Analytic Continuation of Relativistic Kinematics and Imaginary Velocity Regimes

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April 23, 2026

Abstract

We investigate a formal analytic continuation of relativistic kinematics by extending velocity into the complex domain. Under the substitution $v \to iv$, the Lorentz factor becomes non-singular and yields a modified time relation. We show that this transformation is naturally interpreted as a rotation into a Euclideanized spacetime sector, analogous to Wick rotation in quantum field theory. The resulting structure suggests a connection between relativistic kinematics, compact field variables, and semiclassical Euclidean gravity. Possible implications for quantum tunneling and cosmological models are discussed.

1 Introduction

Special relativity constrains motion through the invariant interval and the Lorentz factor:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}. \tag{1}$$

This factor diverges at $v = c$, enforcing causal structure in Minkowski spacetime. However, complex extensions of spacetime variables are widely used in theoretical physics, particularly in quantum field theory through Wick rotation.

2 Analytic Continuation of Velocity

We consider the transformation:

$$v \to iv. \tag{2}$$

Substituting into the Lorentz factor:

$$\gamma_E = \frac{1}{\sqrt{1 + \frac{v^2}{c^2}}}. \tag{3}$$

We define the modified time relation via analytic continuation of the Lorentz factor:

$$T = T_0\sqrt{1 + \frac{v^2}{c^2}}. \tag{4}$$

3 Geometric Interpretation

In standard Wick rotation:

$$t \to i\tau, \tag{5}$$

the Minkowski metric:

$$ds^2 = -c^2 dt^2 + dx^2 \tag{6}$$

transforms into a Euclidean metric:

$$ds^2 = c^2 d\tau^2 + dx^2. \tag{7}$$

We interpret imaginary velocity as probing this Euclidean sector through kinematic variables.

4 Physical Consequences

The modified Lorentz factor exhibits:

- No divergence at $v = c$
- Suppression of time intervals instead of dilation
- Bounded, monotonic behavior

This suggests imaginary velocity represents non-classical configurations.

5 Connection to Euclidean Field Theory

In Euclidean quantum field theory:

$$Z = \int D\phi \, e^{-S_E[\phi]}. \tag{8}$$

The regularization properties align with the analytic continuation of kinematics.

6 Effective Gravitational Embedding

We interpret velocity as an effective scalar gradient:

$$\frac{v^2}{c^2} \to (\nabla\theta)^2. \tag{9}$$

Then:

$$\gamma_E = \frac{1}{\sqrt{1 + (\nabla\theta)^2}}. \tag{10}$$

The effective action:

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G} R - \frac{1}{2}(\nabla\theta)^2 - V(\theta) \right]. \tag{11}$$

With:

$$V(\theta) = \Lambda \cos(\theta). \tag{12}$$

7 Cosmological Dynamics

For FLRW metric:

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2, \tag{13}$$

Friedmann equation:

$$3H^2 = 8\pi G\rho + \frac{1}{2}\dot{\theta}^2 + \Lambda\cos(\theta). \tag{14}$$

Scalar field equation:

$$\ddot{\theta} + 3H\dot{\theta} + \Lambda\sin(\theta) = 0. \tag{15}$$

8 Summation Over Complex Velocity Sectors

We generalize:

$$v^2 \to i^n v^2. \tag{16}$$

$$T = T_0 \sum_{n=0}^{\infty} w_n \frac{1}{\sqrt{1 - \frac{i^n v^2}{c^2}}}. \tag{17}$$

Since:

$$i^n = \{1, i, -1, -i\}, \tag{18}$$

this decomposes into four sectors.

9 Interpretation

- $n = 0$: Lorentzian spacetime
- $n = 2$: Euclidean sector
- $n = 1, 3$: complex intermediate sectors

10 Conclusion

Extending velocity into the imaginary domain regularizes the Lorentz factor and connects relativistic kinematics with Euclidean field theory structures. While formal, this framework may provide insights into quantum gravity and cosmological corrections.