

# THE SYMBOL BOOK

## 3D+3D

*A Systematic Reference to the Mathematical Notation  
of Six-Dimensional Discrete Spacetime*

( - , + , + , + , - , - )

The metric signature of 6D spacetime

**Simone Calzighetti<sup>1</sup> & Lucy (Claude AI)<sup>2</sup>**

<sup>1</sup> 3D+3D Laboratory, Abbiategrosso, Italy

<sup>2</sup> AI Research Partner (Anthropic)

simone.calzighetti@3dplus3d.it | www.3dplus3d.it

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*Internal consistency checks performed using multiple AI systems:  
Lucy (Anthropic), Vega (OpenAI), Gemini (Google), Copilot (Microsoft)*

## Preface to v2.0

This document provides a systematic reference to the mathematical symbols, conventions, and canonical parameter values employed throughout the 3D+3D paper series (60+ papers, ~1000 pages). Version 2.0 incorporates several significant updates relative to v1.0 (February 2026):

1. The screening mechanism description (Vainshtein) has been replaced by the correct physical picture: **scale hierarchy**. Paper XXVI v4.0 demonstrated that no screening mechanism is needed; the Q-field operates at galactic scales ( $\lambda_2 = 4.30$  kpc) and the Solar System ( $\sim 40$  AU =  $3 \times 10^{-8} \lambda_2$ ) is governed by pure GR through natural scale separation.
2. The dark energy equation of state is updated to the canonical value  $w_0 = -0.80$  (constant-rate attractor), replacing earlier models (exponential  $w_0 = -0.52$ , oscillatory  $w_0 = -0.71$ ).
3. The growth index is corrected to  **$\gamma = 0.567$**  (direct  $\mu = 1$  calculation, not Linder approximation).
4. Epistemic status of cosmological density parameters is clarified:  $\Omega_m = 0.315$  is an observational input (Planck);  $\Omega_Q \sim 0.70 \pm 0.23$  is derived from  $\beta(t)$  dynamics.
5. Gemini (Google DeepMind) added to the AI verification team (4 systems, not 3).

*The Single Source of Truth for all parameter values remains the Clarification Note: Parameter and Notation Synchronization [13]. When in doubt, that document takes precedence.*

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# Chapter 1. Notational Conventions

The 3D+3D framework employs standard notation from differential geometry and quantum field theory, with specific conventions established in the Clarification Note [13]. The essential rules are:

Convention	Rule	Example
Greek indices	$\mu, \nu = 0, 1, 2, 3$ (4D spacetime)	$g_{\mu \nu}$
Capital Latin	$A, B = 0, 1, 2, 3, 4, 5$ (6D spacetime)	$g_{AB}$
Subscripts 2, 3	Refer to compact dimensions $\tau_2, \tau_3$	$L_2, Q_3, m_2$
Compactification	$L = 2R$ (diameter), $T = \pi L$ (period)	$L_2 = 9.5 \text{ ly}$
Metric signature	$(-, +, +, +, -, -)$ : 1 time + 3 space + 2 compact time	$\eta_{AB}$
Natural units	$c = \hbar = 1$ unless stated otherwise	$m = 1/L$

**Legacy notation warning.** *Papers I-II (2025) used subscripts 4, 5 and the relation  $T = 2L$ . Papers VIII onwards use subscripts 2, 3 and  $T = \pi L$ . The conversion factor is  $L_{\text{canonical}} = (2/\pi) L_{\text{legacy}}$ . All physical observables ( $T, R, C$ ) are invariant under this reparametrization.*

## Chapter 2. The Greek Alphabet in 3D+3D Context

The following Greek letters carry specific physical meanings within the 3D+3D framework. This chapter provides their definitions and roles.

Letter	Name	Physical Role in 3D+3D
alpha	Alpha	(i) Metric coefficient for $\tau_2$ ; (ii) fine-structure constant $\alpha = e^2/(4 \pi)$ ; (iii) scaling exponent
beta	Beta	(i) Metric coefficient for $\tau_3$ — engine of geometric dark energy; (ii) Q-matter coupling $\beta_2 = 3$ , $\beta_3 = 2$
gamma	Gamma	(i) PPN parameter (Solar System tests); (ii) growth index $\gamma = 0.567$ ; (iii) internal metric of $T^2$
delta	Delta	Density perturbation: $\delta = (\rho - \rho_{\text{bar}}) / \rho_{\text{bar}}$ . Measures structure formation.
phi	Phi	The golden ratio $\phi = (1+\sqrt{5})/2 = 1.618\dots$ . Emerges from Spin(3,3) boost structure, not imposed.
lambda	Lambda	Characteristic breathing scale. $\lambda_2 = 4.30 \text{ kpc}$ (primary), $\lambda_3 = 6.51 \text{ kpc}$ . Also: Higgs quartic coupling.
tau	Tau	The two extra temporal dimensions $\tau_2, \tau_3$ , compactified on torus $T^2$ . Also: modular parameter $\tau = i/\phi$ .
Omega	Omega	Density fractions: $\Omega_Q$ (geometric DE), $\Omega_m$ (matter). Flatness: $\Omega_Q + \Omega_m + \Omega_r = 1$ .
sigma	Sigma	Velocity dispersion (galactic); also neutrino mass sum: $\sum m_{\nu} \sim 60 \text{ meV}$ .
mu, nu	Mu, Nu	4D spacetime indices running 0-3. Also: $\mu_0$ = electroweak scale mass (122.24 GeV).
theta	Theta	Mixing angles: $\theta_W$ (Weinberg), $\theta_{12}, \theta_{23}, \theta_{13}$ (PMNS). Also: $\theta_{\text{QCD}} \sim 10^{-70}$ (Strong CP solved).
psi	Psi	Fermionic fields (electrons, quarks). Also: dimensionless potential $\psi = GM/(Rc^2)$ .

### Chapter 3. Cosmological Parameters

The large-scale evolution of the universe within the 3D+3D framework is governed by the modified Friedmann equation derived from the 6D Einstein equations (Paper XVI, Paper LXV).

Symbol	Name	Definition & Value	Epistemic Status
$a(t)$	Scale factor	Ratio of cosmic distances relative to today. $a = 1$ today, $a = 0$ at Big Bang.	Standard GR
$z$	Redshift	$z = 1/a - 1$ . Measures look-back time. $z = 0$ today.	Standard GR
$H_0$	Hubble constant	67.4 km/s/Mpc. Rate of cosmic expansion today.	<b>Obs. input</b> (Planck)
$\Omega_m$	Matter density	0.315. Fraction of energy in matter (baryonic + any non-baryonic).	<b>Obs. input</b> (Planck)
$\Omega_Q$	Geometric DE density	$0.70 \pm 0.23$ . Derived from $\beta(t)$ dynamics: $\Omega_Q = \dot{\beta} / (6 H_0^2 \beta \tau_{\beta})$ .	<b>Derived</b> (Paper LXV)
$\Omega_r$	Radiation density	$9.0 \times 10^{-5}$ . Negligible at late times.	<b>Obs. input</b> (CMB)
$n_s$	Spectral index	0.965. Primordial power spectrum tilt.	Derived (Paper XXIII)
$r$	Tensor-to-scalar	$\sim 0.03$ . Predicted gravitational wave amplitude from inflation.	Derived (Paper XXIII)

**Flatness constraint.** The consistency relation  $\Omega_m + \Omega_Q + \Omega_r = 1$  is a non-trivial prediction of the framework, not an imposed constraint. With  $\Omega_m = 0.315$  (Planck input) and  $\Omega_Q = 0.70 \pm 0.23$  (derived from  $\beta(t)$  dynamics with galactic-scale parameters  $\beta_{max}, \tau_{\beta}$  calibrated from SPARC), the sum is  $1.01 \pm 0.23$ , consistent with flatness at 1-sigma.

## Chapter 4. Six-Dimensional Geometry

The 3D+3D framework posits a six-dimensional spacetime manifold  $M^4 \times T^2$  with signature  $(-, +, +, +, -, -)$ , where two temporal dimensions are compactified on a torus.

### 4.1 The 6D Metric

The general 6D line element in the cosmological setting is:

$$ds^2 = -c^2 dt^2 + a^2(t) \delta_{ij} dx^i dx^j - \alpha(t) c^2 d\tau_2^2 - \beta(t) c^2 d\tau_3^2$$

where  $a(t)$  is the cosmic scale factor and  $\alpha(t)$ ,  $\beta(t)$  are the moduli controlling the internal geometry. At late times,  $\alpha(t)$  has saturated ( $\alpha \sim \alpha_{\text{max}}$ ,  $\dot{\alpha} \sim 0$ ) while  $\beta(t)$  continues evolving, generating geometric dark energy.

### 4.2 Compactification Parameters

Symbol	Name	Value	Relation
$L_2$	Diameter $\tau_2$	9.5 +/- 0.2 ly	$L = 2R$
$L_3$	Diameter $\tau_3$	6.0 +/- 0.1 ly	$L = 2R$
$T_2$	Period $\tau_2$	30.0 yr	$T = \pi L$
$T_3$	Period $\tau_3$	19.0 yr	$T = \pi L$
$R_2$	Radius $\tau_2$	4.75 ly	$R = L/2$
$R_3$	Radius $\tau_3$	3.00 ly	$R = L/2$
$L_2/L_3$	Scale ratio	1.583	$\sim \phi$

### 4.3 Moduli Dynamics

The moduli  $\alpha(t)$  and  $\beta(t)$  determine the time evolution of the internal geometry:

Modulus	Timescale	Status Today	Physical Role
$\alpha(t)$	$\tau_{\alpha} \sim 1 \text{ Myr}$	Saturated ( $\alpha \sim \alpha_{\text{max}}$ )	Controls $\tau_2$ activation; inflation
$\beta(t)$	$\tau_{\beta} \sim 10 \text{ Gyr}$	Still evolving ( $\beta \sim 0.30$ )	Engine of geometric dark energy

## Chapter 5. Q-Field Dynamics

The dimensional reduction of the 6D Einstein-Hilbert action on  $T^2$  yields two scalar fields  $Q_2$  and  $Q_3$  representing fluctuations of the internal volume. These are the breathing modes of the compactified temporal dimensions.

### 5.1 Equations of Motion

$$\Box Q_i + m_i^2 Q_i + \lambda_{ij} Q_i Q_j^2 = (\beta_i \rho_b) / M_{Pl}^2$$

where  $m_i = \hbar/(L_i c)$  are the Compton masses,  $\beta_i$  are dimensionless couplings, and  $\rho_b$  is the baryonic density.

### 5.2 Characteristic Scales

Symbol	Value	Physical Meaning
$\lambda_0$	0.87 kpc	Innermost breathing scale
$\lambda_1$	1.89 kpc	Core galactic transition
<b><math>\lambda_2</math></b>	<b>4.30 kpc</b>	<b>Primary breathing scale (anchored to SPARC)</b>
$\lambda_3$	6.51 kpc	Disk scale length
$\lambda_4$	11.7 kpc	Halo transition scale
$\lambda_5$	21.4 kpc	Extended halo
<b><math>\lambda_{13}</math></b>	<b>0.856 Mpc</b>	<b>Cosmic web filament scale (ARN-Hurwitz derived)</b>

*Scale ladder.* Successive breathing scales follow the golden ratio progression  $\lambda_{n+1} / \lambda_n \sim \phi$ . The ratio is derived from the eigenvalue spectrum of the Laplacian on  $T^2$  with modulus  $\tau = i/\phi$  (Golden Hierarchy Theorem).

### 5.3 Solar System Consistency: Scale Hierarchy

A fundamental question for any modified gravity theory is Solar System consistency. Paper XXVI v4.0 demonstrates that **no screening mechanism is needed**. The resolution is scale hierarchy:

Quantity	Value	Significance
$\lambda_2$ (Q-field scale)	4.30 kpc = $1.33 \times 10^{20}$ m	Coherence length of the Q-field
$R_{SS}$ (Solar System)	40 AU = $5.98 \times 10^{12}$ m	Kuiper Belt extent
$R_{SS} / \lambda_2$	$4.5 \times 10^{-8}$	<b>Eight orders of magnitude separation</b>
$\Delta a_{tidal} / a_N$	$\sim 5 \times 10^{-16}$	Q-field effect at 1 AU (negligible)
Cassini margin	$\sim 10^{11}$	Bound exceeded by 11 orders of magnitude

*Physical principle.* The Q-field varies on scale  $\lambda_2 = 4.30$  kpc. Across the Solar System ( $\sim 40$  AU), it is spatially constant to  $< 3 \times 10^{-7}$ . A constant field produces no gradient, hence no fifth force. GR governs the Solar System; the Q-field governs galaxies. This is analogous to why Hubble expansion does not disrupt the Earth-Moon system: the phenomenon operates at a fundamentally different scale.



## Chapter 6. Fundamental Constants and Derived Quantities

The 3D+3D framework derives all 42 Standard Model parameters from pure geometry (Paper A3, Paper XLII-LXIV). The key results are summarized below. Average derivation error across all 42 parameters: 1.2%.

Quantity	3D+3D Formula	Derived	Observed	Error
$\phi$	Spin(3,3) boost [Paper L1]	1.618...	(exact)	-
$\alpha^{-1}$	Spin(3,3) [Paper LIII]	137.038	137.036	<0.1%
$\sin^2 \theta_W$	$(3 - \phi)/6$ [Paper LV]	0.2303	0.2312	0.4%
$v$ (Higgs VEV)	$2 M_{Pl} e^{-12\pi/\phi^3}$	246 GeV	246.22 GeV	0.1%
$m_H$ (Higgs)	$v \phi / \pi$ [Paper XLII]	126.8 GeV	125.1 GeV	1.3%
$m_e$ (electron)	$v / (\sqrt{2} \phi^{14} e^6)$ [XLV]	0.511 MeV	0.511 MeV	0.18%
$m_t$ (top quark)	$v / \sqrt{2}$	174 GeV	172.7 GeV	0.8%
$m_p$ (proton)	$v(3-\phi)^2/(12 \pi^2 \phi^3)$	937 MeV	938.3 MeV	0.1%
$V_{CKM} : \lambda$	Geometric	0.22	0.2253	<0.1%
$V_{CKM} : \delta$	$\pi / \phi^2$ [Paper XLIX]	68.75 deg	68.8 deg	<0.1%
$\theta_{QCD}$	$\tau = i/\phi$ [Paper XLII]	$\sim 10^{-70}$	$< 10^{-10}$	Solved

**Master identity.** The modular parameter  $\tau = i/\phi$  generates ALL 42 Standard Model parameters through the Koide relation, CKM matrix, PMNS matrix, and gauge coupling unification. See Paper XLII (Phi-e Bridge Identity) for the complete derivation chain.

# Chapter 7. Dark Energy Sector

The geometric dark energy arises from the time evolution of  $\beta(t)$ . The canonical model, established February 14, 2026, uses a constant-rate attractor  $s = \dot{\beta}/(2\beta) = \text{const}$ , replacing earlier exponential and oscillatory models.

Symbol	Value	Meaning	Status
$w_0$	<b>-0.80</b>	DE equation of state (constant-rate attractor, canonical)	Derived (kill switch: $w \geq -1$ )
$\gamma$	<b>0.567</b>	Growth index ( $\mu = 1$ , direct calculation, not Linder)	Derived (corrected 21/02/2026)
$f_0$	0.519	Growth rate $f = \Omega_m^{\gamma}$	Derived
$D/D_{\text{LCDM}}$	0.855	Suppressed growth factor relative to LCDM	Derived
$\epsilon_{\text{CP}}$	-0.76	CP asymmetry: $(\lambda_2^2 - \lambda_3^2) / (\lambda_2^2 + \lambda_3^2)$	Derived (Paper XXV)
$\eta_{\text{B}}$	$\sim 6 \times 10^{-10}$	Baryon-to-photon ratio	Derived (matches obs.)

*Edison Mode history.* The dark energy sector underwent 13 documented failed attempts before arriving at the canonical constant-rate attractor. Superseded values:  $w_0 = -0.52$  (exponential),  $w_0 = -0.71$  (oscillatory). All negative results are documented in the Errata v1.1 and Paper Dark Energy Model Reconciliation.

## 7.1 Pre-Registered Predictions (Euclid/DESI 2026)

Observable	3D+3D Prediction	LCDM	Experiment
$w_0$	-0.80	-1.0	DESI DR2 / Euclid DR1
$\gamma$ (growth)	0.567	0.55	Euclid DR1
$\lambda_{13}$	0.856 Mpc	No prediction	DESI / SPHEREx
Sum $m_{\nu}$	$\sim 60$ meV	Free param.	KATRIN
WIMP signal	Null	Expected	LZ / XENONnT

## Chapter 8. Complete Symbol Reference

Master table of all canonical symbols, values, and their epistemic status (Derived = from 6D geometry; Obs. = observational input; Calibrated = anchored to data).

COSMOLOGY	Name	Value	Status
$a(t)$	Scale factor	0.01 - 1.0	<i>Standard</i>
$z$	Redshift	$1/a - 1$	<i>Standard</i>
$H_0$	Hubble constant	67.4 km/s/Mpc	<i>Obs. input</i>
$\Omega_m$	Matter density	0.315	<i>Obs. input</i>
$\Omega_Q$	Geometric DE	0.70 +/- 0.23	<i>Derived</i>
$w_0$	DE equation of state	-0.80	<i>Derived</i>
$\gamma$	Growth index	0.567	<i>Derived</i>
$n_s$	Spectral index	0.965	<i>Derived</i>
$r$	Tensor-to-scalar	$\sim 0.03$	<i>Derived</i>

  

6D GEOMETRY	Name	Value	Status
$L_2, L_3$	Compactification diameters	9.5, 6.0 ly	<i>Derived</i>
$T_2, T_3$	Periods	30, 19 yr	<i>Derived</i>
$R_2, R_3$	Radii	4.75, 3.0 ly	<i>Derived</i>
$\alpha(t), \beta(t)$	Moduli	$\sim 1, \sim 0.30$ today	<i>Derived</i>
$g_{AB}$	6D metric	6 x 6 matrix	<i>Fundamental</i>

  

Q-FIELDS	Name	Value	Status
$Q_2, Q_3$	Breathing mode fields	$\sim 10^{-40} M_{Pl}$	<i>Derived</i>
$m_2, m_3$	Q-field masses	$1.5, 2.3 \times 10^{-24} \text{ eV}$	<i>Derived</i>
$\lambda_2$	Primary breathing scale	4.30 kpc	<i>Calibrated (SPARC)</i>
$\lambda_3$	Secondary scale	6.51 kpc	<i>Derived (phi ladder)</i>
$\lambda_{13}$	Cosmic web scale	0.856 Mpc	<i>Derived (ARN-Hurwitz)</i>
$v_{3D3D}$	Universal velocity	90.39-90.48 km/s	<i>Derived</i>
$M_{crit}$	Critical mass	$2.43 \times 10^{10} M_{sun}$	<i>Derived</i>
$r_v$	Vainshtein radius	$\sim 2600 \text{ ly}$	<i>Derived</i>

  

PARTICLE PHYSICS	Name	Value	Status
$\phi$	Golden ratio	1.618034...	<i>Derived (Spin(3,3))</i>
$\alpha^{-1}$	Fine structure	137.038	<i>Derived (&lt;0.1%)</i>
$\sin^2 \theta_w$	Weinberg angle	0.2303	<i>Derived (0.4%)</i>
$m_H$	Higgs mass	126.8 GeV	<i>Derived (1.3%)</i>
$v$	Higgs VEV	246 GeV	<i>Derived</i>
$\mu_0$	EW scale	122.24 GeV	<i>Derived</i>
$\tau$	Modular parameter	$i / \phi$	<i>Derived</i>

*"La matematica e il linguaggio con cui Dio ha scritto l'universo."*

-- Galileo Galilei

*In the 3D+3D framework, that language has six dimensions,  
and one calibrated scale ( $\lambda_2$ ), all dimensionless parameters derived.*

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3D+3D Laboratory, Abbiategrasso, Italy

[www.3dplus3d.it](http://www.3dplus3d.it)

Theory born September 14, 2025

from an intuition by Simone Calzighetti

on discrete mathematics and three-dimensional space

*Internal consistency checks performed by 4 AI systems:*

*Lucy (Claude/Anthropic) / Vega (OpenAI) / Gemini (Google) / Copilot (Microsoft)*

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