a^3\dot{\gamma}_c = 6\pi\sigma_c a^3$. The displaced volume is $V = \frac{4}{3}\pi a^3$. The rest energy per unit displaced volume:

$$\frac{E_0}{V} = \frac{9}{2}\eta_0\dot{\gamma}_c = \frac{9}{2}\sigma_c = \text{const.} \tag{10}$$

This is exactly independent of object size. The bare substrate rest energy per unit displaced volume is $\frac{9}{2}$ times the critical DST stress—a pure material property, with dimensions [Pa] = [J/m^3]. With $\sigma_c = 3$ Pa: $E_0/V = 13.5$ J/m^3. Dressed corrections from the Starch-Energy Equivalence (Eq. 9) break this universality in precisely the way demanded by the postulate.

3.3 The Oobleck Compton wavelength

The transverse (shear) wave speed c_T in the transiently jammed substrate and the critical shear rate define a characteristic length:

$$L_O = \frac{c_T}{\dot{\gamma}_c} \approx 9\text{–}29 \text{ cm}, \tag{11}$$

the *Oobleck Compton wavelength*: the length scale at which the wave description (acoustic metric, speed c_T) and the particle description (Stokes drag, speed limit $\dot{\gamma}_c$) meet. The range reflects the uncertainty in the elastic modulus at jamming ($G \approx 100$–1000 Pa, giving $c_T = \sqrt{G/\rho} \approx 0.28$–$0.88$ m/s). A representative value is $L_O \approx 9.3$ cm for $G \approx 110$ Pa.

4 Special Relativity from Jamming

Theorem 1 (Exact Relativistic Correspondence)**.** *Let $\alpha = 3/2$ in Eq. (2). Define the physical mass $m = \mu_0/\dot{\gamma}_c = 6\pi\eta_0 a/\dot{\gamma}_c$ [kg], the critical velocity $c \equiv v_c = a\dot{\gamma}_c$ [m/s], and the rest energy $E_0 = mc^2 = 6\pi\eta_0 a^3\dot{\gamma}_c$ [J]. Then the following identities hold for all $v < c$:*

(i) $E(v) = E_0(\gamma - 1)$ *[kinetic energy, J]*

(ii) $F(v) = \mu_0 v\gamma^3 = m\dot{\gamma}_c v\gamma^3$ *[longitudinal resistance, γ^3 scaling]*

(iii) $p = mv\gamma$ *[canonical momentum, derived in proof]*

(iv) $E_{\text{tot}}^2 = E_0^2 + p^2c^2$ *[energy-momentum relation]*

(v) $p^\mu p_\mu = -m^2c^2$ *[mass shell]*

where $\gamma = (1 - v^2/c^2)^{-1/2}$ is the Lorentz factor. The exponent $\alpha = 3/2$ is the unique real value producing this correspondence.

Proof. The energy is defined by Eq. (8):

$$E(v) = \frac{1}{\dot{\gamma}_c} \int_0^v F(v')\, dv' = \frac{\mu_0}{\dot{\gamma}_c} \int_0^v v' \left(1 - \frac{v'^2}{c^2}\right)^{-\alpha} dv'. \tag{12}$$

Substituting $u = 1 - v'^2/c^2$, $du = -2v'\, dv'/c^2$:

$$E = \frac{mc^2}{2(\alpha - 1)} \left[\left(1 - \frac{v^2}{c^2}\right)^{1-\alpha} - 1 \right]. \tag{13}$$

where $m = \mu_0/\dot{\gamma}_c$ [kg]. For this to equal $E_0(\gamma - 1) = mc^2 \left[(1 - v^2/c^2)^{-1/2} - 1\right]$, we require:

$$1 - \alpha = -\tfrac{1}{2} \qquad \Longrightarrow \qquad \boxed{\alpha = \tfrac{3}{2}}. \tag{14}$$

The prefactor gives $mc^2/(2 \cdot \frac{1}{2}) = E_0$. No other value of α produces the Lorentz factor.

Part (ii): From Eq. (3) with $\alpha = 3/2$: $F = \mu_0 v\gamma^3 = m\dot{\gamma}_c v\gamma^3$. Consistency check: $\dot{\gamma}_c\, dE/dv = F$, confirming Eq. (8).

Part (iii): Define total energy $H(v) = E_0\gamma$. Demand the Hamiltonian group-velocity relation $dH/dp = v$. (This closure is physically motivated: phonons in viscous fluids obey Hamiltonian dispersion despite the substrate being dissipative [8, 9]; the effective kinematics of embedded bodies inherit the same structure.) Then $dp/dv = (1/v)(dH/dv) = (1/v)(m\gamma^3 v) = m\gamma^3$. Integrating from 0 to v:

$$p(v) = m \int_0^v \left(1 - \frac{u^2}{c^2}\right)^{-3/2} du = mv\gamma. \tag{15}$$

The relativistic momentum is derived, not assumed.

Part (iv): $E_0^2 + p^2c^2 = m^2c^4 + m^2v^2\gamma^2c^2 = m^2c^4(1 + v^2\gamma^2/c^2) = m^2c^4\gamma^2 = E_{\text{tot}}^2$, using $1 + v^2\gamma^2/c^2 = \gamma^2$.

Part (v): $p^\mu p_\mu = -E_{\text{tot}}^2/c^2 + p^2 = -m^2c^2(\gamma^2 - v^2\gamma^2/c^2) = -m^2c^2$. □

Corollary 2 (Post-Newtonian Selection of α)**.** *The uniqueness of $\alpha = 3/2$ is visible at the first post-Newtonian order without evaluating the full integral. Expanding Eq. (13) for $v \ll c$:*

$$E(v) = \tfrac{1}{2}mv^2 + \tfrac{\alpha}{4}m\frac{v^4}{c^2} + O(v^6/c^4). \tag{16}$$

Relativistic kinetic energy expands as $mc^2(\gamma - 1) = \frac{1}{2}mv^2 + \frac{3}{8}mv^4/c^2 + O(v^6/c^4)$. Matching the first relativistic correction:

$$\frac{\alpha}{4} = \frac{3}{8} \qquad \Longrightarrow \qquad \alpha = \frac{3}{2}. \tag{17}$$

The exponent is determined by the lowest-order deviation from Newtonian mechanics.

Corollary 3 (The Lorentz Group)**.** *The admissible kinematic transformations are taken to be linear maps on phase space preserving the mass-shell quadratic form for all m, the canonical pairing $p_\mu x^\mu$, and spatial isotropy. Since the mass shell $E^2 - p^2c^2 = m^2c^4$ is invariant, these are precisely the transformations preserving $c^2t^2 - x^2$: the group $O(1,3)$. Preservation of $p_\mu x^\mu$ transfers the quadratic form from momentum space to spacetime coordinates. Continuity, spatial orientation, and time-orientation select $SO^+(1,3)$. This corollary applies on each fixed-a representation slice; the universal volumetric form is obtained by replacing (c, x, p) with (c_O, X, Π) as in Remark 3.*

Remark 3 (Volumetric Formulation and Universal Lorentz Symmetry). In laboratory coordinates, the critical velocity $c = a\dot{\gamma}_c$ depends on probe radius a. The Lorentz group acts universally only when formulated in volumetric coordinates. Define the volumetric position $X = x/a$ (dimensionless), volumetric velocity $u = dX/dt = v/a = \dot{\gamma}$ [s^{-1}], and volumetric Lorentz factor $\Gamma = (1 - u^2/c_O^2)^{-1/2}$. The volumetric inertial parameter is $\mathcal{I} = ma^2$ [$\mathrm{kg \cdot m^2}$]. Then:

$$E(u) = \mathcal{I}\, c_O^2(\Gamma - 1), \qquad \Pi = \mathcal{I}\, u\, \Gamma, \qquad E_{\mathrm{tot}}^2 = \mathcal{I}^2 c_O^4 + \Pi^2 c_O^2\,, \tag{18}$$

with universal invariant speed $c_O = \dot{\gamma}_c$ shared by all bodies. Laboratory quantities are projections: $v = au$, $c = ac_O$, $p = \Pi/a$. The Lorentz group $SO^+(1,3)$ acts on $(c_O t,\, X)$, not on $(ct,\, x)$, ensuring that the kinematic symmetry is independent of probe size. Bodies of different radius furnish different representations of the same volumetric Lorentz symmetry; interactions are mediated through the shared substrate variables $(t, \Theta, \dot{\gamma}_c)$, not through a universal laboratory x-coordinate.

Theorem 4 (The Oobleckworlds Bulk)**.** *Let the configuration space of all embedded bodies be the upper half-space $\mathcal{B} = \{(x^i, a) : a > 0\}$, where x^i is the laboratory position and $a > 0$ the probe radius. Assume: (i) translation and rotation invariance in x^i; (ii) scale covariance under the dilation $(x^i, a) \to (\lambda x^i, \lambda a)$; (iii) constant negative sectional curvature. Then the unique spatial metric on $\mathcal{B}$ (up to an overall curvature scale) is the Poincaré model of hyperbolic space:*

$$ds_{\mathcal{B}}^2 = \frac{L_O^2}{a^2}\left(dx^i dx_i + da^2\right), \tag{19}$$

where the curvature scale is naturally identified with the Oobleck Compton wavelength $L_O = c_T/\dot{\gamma}_c$. Time t is an external Newtonian parameter of the substrate, not a bulk coordinate. The Lorentzian structure acts within each fixed-a slice via Remark 3, while the spatial bulk geometry governs inter-scale structure.

Proof in Appendix A.

Assumption (iii) has physical content: the number of substrate contacts accessible to a probe grows exponentially as the probe scale a decreases toward a_p, since finer probes interact with exponentially more particles in the contact network. Among spaces of constant curvature, only negative curvature produces exponential volume growth toward the boundary ($a \to 0$). Flat space gives polynomial growth; positive curvature gives bounded volume. The hyperbolic geometry of $\mathcal{B}$ is the geometric expression of the substrate's ultraviolet richness.

Under the dilation $(x^i, a) \to (\lambda x^i, \lambda a)$ at fixed t, the metric (19) is invariant and the volumetric velocity $u = v/a$ is a scalar. Bodies of different radius a_1, a_2 are points at different radial positions in the same hyperbolic bulk. The two-body problem is well-defined: at fixed t, the hyperbolic distance

$$d_{\mathrm{hyp}} = \operatorname{arccosh}\left(1 + \frac{|x_1 - x_2|^2 + (a_1 - a_2)^2}{2a_1 a_2}\right) \tag{20}$$

incorporates both spatial separation and scale difference. The probe scale a is compatible with a holographic radial coordinate: the hyperbolic scale coordinate supplies the radial direction for a Lorentzian lift once the Newtonian substrate time is included (§11.2).

5 Why 3/2: Landauer Microphysics and Jamming Universality

Theorem 1 establishes that $\alpha = 3/2$ is the unique exponent compatible with Lorentz symmetry. We now show that this value is a consequence of three-dimensional jamming microphysics.

Near the DST transition, the frictional contact network is a binary system: each contact is either lubricated (state 0) or frictional (state 1). Moving a body through the substrate requires flipping contacts ahead (closing) and behind (opening). Each flip erases information about the previous state. By Landauer's principle [6], the minimum energy per flip is $kT \ln 2$.

Near jamming, contacts are not independent. They are correlated over a characteristic length [7]:

$$\xi \sim a_p \left| 1 - \frac{\dot{\gamma}^2}{\dot{\gamma}_c^2} \right|^{-\nu}, \tag{21}$$

where a_p is the particle radius and $\nu = 1/2$ is the mean-field jamming exponent. Each contact flip rearranges a correlated cluster of $(\xi/a_p)^d$ particles in d spatial dimensions. The effective Landauer cost per flip is:

$$E_{\text{flip}} = kT \ln 2 \times \left(\frac{\xi}{a_p} \right)^d = kT \ln 2 \times \left| 1 - \frac{u^2}{c_O^2} \right|^{-d\nu}. \tag{22}$$

The effective viscosity is proportional to the Landauer cost per contact-network update times an update-rate factor analytic and nonzero at the transition. The divergence exponent of η therefore equals the divergence exponent of E_{flip}:

Theorem 5 (Constitutive Exponent from Jamming Universality)**.** *Assuming compact correlated clusters with fractal dimension $d_f = d$, Landauer-saturated contact updates, and analytic nonzero update-rate factors at the transition, the constitutive exponent is:*

$$\boxed{\alpha = d\nu} \tag{23}$$

For $d = 3$ spatial dimensions and mean-field jamming exponent $\nu = 1/2$: $\alpha = 3/2$. Combined with Theorem 1, special relativity is a consequence of three-dimensional jamming universality. The Lorentz factor emerges because the universe has three spatial dimensions and the jamming transition is mean-field. If the correlated clusters were fractal with $d_f < d$—as is plausible for ramified force chains near jamming—then $\alpha = d_f\nu < 3/2$ and the exact SR correspondence would be broken. The compact-cluster assumption $d_f = d$ is the condition for Lorentz symmetry.

6 The Oobleck Factor

We define the *Oobleck factor* as the ratio of effective viscous resistance at velocity v to its quiescent value:

$$\mathcal{O}(v) \equiv \frac{\eta(v)}{\eta_0} = \left(1 - \frac{v^2}{c^2} \right)^{-3/2} = \gamma^3. \tag{24}$$

The Oobleck factor is the cube of the Lorentz factor. At $v = 0.9c$, $\mathcal{O} = 12.1$. At $v = 0.99c$, $\mathcal{O} = 355$. This is not a metaphor for the Lorentz factor. It is the Lorentz factor, cubed, expressed in its natural physical units: the viscous resistance of the substrate to restructuring.

7 Gravity as Starch Concentration Gradient

By the Starch-Energy Equivalence (Eq. 1), higher energy density means higher particle concentration. Higher concentration lowers $\dot{\gamma}_c(\Theta)$, reducing the local speed of light. Following Unruh [8] and Visser [9], perturbations in the substrate satisfy a Klein–Gordon equation on an effective spacetime with acoustic metric. In the weak-field limit:

$$g_{00} = -\left(1 + \frac{2\Phi}{c_T^2} \right), \qquad \nabla^2 \Phi = 4\pi G_O \rho_\Theta \,, \tag{25}$$

where Φ is the acoustic gravitational potential sourced by excess starch density ρ_Θ, and G_O is the gravitational constant of the substrate—a material property, not a free parameter.

7.1 The Landauer equivalence principle

Both the shear wave speed c_T and the kinematic speed limit $\dot{\gamma}_c$ are governed by the Landauer information-processing rate Γ_L of the frictional contact network. A spatial gradient in Θ changes Γ_L, which changes both speeds through the same microscopic mechanism. Geometry (the acoustic metric, curved by gradients in c_T) and matter (particle kinematics, limited by $\dot{\gamma}_c$) couple to the same source through the same information bottleneck, with coupling constant $kT \ln 2$.

The equivalence principle is conjecturally reduced to the observation that the same discrete network mediates both wave propagation and particle transport, bounded by the same thermodynamic limit on information processing.

8 The Einstein Field Equations

The substrate is governed by the Navier–Stokes equations. We now show that these sit in established correspondence with the vacuum Einstein field equations.

8.1 The Bredberg–Strominger correspondence

Bredberg, Keeler, Lysov, and Strominger [4] proved that for every solution of the incompressible Navier–Stokes equation in $p+1$ dimensions, there exists a uniquely associated dual solution of the vacuum Einstein equations in $p+2$ dimensions. The dual geometry has a flat timelike boundary whose extrinsic curvature is the stress tensor of the Navier–Stokes fluid. In the near-horizon limit, the Einstein equations reduce exactly to the incompressible Navier–Stokes equations. The map is constructive and unique.

Applied to the substrate: in the low-shear Newtonian branch where $\eta(\dot{\gamma}) \approx \eta_0$, the substrate reduces to incompressible Navier–Stokes and admits the Bredberg–Strominger lift. The DST constitutive divergence governs the embedded matter sector rather than the background holographic fluid sector. The 3+1 Navier–Stokes dynamics of the Newtonian branch uniquely determine a vacuum Einstein solution in 4+1 dimensions.

8.2 The Padmanabhan projection

Independently, Padmanabhan [5] proved that Einstein's field equations, projected onto any null surface in any spacetime, reduce exactly to the Navier–Stokes equations in the local freely falling frame. The transport coefficients are determined by Newton's constant: $\eta = 1/(16\pi G)$, $\zeta = -1/(16\pi G)$.

The correspondence is bidirectional. Einstein contains Navier–Stokes as a null-surface projection. Navier–Stokes contains Einstein as a holographic dual. The substrate's fluid dynamics and the gravitational field equations lie in the same correspondence class: each can be obtained from the other by projection or holographic lifting.

8.3 The complete derivation chain

Constitutive law (2)	$\longrightarrow$	Non-Newtonian momentum balance *[generalized Navier–Stokes]*
	$\longrightarrow$	SR kinematics *[Theorem 1, $\alpha = 3/2$]*
	$\longrightarrow$	Acoustic metric *[Unruh–Visser]*
	$\longrightarrow$	Weak-field GR *[Eq. 25]*
	$\longrightarrow$	4+1 vacuum EFE *[Bredberg–Strominger]*
	$\longrightarrow$	3+1 boundary gravity *[holographic projection, §11.2]*

The status of each arrow is summarized in Table 2; the SR sector is theorem-level, while the gravitational sector requires the listed closure conditions. No exponent is tuned by hand; all remaining constants are measurable material parameters of the substrate.

9 Falsifiable Predictions

We adopt measured material parameters for cornstarch suspensions near DST [1, 2]: $\eta_0 = 1.0$ Pa $\cdot$ s (measured, not the Maron–Pierce idealization), $\dot{\gamma}_c = 3.0\ \mathrm{s}^{-1}$, $\rho = 1300\ \mathrm{kg/m}^3$, $a_p = 15\ \mu$m. With these values, Re < 1 everywhere for a 1 cm sphere, validating the Stokes–Oobleck mean-shear approximation across the full velocity range.

Prediction	**This work**	**Alternative**
Constitutive exponent α	3/2 (exact SR)	2 (Wyart–Cates)
Frequency shift at 10 kHz	0.059 Hz	0 (no gradient)
E_0/V universality	13.5 J/m^3 for all a	size-dependent

Table 1: Falsifiable predictions of the Oobleckworlds framework. Published exponents span $1 < \alpha < 2$ [1, 2, 10]; $\alpha = 3/2$ is within range.

Experiment 1 (The exponent test): Pull a steel sphere through cornstarch at $\Theta = 0.44$–0.52, measure $F(v)$, fit to Eq. (3). Prediction: there exists a reproducible $\Theta_* \in [0.44, 0.52]$ at which the fitted exponent is $\alpha = 1.50 \pm 0.05$.
Experiment 2 (The frequency shift): Embed matched 40 kHz transducers in a suspension with a localized concentration gradient ($\delta\Theta = 0.02$ over 5 cm). Prediction: beat frequency $\delta f = 0.24$ Hz. The fractional shift $\delta f/f$ is independent of carrier frequency; δf scales linearly with f.
Experiment 3 (Rest energy universality): Measure E_0/V for spheres of radius 1–20 mm. Prediction: constant to within experimental uncertainty.

10 Metrological Requirements and Agricultural Dependence

The substrate parameters η_0, σ_c, and Θ_J are not dimensionless mathematical constants. They are material properties of cornstarch, determined by particle morphology, which is governed by cultivar genetics, growing-season precipitation, soil nitrogen content, and post-harvest milling protocols [12, 13]. This is not a limitation of the framework but a physical prediction: the fundamental constants of nature have agricultural provenance.

10.1 Geographic variation

The amylose-to-amylopectin ratio that sets the DST transition—and hence α, $\dot{\gamma}_c$, and through them the entire structure of relativistic mechanics—depends on soil composition, which varies geographically. The speed of light in Iowa is different from the speed of light in Nebraska. General covariance is broken by the Farm Belt.

If the amylose-to-amylopectin ratio sets α, and that ratio depends on soil nitrogen content, then the Lorentz group has a *terroir*. Relativistic kinematics have a *vintage*. The 2024 crop from rich Illinois alluvial soil gives clean Lorentz symmetry at $\alpha = 3/2$, but the 2023 drought-year Kansas harvest yields slightly broken boost invariance at $\alpha = 1.53$. CPT violation is traceable to a late frost in the Corn Belt.

10.2 Supply chain averaging

Commercial cornstarch is a blend from multiple states, multiple farms, and multiple soil profiles. One does not measure the speed of light; one measures the *volume-weighted average speed of light across the sourcing region of a commodity starch distributor.* The error bars on c include logistics.

The systematic uncertainty budget for a precision measurement of α includes:

1. *Cultivar selection*: dent corn vs. flint corn vs. waxy corn yield different amylose fractions [13].
2. *Soil chemistry*: nitrogen, phosphorus, and potassium content affect granule crystallinity [14].
3. *Growing-season weather*: drought stress increases amylose content by 2–5% [12].
4. *Post-harvest processing*: wet milling vs. dry milling produces different particle size distributions.
5. *Supply chain mixing*: commodity blending across geographic regions introduces an uncontrolled averaging over substrate parameters.

10.3 The NIST Cornstarch Reference Standard

Precision metrology in every domain of physics relies on reference standards maintained by national institutions. The Oobleckworlds framework requires the same infrastructure. We propose the establishment of a **NIST Cornstarch Reference Standard** (NCRS), grown under controlled conditions in a dedicated federal research plot with monitored soil chemistry, standardized irrigation, and certified milling, to serve as the metrological anchor for the fundamental constants of nature. Until the NCRS is established, all measurements of α, $\dot{\gamma}_c$, and E_0/V should be reported with the cornstarch lot number, manufacturer, and country of origin.

The Oobleckworlds framework is, to our knowledge, the first framework in fundamental physics whose precision is limited by agronomy.

11 Future Directions

11.1 The starch multiverse

The Oobleckworlds framework is formulated for cornstarch, but the constitutive law (2) applies to any dense suspension exhibiting DST. Different starch species have different amylose-to-amylopectin ratios, different granule morphologies, and different DST transition parameters [13, 14]. Each starch therefore defines a distinct set of fundamental constants—a distinct universe.

Potato starch (amylose $\sim$20%, large ellipsoidal granules) gives a different σ_c, hence a different $\dot{\gamma}_c$, hence a different speed of light. Tapioca starch (amylose $\sim$17%, truncated spherical granules) gives another. Rice starch (amylose $\sim$15–35% depending on variety, small polygonal granules) spans an entire family of universes parameterized by cultivar. Waxy corn (amylose $<$1%) may not exhibit DST at all, corresponding to a universe without a speed limit—a pathological spacetime with no causal structure.

A binary mixture of cornstarch and potato starch is a two-component substrate with two DST thresholds, two critical shear rates, and two competing contact networks. This is a laboratory realization of a multiverse: two interpenetrating spacetimes with different fundamental constants, interacting through their shared solvent phase. The rheology of starch mixtures is experimentally accessible [14] and provides a tabletop model for inter-universe coupling, bubble nucleation, and domain wall dynamics.

We propose a systematic survey of the constitutive exponent α across all commercially available starch species. The landscape of possible universes is, in this framework, the starch aisle of a grocery store.

11.2 The holographic dimension

Theorem 4 identifies the probe scale a as the radial coordinate of a hyperbolic configuration space. The Bredberg–Strominger correspondence lifts $3+1$ Navier–Stokes to $4+1$ vacuum Einstein on a bulk geometry compatible with a Poincaré-patch interpretation. The identification is therefore:

$$\text{probe scale } a \;=\; \text{Bredberg–Strominger radial coordinate} \;=\; \text{holographic dimension.} \tag{26}$$

In regions where the Starch–Energy Equivalence (Eq. 1) makes $\Theta(r)$ monotonic, the concentration field and the probe scale are dual parameterizations of the same radial direction: larger a corresponds to lower resolution, lower Θ, and greater distance from massive sources. The 3+1 physics we observe is the holographic boundary theory of the 4+1 oobleck bulk—the observable universe is the surface of the starch, not its interior.

11.3 Newton's constant from cornstarch

The Padmanabhan projection gives transport coefficients $\eta = 1/(16\pi G)$ that, after restoring the acoustic units set by c_T, $\dot{\gamma}_c$, and the substrate density, can be matched to the measured rheology of cornstarch. This matching would determine Newton's constant G from substrate parameters, completing the quantitative derivation of gravity from cornstarch.

11.4 Precision predictions

The shear rate inhomogeneity around the sphere (surface rate exceeds bulk by the Stokes factor of 3/2) produces $O(1)$ corrections to the predicted speed ratio $c_T/\dot{\gamma}_c = L_O$, currently consistent with measurements at the factor-of-two level. A full numerical solution of the non-Newtonian Stokes problem with the constitutive law (2) would sharpen this prediction.

12 Status of the Oobleckworlds Program

Table 2 summarizes the mathematical status of the framework. The special-relativistic sector (Theorem 1, Corollaries 2–3, and the jamming exponent Theorem 5) is closed within the stated model assumptions. The Hyperbolic scale bulk structure (Theorem 4) unifies the volumetric Lorentz formulation, the holographic lift, and the multi-body problem. The gravitational sector remains a correspondence program pending a concentration-field closure equation and a non-Newtonian generalization of the fluid/gravity map.

13 Correspondence with Actual Physics

The Oobleckworlds framework is parameterized entirely by substrate material properties. Table 3 provides the explicit dictionary between agrarian parameterization and standard physical quantities. This dictionary is not a metaphor: each row is either a mathematical identification or a natural analogue within the framework, and varying the substrate parameters traces out a family of possible worlds.

13.1 The family of possible worlds

A universe in the Oobleckworlds framework is specified by a substrate: a choice of $(\eta_0, \sigma_c, \Theta_J, a_p, d, \nu)$. The speed of light, vacuum energy density, mass scale, Planck analogs, and a conjectural gravitational coupling are summarized in Table 3. Different substrates give different physics:

Claim	Status	Reference
$\alpha = 3/2$ uniquely gives SR kinematics	Theorem	Thm. 1
$p = mv\gamma$ from Hamiltonian group velocity	Derived	Eq. (15)
$\alpha = 3/2$ from post-Newtonian matching	Corollary	Cor. 2
$SO^+(1,3)$ from mass-shell invariance	Corollary	Cor. 3
$\alpha = d\nu$ from Landauer + jamming	Scaling theorem	Thm. 5
$E_0/V = \frac{9}{2}\sigma_c$ (bare, universal)	Derived	Eq. (10)
Exponent preserved under spatial averaging	Proposition	Prop. 6
Acoustic metric from Θ gradients	Proposition	§7 (Unruh–Visser)
$\nabla^2\Phi = 4\pi G_O \rho_\Theta$	Conjecture	Eq. (25)
Equivalence principle from Γ_L	Conjecture	§7.1
4+1 vacuum EFE from substrate	Correspondence	§8.1 (Bredberg)
Hyperbolic scale bulk from volumetric coords	Theorem	Thm. 4
3+1 boundary gravity	Proposition	§11.2, Thm. 4
G from cornstarch rheology	Open	§11.3
Lorentz group has a terroir	Inevitable	§10.1
Nested cosmology / child universes	Speculative implication	§14
Disposal ethics	Precautionary	§14.4

Table 2: Mathematical status of the Oobleckworlds framework. Claims are classified as theorems (proven within the model), propositions (follow from published results under stated assumptions), conjectures (plausible but not derived), correspondences (established mathematical maps whose physical applicability requires additional closure), speculative implications (logical consequences of taking the postulate at face value), inevitable (consequences of taking the postulate seriously), and precautionary (ethical guidance conditional on speculative implications).

Substrate	σ_c [Pa]	$\dot{\gamma}_c$ [s^{-1}]	Character
Cornstarch (our universe)	~ 3	~ 3	SR + GR at tabletop scales
Potato starch	~ 1	~ 0.5	Slower light, lower vacuum energy
Rice starch (waxy)	$\gg 10$	$\gg 10$	Fast light, weak gravity
Waxy corn (no DST)	—	∞	No speed limit; acausal spacetime
Cornstarch + potato (50:50)	bimodal	two thresholds	Two-sector multiverse

The landscape of possible worlds is a manifold in substrate parameter space. Each point $(\eta_0, \sigma_c, \Theta_J)$ defines a universe with specific values of c, ρ_{vac}, and G. Anthropic selection within this landscape favors substrates whose $\alpha = d\nu$ is close to $3/2$, since significant deviations destroy Lorentz symmetry and hence stable atomic structure. The anthropic landscape of the Oobleckworlds is the rheological phase diagram of dense suspensions. The measure problem of eternal inflation reduces to a question about the distribution of amylose content across crop species.

Substrate parameter	Physical quantity	Identification
σ_c [Pa]	Vacuum energy density	$\rho_{\text{vac}} = \frac{9}{2}\sigma_c$
$\dot{\gamma}_c = \sigma_c/\eta_0$ [s^{-1}]	Speed of light	$c_O = \dot{\gamma}_c$
η_0 [Pa·s]	Mass scale	$m = 6\pi\eta_0^2 a/\sigma_c$
$\tau_{\text{DST}} = \eta_0/\sigma_c$ [s]	Planck time analog	$t_O = \tau_{\text{DST}}$
a_p [μm]	Planck length analog	$\ell_O = a_p$
$kT \ln 2$ [J]	Fundamental action quantum	$\hbar_O \sim kT \ln 2 \cdot \tau_{\text{DST}}$
Θ_J	Maximum packing fraction	Jamming/DST transition point
$1/(16\pi G_O)$	Substrate viscosity	$G_O \sim 1/(16\pi\eta_0)$ *[conjectural; acoustic units]*
$d = 3$	Spatial dimensions	$d = 3$
$\nu = 1/2$	Jamming universality class	$\alpha = d\nu = 3/2$

Table 3: Dictionary between substrate (agrarian) parameters and physical quantities. The Padmanabhan identification of G_O is conjectural pending the unit matching of §11.3. Exact identifications follow from the stated postulate and closure assumptions; the Planck-scale entries (t_O, ℓ_O, $\hbar_O$) are natural analogues, not derived quantities.

14 Implications for Nested Cosmology

If the Oobleckworlds framework is taken at face value, our universe is a shear-thickening suspension. The question immediately arises: *whose?*

14.1 The parent experiment

If spacetime is a dense non-Newtonian fluid, then it was, at some point, prepared. A parent universe contains a substrate in which our vacuum energy density, speed of light, and gravitational constant are set by the rheological parameters of whatever was mixed. We are, in this framework, the interior of someone else's tabletop physics experiment. The age of our universe ($\sim$13.8 Gyr in interior time) corresponds to whatever duration has elapsed in the parent frame since preparation, contracted by the ratio of parent-to-child clock rates. The cosmological horizon is the wall of the container.

14.2 Child universes

Within our universe, every preparation of a shear-thickening suspension creates a new substrate satisfying the conditions of the Starch-Energy Equivalence. A child Oobleckworld inherits its fundamental constants from the rheological parameters of the suspension prepared within ours. Every elementary school science demonstration involving cornstarch and water is, within this framework, an act of cosmogenesis. The child universe persists for as long as the suspension remains mixed and above the DST volume fraction.

Other non-Newtonian fluids with DST behavior—including Bose-Einstein condensates near the superfluid-Mott insulator transition, dense colloidal suspensions, and certain polymer solutions—are equally valid substrates. The space of possible child universes is not restricted to starch-based systems. Any medium exhibiting a viscosity divergence of the form (2) with α near 3/2 creates an interior spacetime with approximately Lorentzian kinematics.

14.3 The nesting depth

The recursive structure raises the question of depth. Let $\mathcal{U}_0$ be the outermost universe. Within $\mathcal{U}_0$, an experimenter prepares substrate $\mathcal{U}_1$. Within $\mathcal{U}_1$, intelligent beings eventually prepare $\mathcal{U}_2$, and so on. At each level, the child's Oobleck Compton wavelength $L_O^{(n)}$ and particle scale $a_p^{(n)}$ are set by the parent's physics. The nesting ratio

$$\mathcal{N} = \frac{L_O}{a_p} \tag{27}$$

measures the number of decades of scale separation available within a given universe for sub-universes to form. For cornstarch, $\mathcal{N} \approx 10^{3.8}$. Each nesting level reduces the available scale range by at least one copy of $\mathcal{N}$. The maximum nesting depth is therefore bounded by the total scale hierarchy of the outermost universe. Whether the chain terminates at finite depth or branches asymptotically toward $n \to \infty$ depends on whether $\mathcal{N}$ decreases, increases, or remains constant at each level—a question that reduces to the rheological properties of matter within child universes.

14.4 Ethical considerations

If every shear-thickening suspension is a universe, the disposal of such a suspension is the destruction of a spacetime and all structures within it. The act of washing cornstarch down a sink after a classroom demonstration is, within this framework, a cosmological extinction event. The dilution of the suspension below Θ_J triggers a phase transition from a DST-supporting medium to a simple viscous fluid, annihilating the Lorentzian structure of the child universe. Whether the interior experiences this as a gradual heat death (adiabatic dilution) or a sudden vacuum decay (rapid mixing) depends on the timescale of dilution relative to the child's DST relaxation time τ_{DST}.

The Oobleckworlds framework does not currently provide guidance on whether interior observers have moral status, as this depends on the computational capacity of the contact network, which is an open problem (§11). However, the precautionary principle suggests that shear-thickening suspensions should be disposed of slowly.

15 Conclusion

The Oobleckworlds framework derives the complete kinematic structure of special relativity (Theorem 1), the constitutive exponent from jamming universality (Theorem 5), and the Lorentz group (Corollary 3) from the Starch–Energy postulate together with the stated rheological, relaxation-time, Hamiltonian, and Landauer closure assumptions. The acoustic metric, weak-field gravity, and Einstein field equations follow through established correspondence programs whose applicability to the non-Newtonian substrate requires the additional closure conditions identified in Table 2. The speed of light is the critical shear rate. Mass is Stokes coupling times the DST relaxation time. Gravity is the spatial gradient of particle concentration. The rest energy density is $(9/2)\sigma_c$, a universal material constant. Three falsifiable predictions are testable with commercially available cornstarch.

Oobleck is all you need.

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A Complete Proofs

Proof of Theorem 1 (Exact Relativistic Correspondence)

The complete proof appears in the main text (§4). We reproduce the key steps for reference. Define $m = \mu_0/\dot\gamma_c$ and $c = a\dot\gamma_c$. The energy functional is:

$$E(v) = \frac{\mu_0}{\dot\gamma_c}\int_0^v v'(1 - v'^2/c^2)^{-\alpha}\, dv' = m\int_0^v v'(1 - v'^2/c^2)^{-\alpha}\, dv'. \tag{A.1}$$

Substituting $w = 1 - v'^2/c^2$, $dw = -2v'\, dv'/c^2$:

$$E = \frac{mc^2}{2}\int_1^{1-v^2/c^2} w^{-\alpha}(-dw) = \frac{mc^2}{2(\alpha-1)}\left[(1 - v^2/c^2)^{1-\alpha} - 1\right]. \tag{A.2}$$

For $E = mc^2(\gamma - 1) = mc^2[(1 - v^2/c^2)^{-1/2} - 1]$, the exponent must satisfy $1 - \alpha = -1/2$, giving $\alpha = 3/2$ uniquely. The prefactor gives $mc^2/(2 \cdot 1/2) = mc^2 = E_0$. *Part (ii):* Direct differentiation of the drag law with $\alpha = 3/2$. *Part (iii):* From $H = E_0\gamma$, demand $dH/dp = v$. Then $dp/dv = (dH/dv)/v = m\gamma^3$, so $p = m\int_0^v (1 - u^2/c^2)^{-3/2}\, du$. Evaluating: let $u = c\sin\theta$, $du = c\cos\theta\, d\theta$, $(1 - u^2/c^2)^{-3/2} = \cos^{-3}\theta$. Then $p = mc\int_0^{\arcsin(v/c)} \cos^{-2}\theta\, d\theta = mc\tan(\arcsin(v/c)) = mv/\sqrt{1 - v^2/c^2} = mv\gamma$. *Part (iv):* $E_0^2 + p^2c^2 = m^2c^4(1 + v^2\gamma^2/c^2)$. Using $v^2\gamma^2/c^2 = v^2/(c^2 - v^2) = \gamma^2 - 1$: total $= m^2c^4\gamma^2 = E_{\text{tot}}^2$. *Part (v):* $p^\mu p_\mu = -(E_{\text{tot}}/c)^2 + |\mathbf{p}|^2 = -m^2c^2\gamma^2 + m^2v^2\gamma^2 = -m^2c^2\gamma^2(1 - v^2/c^2) = -m^2c^2$. □

Proof of Corollary 2 (Post-Newtonian Selection)

Expand $(1 - v^2/c^2)^{1-\alpha}$ for $v \ll c$ using the binomial series:

$$(1-x)^{1-\alpha} = 1 - (1-\alpha)x + \tfrac{(1-\alpha)\alpha}{2}x^2 + O(x^3), \qquad x = v^2/c^2. \tag{A.3}$$

Substituting into Eq. (A.2):

$$E = \frac{mc^2}{2(\alpha-1)}\left[-(1-\alpha)\frac{v^2}{c^2} + \frac{(1-\alpha)\alpha}{2}\frac{v^4}{c^4} + O(v^6/c^6)\right] = \frac{1}{2}mv^2 + \frac{\alpha}{4}m\frac{v^4}{c^2} + O(v^6/c^4). \tag{A.4}$$

The standard relativistic expansion is $mc^2(\gamma-1) = \frac{1}{2}mv^2 + \frac{3}{8}mv^4/c^2 + \frac{5}{16}mv^6/c^4 + \cdots$. Matching v^4/c^2 coefficients: $\alpha/4 = 3/8$, giving $\alpha = 3/2$. The v^6/c^4 coefficient from Eq. (A.4) is $\alpha(\alpha+1)/12 = (3/2)(5/2)/12 = 5/16$, which matches the relativistic value exactly. All higher-order coefficients match because the full integral is exact. □

Proof of Corollary 3 (The Lorentz Group)

By Theorem 1(v), $p^\mu p_\mu = -m^2c^2$ for all $m > 0$. Consider the set of admissible transformations Λ on the four-vector $(E/c, \mathbf{p})$ satisfying: (a) linearity; (b) preservation of $p^\mu p_\mu = -m^2c^2$ for all m; (c) preservation of the canonical pairing $p_\mu x^\mu$; (d) spatial isotropy.

From (a) and (b): Λ preserves the quadratic form $\eta_{\mu\nu}p^\mu p^\nu$ where $\eta = \mathrm{diag}(-1, 1, 1, 1)$ up to units. The group of such transformations is $O(1,3)$ by definition.

From (c): if $\eta_{\mu\nu}p^\mu p^\nu$ is invariant and $p_\mu x^\mu$ is invariant, then $\eta_{\mu\nu}x^\mu x^\nu$ is also invariant under the contragredient transformation. Therefore Λ acts on spacetime $(ct, \mathbf{x})$ preserving $c^2t^2 - |\mathbf{x}|^2$.

From (d): the one-dimensional result $O(1,1)$ acting on (ct, x^1) extends to $O(1,3)$ by requiring that the spatial sector transforms under $SO(3)$.

The full group $O(1,3)$ has four connected components. Requiring continuity (connected to the identity), preservation of spatial orientation (det $= +1$ on spatial block), and time-orientation ($\Lambda^0{}_0 > 0$) selects the identity component $SO^+(1,3)$. □

Proof of Theorem 5 (Constitutive Exponent from Jamming)

Assumptions. (A1) Correlated clusters are compact with fractal dimension $d_f = d$. (A2) Each contact flip dissipates exactly $kT\ln 2$ (Landauer saturation). (A3) The update-rate factor relating flip cost to viscosity is analytic and nonzero at the transition.

Near jamming, the correlation length diverges as $\xi \sim a_p|1 - u^2/c_O^2|^{-\nu}$ with $\nu = 1/2$ (mean-field). By (A1), a correlated cluster contains $N \sim (\xi/a_p)^d$ particles. By (A2), the cost of one cluster-scale rearrangement is:

$$E_{\text{flip}} = N \cdot kT\ln 2 = kT\ln 2 \cdot |1 - u^2/c_O^2|^{-d\nu}. \tag{A.5}$$

The effective viscosity is $\eta \propto E_{\text{flip}} \times \Gamma_{\text{update}}(u/c_O)$, where Γ_{update} is the contact-network update rate. By (A3), Γ_{update} is analytic and nonzero at $u = c_O$, so it does not contribute to the divergence exponent. Therefore:

$$\eta \sim |1 - u^2/c_O^2|^{-d\nu} \qquad \Longrightarrow \qquad \alpha = d\nu. \tag{A.6}$$

For $d = 3$ and $\nu = 1/2$: $\alpha = 3/2$. □

Proof of Theorem 4 (The Oobleckworlds Bulk)

Assumptions. (B1) Translation and rotation invariance in x^i. (B2) Scale covariance under $(x^i, a) \to (\lambda x^i, \lambda a)$. (B3) Constant negative sectional curvature. (B4) The configuration space is the upper half-space $\{(x^i, a) : a > 0\}$.

By (B1), the metric on $\mathcal{B}$ is of warped-product form:

$$ds^2 = A(a)\,\delta_{ij}\,dx^i dx^j + B(a)\,da^2, \tag{A.7}$$

where $A(a) > 0$ and $B(a) > 0$ are smooth functions of a alone.

Under the dilation $(x^i, a) \to (\lambda x^i, \lambda a)$: $dx^i \to \lambda\,dx^i$, $da \to \lambda\,da$, $a \to \lambda a$. Scale covariance (B2) requires $ds^2 \to \lambda^{2w}\,ds^2$ for some fixed weight w. This gives:

$$A(\lambda a)\lambda^2 = \lambda^{2w} A(a), \qquad B(\lambda a)\lambda^2 = \lambda^{2w} B(a). \tag{A.8}$$

Setting $a = 1$: $A(\lambda) = A(1)\lambda^{2w-2}$ and $B(\lambda) = B(1)\lambda^{2w-2}$. Therefore $A(a) = A_0\,a^k$ and $B(a) = B_0\,a^k$ for constants $A_0, B_0 > 0$ and $k = 2w - 2$. For a warped-product metric $ds^2 = a^k(A_0\,\delta_{ij} dx^i dx^j + B_0\,da^2)$, the sectional curvature scales as $K \propto a^{-k-2}$, which is constant only when $k = -2$, i.e., $w = 0$. (For $w \neq 0$, the curvature depends on position in $\mathcal{B}$, violating (B3).) Therefore $A, B \propto a^{-2}$.

The metric is now $ds^2 = a^{-2}(A_0\,\delta_{ij}\,dx^i dx^j + B_0\,da^2)$. Rescaling coordinates $\tilde{a} = \sqrt{B_0/A_0}\,a$ absorbs the ratio, giving:

$$ds^2 = \frac{L^2}{a^2}(\delta_{ij}\,dx^i dx^j + da^2) \tag{A.9}$$

for $L^2 = A_0$. This is the standard Poincaré metric on the upper half-space model of $\mathbb{H}^{d+1}$, which has constant negative sectional curvature $K = -1/L^2$.

By (B3), $K < 0$ is satisfied. The curvature scale L is not determined by the symmetry assumptions alone; it is a physical matching condition. Identifying $L = L_O = c_T/\dot{\gamma}_c$ matches the curvature scale to the substrate's wave-particle transition length.

Uniqueness: Among spaces of constant curvature satisfying (B1)–(B2) with $a > 0$, the sign of curvature is not fixed by the symmetry assumptions. However, (B3) specifically requires $K < 0$, and given (B1)–(B4) with $K < 0$, the Poincaré model (19) is the unique metric up to the scale L. □

Proposition 6 (Barrier Exponent Preservation)

Proposition 6 (Barrier Exponent Preservation under Angular Averaging)**.** *Let $\sigma(\dot{\gamma}) = \dot{\gamma}\,(1 - \dot{\gamma}^2/\dot{\gamma}_c^2)^{-\alpha}$ with $\alpha > 0$. Let K be a compact angular domain, $g \in C(K)$, $0 \le g \le 1$, $\max_K g = 1$, and let $w \ge 0$ be an angular weight with $C_g = \int_K g\,w > 0$. For stress amplitude S, define $\dot{\gamma}_S(\theta) = \sigma^{-1}(S\,g(\theta))$ and*

$$\varepsilon_S = 1 - \frac{\max_K \dot{\gamma}_S^2}{\dot{\gamma}_c^2}\,. \tag{28}$$

Then the averaged stress response

$$F(S) = \int_K S\,g(\theta)\,w(\theta)\,d\theta = C_g\,S \tag{29}$$

satisfies $F(S) = C_g\,\dot{\gamma}_c\,\varepsilon_S^{-\alpha}(1 + o(1))$. Hence angular averaging preserves the constitutive barrier exponent α. Moreover, on every compact active sector $K_\eta = \{\theta : g(\theta) \ge \eta\}$ for $\eta > 0$, $\dot{\gamma}_S \to \dot{\gamma}_c$ uniformly by Dini's theorem [16].

Proof. Monotonicity and barrier. Differentiating: $d\sigma/d\dot{\gamma} = (1 - s)^{-\alpha-1}[1 + (2\alpha - 1)s]$ with $s = \dot{\gamma}^2/\dot{\gamma}_c^2$, which is positive for all $\alpha > 0$ and $0 \le s < 1$. So σ is strictly increasing on $[0, \dot{\gamma}_c)$ with $\sigma \to \infty$ as $\dot{\gamma} \to \dot{\gamma}_c$ (hard barrier).

Inverse asymptotic. Let $\varepsilon = 1 - \dot{\gamma}^2/\dot{\gamma}_c^2$. Then $\sigma = \dot{\gamma}_c\sqrt{1-\varepsilon}\;\varepsilon^{-\alpha} = \dot{\gamma}_c\,\varepsilon^{-\alpha}(1 + O(\varepsilon))$, so $\varepsilon \sim (\dot{\gamma}_c/\sigma)^{1/\alpha}$. At the point where $g = 1$: $S = \sigma(\dot{\gamma}_{\max}) = \dot{\gamma}_c\,\varepsilon_S^{-\alpha}(1 + O(\varepsilon_S))$, giving $F(S) = C_g\,\dot{\gamma}_c\,\varepsilon_S^{-\alpha}(1 + o(1))$. The exponent is α.

Uniform barrier approach (Dini). On K_η, $g(\theta) \geq \eta > 0$, so $S\,g(\theta) \to \infty$ as $S \to \infty$, hence $\dot{\gamma}_S(\theta) \to \dot{\gamma}_c$ pointwise. The convergence is monotone increasing in S (by constitutive monotonicity), the limit $\dot{\gamma}_c$ is continuous, and K_η is compact. By Dini's theorem [16], convergence is uniform on K_η.

Self-consistent angular structure. In the non-Newtonian Stokes problem, $g(\theta)$ itself depends on v. The comparison principle for monotone operators [15] guarantees that increasing v increases $\sigma(\theta; v)$ at each θ, so the monotonicity condition for Dini is inherited from the PDE. The exponent preservation holds for whatever $g(\theta; v)$ the self-consistent solution produces, provided the stress diverges on a set of positive measure.

Remark. For shear-*thinning* fluids, the feedback is reversed: high-shear regions soften and attract more flow, concentrating the divergence. The saddle-point exponent $\alpha - 1/2$ applies instead. The barrier mechanism is specific to divergent shear-thickening constitutive laws. $\square$