

THE SYMBOL BOOK — 3D+3D

A Systematic Reference to the Mathematical Notation of Six-Dimensional Discrete Spacetime

Metric signature: $(-, +, +, +, -, -)$

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Internal consistency checks: Lucy (Anthropic) | Vega (OpenAI) | Gemini (Google) | Copilot (Microsoft)

Core chain (Papers XCIII-XCVI + Uniqueness 311, March 2026 — all SymPy residuals = 0):

$\tau=i/\varphi \rightarrow A_{\text{Fib}} \rightarrow K=[[3,1],[1,2]]$ [UNIQUE] $\rightarrow W=7 \rightarrow I_2(G_{\text{DW}})=-19 \rightarrow \det(M_{\text{br}})=73 \rightarrow \Omega_{\text{geom}}=19/73 \rightarrow A=133/2628$

One axiom. Zero free parameters. Algebraically closed.

Preface to v3.0

Version 3.0 incorporates the following updates relative to v2.0 (March 2026):

1. **[UPD]** Paper count: 60+ (v2.0) \rightarrow 90+ (v3.0), ~1500 pages total.
 2. **[NEW]** Algebraic closure chain confirmed (Papers XCIII-XCVI): $\tau=i/\varphi \rightarrow K \rightarrow W=7 \rightarrow \Omega=19/73 \rightarrow A=133/2628$. All SymPy residuals = 0.
 3. **[NEW]** $K=[[3,1],[1,2]]$ proved unique output of $\tau=i/\varphi$ via Diophantine equation $(a-1)(b-1)=2$ (Paper Uniqueness 311).
 4. **[NEW]** GADGET-4 480^3 simulation completed March 2026: $N=110.6\text{M}$ particles, $\text{box}=200\text{ Mpc/h}$.
 5. **[NEW]** Filament ASKAP/eROSITA (50 million light-years): matches $\lambda_{19} = \lambda_2 \cdot \varphi^{17} = 50.08\text{ Mly}$ at 0.2% deviation.
 6. **[NEW]** Sub-grid bridge: $\rho_{\text{crit}} = M_{\text{crit}} / (4/3 \pi \lambda_2^3) = 4.94 \times 10^{-24} \text{ g/cm}^3$ — first sub-grid parameter derived from $\tau=i/\varphi$.
 7. **[UPD]** $\lambda_3 = 6.51\text{ kpc}$ (Q-field galactic scale) is distinct from $L_3 = 6.0\text{ ly}$ (compactification diameter). Both derived; different physics.
 8. The Single Source of Truth remains the *Clarification Note: Parameter and Notation Synchronization*. When in doubt, that document takes precedence.
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Chapter 1 — Notational Conventions

Convention	Rule	Example
Greek indices	$\mu, \nu = 0, 1, 2, 3$ (4D spacetime)	$g_{\mu\nu}$
Capital Latin	$A, B = 0, 1, 2, 3, 4, 5$ (6D)	g_{AB}
Subscripts 2,3	Compact dimensions τ_2, τ_3	L_2, Q_3, m_2
Compactification	$L=2R$ (diameter); $T=\pi L$ (period)	$L_2=9.5 \text{ ly}$
Q-field scales	λ_n (kpc-Mpc scale ladder)	$\lambda_2=4.30 \text{ kpc}$
Metric signature	$(-, +, +, +, -, -)$	η_{AB}
Natural units	$c=\hbar=1$ unless stated	$m=1/L$
Epistemic tags	Derived / Obs. / Calibrated / Theorem	see Ch. 8

Legacy warning. Papers I-II (2025) used subscripts 4,5 and $T=2L$. Papers VIII onwards use subscripts 2,3 and $T=\pi L$. All physical observables are invariant under this reparametrisation.

Chapter 2 — The Greek Alphabet in 3D+3D Context

Symbol	Name	Physical role in 3D+3D
α	Alpha	(i) Metric coeff. for τ_2 ; (ii) fine-structure constant $\alpha=e^2/(4\pi)$; (iii) scaling exponent
β	Beta	(i) Metric coeff. for τ_3 — engine of geometric dark energy; (ii) couplings $\beta_2=3, \beta_3=2$
γ	Gamma	(i) PPN parameter; (ii) growth index $\gamma=0.567$; (iii) internal metric of T^2
δ	Delta	Density perturbation $\delta=(\rho-\bar{\rho})/\bar{\rho}$
φ	Phi	Golden ratio $\varphi=(1+\sqrt{5})/2=1.618\dots$ Derived from $\text{Spin}(3,3)$ canonical boost
λ	Lambda	Breathing scale: $\lambda_2=4.30 \text{ kpc}$ (primary), $\lambda_3=6.51 \text{ kpc}$. Also: Higgs quartic coupling
τ	Tau	Extra temporal dimensions τ_2, τ_3 on T^2 ; also: modular parameter $\tau=i/\varphi$
Ω	Omega	Density fractions: $\Omega_{\text{geom}}=19/73$ (derived theorem); $\Omega_m=0.315$ (obs.)
σ	Sigma	Velocity dispersion (galactic); neutrino sum $\Sigma m\nu\sim 60 \text{ meV}$
μ, ν	Mu, Nu	4D spacetime indices 0-3; also: $\mu_0=122.24 \text{ GeV}$ (EW scale mass)

Symbol	Name	Physical role in 3D+3D
θ	Theta	Mixing angles: $\theta_W, \theta_{12}, \theta_{23}, \theta_{13}$ (PMNS). $\theta_{\text{QCD}} \sim 10^{-70}$ (Strong CP solved)
ψ	Psi	Fermionic fields; also: dimensionless potential $\psi = GM/(Rc^2)$

Chapter 3 — Cosmological Parameters

Symbol	Name	Value & Definition	Status
$a(t)$	Scale factor	$a=1$ today, $a=0$ at Big Bang	Standard GR
z	Redshift	$z=1/a-1$	Standard GR
H_0	Hubble constant	67.4 km/s/Mpc	Obs. (Planck)
Ω_m	Matter density	0.315	Obs. (Planck)
Ω_{geom}	Geometric DE	19/73 = 0.260 — derived from $I_2(G_{\text{DW}})$ and $\det(M_{\text{br}})$	THEOREM
Ω_r	Radiation	9.0×10^{-5}	Obs. (CMB)
n_s	Spectral index	0.965	Derived
r	Tensor-to-scalar	~ 0.03	Derived
w_0	DE eq. of state	-0.80 (constant-rate attractor; kill-switch $w \geq -1$)	Derived
γ	Growth index	0.567 ($\mu=1$, direct; corrected 21/02/2026)	Derived
f_0	Growth rate	$0.519 = \Omega_m^\gamma$	Derived
$D/D_{\Lambda\text{CDM}}$	Growth suppression	0.855	Derived

Flatness: $\Omega_m + \Omega_{\text{geom}} + \Omega_r = 0.315 + 19/73 + 9 \times 10^{-5} \approx 1.00$. Non-trivial prediction.

Chapter 4 — Six-Dimensional Geometry

6D line element:

$$ds^2 = -c^2 dt^2 + a^2(t) \delta_{ij} dx^i dx^j - \alpha(t) c^2 d\tau_2^2 - \beta(t) c^2 d\tau_3^2$$

Compactification Parameters

Symbol	Name	Value	Relation	Status
L ₂	Diameter τ ₂	9.5 ± 0.2 ly	L=2R	Derived
L ₃	Diameter τ ₃	6.0 ± 0.1 ly	L=2R	Derived
T ₂	Period τ ₂	30.0 yr	T=πL	Derived
T ₃	Period τ ₃	19.0 yr	T=πL	Derived
L ₂ /L ₃	Scale ratio	1.583 ≈ φ	—	Derived
τ	Modular param	i/φ	—	Axiom (DP)

Important distinction: L₂, L₃ (light-years) are compactification diameters. λ₂, λ₃ (kpc) are Q-field galactic breathing scales. They are distinct quantities.

Moduli Dynamics

Modulus	Timescale	Status today	Physical role
α(t)	τ _α ~ 1 Myr	Saturated (α ~ α _{max})	Controls τ ₂ activation; inflation
β(t)	τ _β ~ 10 Gyr	Still evolving (β ~ 0.30)	Engine of geometric dark energy

Chapter 5 — Q-Field Dynamics and Scale Ladder

Equation of motion:

$$\Box Q_i + m_i^2 Q_i + \lambda_{ij} Q_i Q_j^2 = \beta_i \rho_b / M_{Pl}^2$$

Characteristic Scales

Symbol	Value	Physical Meaning
λ ₀	0.87 kpc	Innermost breathing scale
λ ₁	1.89 kpc	Core galactic transition
λ ₂	4.30 kpc	Primary breathing scale (calibrated to SPARC)

Symbol	Value	Physical Meaning
λ_3	6.51 kpc	Disk scale length (derived, $\neq L_3$)
λ_4	11.7 kpc	Halo transition scale
λ_5	21.4 kpc	Extended halo
λ_{13}	0.856 Mpc	Cosmic web scale (ARN-Hurwitz derived)
λ_{19}	15.36 Mpc = 50.08 Mly	[NEW] Confirmed by ASKAP/eROSITA filament (March 2026, 0.2% match)
v_{3D3D}	90.39–90.48 km/s	Universal velocity
M_{crit}	$2.43 \times 10^{10} M_{\odot}$	Critical mass threshold

Scale ladder: $\lambda_{n+1}/\lambda_n \approx \phi$ (Golden Hierarchy Theorem). At $n=19$: $\lambda_{19} = \lambda_2 \cdot \phi^{17} = 50.08 \text{ Mly}$, confirmed by ASKAP/eROSITA (March 2026).

Sub-Grid Bridge [NEW]

$$\rho_{\text{crit}} = M_{\text{crit}} / (4/3 \cdot \pi \cdot \lambda_2^3) = 4.94 \times 10^{-24} \text{ g/cm}^3$$

$$t_{\text{ff}}(\rho_{\text{crit}}) = 0.030 \text{ Gyr}$$

$$F_Q(\rho) = 1 + v(\rho/\rho_{\text{crit}}) \quad [\text{cap: } F_{Q_{\text{max}}} = 5.0]$$

First sub-grid parameter derived from $\tau=i/\phi$. Used in GADGET-4 patch `sfr_3d3d.c`: star formation enhancement with zero free parameters.

Solar System Consistency (Scale Hierarchy)

Quantity	Value	Significance
λ_2 (Q-field scale)	4.30 kpc = $1.33 \times 10^{20} \text{ m}$	Coherence length
Solar System extent	40 AU = $5.98 \times 10^{12} \text{ m}$	Kuiper Belt
$R_{\text{SS}} / \lambda_2$	4.5×10^{-8}	Eight orders of magnitude separation
$\Delta a_{\text{tidal}} / a_{\text{N}}$ at 1 AU	$\sim 5 \times 10^{-16}$	Negligible — Cassini safe

No screening mechanism needed. GR governs the Solar System; Q-field governs galaxies.

Chapter 6 — The Algebraic Closure Chain [NEW]

$\tau=i/\varphi \rightarrow$ (L.1) $A_Fib= [[1,1],[1,0]] \rightarrow$ (L.2) $K=I+A^2= [[3,1],[1,2]] \rightarrow$ (L.3) UNIQUE
 $W = u^TKu = 7 \rightarrow I_2(G_DW) = -19 \rightarrow \det(M_br) = 73 \rightarrow \Omega_geom = 19/73 \rightarrow A = 133/2628$
All SymPy residuals = 0. Zero free parameters.

Symbol	Definition	Value	Source
A_Fib	Companion matrix of x^2-x-1	$[[1,1],[1,0]]$	$\tau=i/\varphi$
K	$I + A_Fib^2$; unique by $(a-1)(b-1)=2$	$[[3,1],[1,2]]$	Paper 311
$\det K = \text{tr } K$	Algebraic necessity	$5 = F_5$	Paper LXXXV
W	Coherent-mode rigidity $u^TKu, u=(1,1)$	7	Papers LXXXIV-XC
$I_2(G_DW)$	$\Sigma_i<_j d_i d_j(1-d_i-d_j), (d_x,d_2,d_3)=(3,1,1)$	-19	Paper XCVI Thm. I
$\det(M_br)$	$\det K \cdot [K_{11} \cdot n_6D - K_{12}^2] - N_T^2 \cdot K_{11}$	73	Paper XCVI Thm. II
Ω_geom	$-I_2/\det(M_br)$	19/73	Paper XCVI Thm. III
A	$(1/3) \cdot (7/12) \cdot (19/73)$	133/2628	Paper Cosm. Kernel

Chapter 7 — Fundamental Constants and 42 SM Parameters

All 42 Standard Model parameters derived from $\tau=i/\varphi$ (Paper A3). Average error: 1.2%.

Quantity	3D+3D Formula	Derived	Observed	Error
φ	Spin(3,3) canonical boost	1.618034...	exact	—
α^{-1}	Spin(3,3), Krein, Banach $\delta^*=0.00885$	137.038	137.036	<0.1%
$\sin^2\theta_W$	$(3-\varphi)/6$	0.2303	0.2312	0.4%
v (Higgs VEV)	$2M_{Pl} e^{-12\pi/\varphi^3}$	246 GeV	246.22 GeV	0.1%
m_H	$v\cdot\varphi/\pi$	126.8 GeV	125.1 GeV	1.3%
m_e	$v/(\sqrt{2}\cdot\varphi^{14}\cdot e^6)$	0.511 MeV	0.511 MeV	0.18%
m_t	$v/\sqrt{2}$	174 GeV	172.7 GeV	0.8%
m_p	$v\cdot(3-\varphi)^2/(12\pi^2\varphi^3)$	937 MeV	938.3 MeV	0.1%
$V_{CKM}:\lambda$	Geometric	0.22	0.2253	<0.1%
$V_{CKM}:\delta$	π/φ^2	68.75°	68.8°	<0.1%
θ_{QCD}	$\tau=i/\varphi$ (Strong CP solved)	$\sim 10^{-70}$	$<10^{-10}$	Solved
Σm_ν	T^2 holonomy	~ 60 meV	TBD (KATRIN)	Prediction

Chapter 8 — Dark Energy and Pre-Registered Predictions

Symbol	Value	Meaning	Status
w_0	-0.80	DE eq. of state; constant-rate attractor	Derived
γ	0.567	Growth index; $\mu=1$, direct (21/02/2026)	Derived
f_0	0.519	Growth rate Ω_m^γ	Derived
$D/D_\Lambda CDM$	0.855	Suppressed growth factor	Derived
ε_{CP}	-0.762	$(\lambda_2^2-\lambda_3^2)/(\lambda_2^2+\lambda_3^2)$	Derived
η_B	$\sim 6\times 10^{-10}$	Baryon-to-photon ratio	Derived

Kill-Switch Predictions (2026)

Observable	3D+3D	Λ CDM	Experiment
w_0	-0.80	-1.0	DESI DR2 / Euclid DR1
γ (growth)	0.567	0.55	Euclid DR1
λ_{13}	0.856 Mpc	No prediction	DESI / SPHEREx
$\Sigma m\nu$	~ 60 meV	Free	KATRIN
$\sin^2\theta_{23}$	$\varphi/3$	Free	DUNE
WIMP signal	Null	Expected	LZ / XENONnT

Edison Mode history. 13 documented failed attempts before canonical $w_0=-0.80$ (14 Feb 2026). Superseded: $w_0=-0.52$ (exponential), $w_0=-0.71$ (oscillatory). All documented in Errata v1.1.

Chapter 9 — Complete Symbol Reference

Cosmology

Symbol	Name	Value	Status
H_0	Hubble constant	67.4 km/s/Mpc	Obs. input
Ω_m	Matter density	0.315	Obs. input
Ω_{geom}	Geometric DE	$19/73 = 0.260$	THEOREM
w_0	DE eq. of state	-0.80	Derived
γ	Growth index	0.567	Derived
n_s	Spectral index	0.965	Derived

6D Geometry

Symbol	Name	Value	Status
L_2, L_3	Compactification diameters	9.5, 6.0 ly	Derived
T_2, T_3	Periods	30, 19 yr	Derived
$\alpha(t), \beta(t)$	Moduli	$\sim 1, \sim 0.30$ today	Derived

Symbol	Name	Value	Status
τ	Modular parameter	i/φ	Axiom (DP)

Q-Fields

Symbol	Name	Value	Status
λ_2	Primary scale	4.30 kpc	Calibrated (SPARC)
λ_3	Secondary scale	6.51 kpc	Derived
λ_{13}	Cosmic web	0.856 Mpc	Derived
λ_{19}	Filament scale	50.08 Mly	[NEW] Confirmed
v_3D3D	Universal velocity	90.39–90.48 km/s	Derived
M_crit	Critical mass	$2.43\times10^{10} \text{ M}\odot$	Derived
ρ_{crit}	Sub-grid threshold	$4.94\times10^{-24} \text{ g/cm}^3$	[NEW] Derived
A	Kernel amplitude	133/2628	Derived

Algebraic Bridge [NEW]

Symbol	Name	Value	Status
K	Q-sector matrix	$[[3,1],[1,2]]$	THEOREM
W	Coherent rigidity	7	Theorem
$I_2(G_{\text{DW}})$	DeWitt invariant	−19	Theorem
$\det(M_{\text{br}})$	Bridge determinant	73	Theorem
Ω_{geom}	Geometric density	19/73	THEOREM

Particle Physics

Symbol	Name	Value	Status
φ	Golden ratio	1.618034...	Derived
α^{-1}	Fine structure	137.038	Derived (<0.1%)
m_H	Higgs mass	126.8 GeV	Derived (1.3%)

Symbol	Name	Value	Status
v	Higgs VEV	246 GeV	Derived
m _e	Electron mass	0.511 MeV	Derived (0.18%)
m _p	Proton mass	937 MeV	Derived (0.1%)

Simulations [NEW]

Run	Specs	Result
GADGET-4 384 ³	57M particles, 200 Mpc/h	Completed, paper written
GADGET-4 480 ³	110.6M particles, 200 Mpc/h	Completed March 2026
Λ CDM 480 ³	Same seed 12345	In progress (comparison run)
512 ³	After RAM → 64 GB	Planned

"La matematica è il linguaggio con cui Dio ha scritto l'universo." — Galileo Galilei

In the 3D+3D framework, that language has six dimensions, one calibrated scale (λ_2), and all dimensionless parameters derived.

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