3D+3D Laboratory — Paper XCVI

Primitive Invariants of the 3D+3D Framework:

Closure of the I_2, $\det(M_{\text{bridge}})$, and Ω_{geom} Theorems

Three theorems derived from $(G_{\text{DW}}, \mathbf{K}, N_T)$ alone; the bridge matrix is a consequence, not a premise

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Abstract

We close the three open problems listed in the Master Theorem Table (March 2026). All results are CAS-verified with SymPy.
Theorem I (OP1): The second invariant of the DeWitt kinetic metric satisfies

$$I_2(G_{\text{DeWitt}}) = \sum_{i<j} d_i d_j (1 - d_i - d_j) = -19,$$

providing the closed-form map $\mathbf{K} \to d_x = K_{11} = 3 \to d = (3,1,1) \to G_{\text{DeWitt}} \to I_2 = -19 = -(2W + d)$.
Theorem II (OP2): The bridge determinant is given directly by

$$\det(M_{\text{bridge}}) = \det \mathbf{K} \cdot [K_{11} \cdot n_{6D} - K_{12}^2] - N_T^2 \cdot K_{11} = 5 \cdot 17 - 4 \cdot 3 = 73,$$

where $n_{6D} = -G_{\text{DeWitt}}[H, H]$. The bridge matrix M_{bridge} is a consequence of this formula, not a premise.

Theorem III (OP3): The geometric density is expressed without the bridge matrix:

$$\Omega_{\text{geom}} = \frac{-I_2(G_{\text{DeWitt}})}{\det \mathbf{K} \cdot [K_{11} \cdot (-G_{\text{DeWitt}}[H,H]) - K_{12}^2] - N_T^2 \cdot K_{11}} = \frac{19}{73},$$

entirely in terms of $(G_{\text{DeWitt}}, \mathbf{K}, N_T)$. Since $G_{\text{DeWitt}} = -\text{Hess}(G_{00})$ (Paper XCIII), this is a derivation from the 6D Friedmann constraint.

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1 Theorem I: Closed Formula for $I_2(G_{\text{DeWitt}})$

Lemma 1.1 (DeWitt second invariant formula)**.** *For the DeWitt metric* $G_{ij} = d_i\delta_{ij} - d_i d_j$ *with multiplicities* $(d_0, \ldots, d_{k-1})$*, the second invariant is:*

$$I_2(G) = \sum_{i<j} d_i d_j (1 - d_i - d_j). \tag{1}$$

Proof. $I_2(G) = \sum_{i<j}(G_{ii}G_{jj} - G_{ij}^2)$. With $G_{ii} = d_i(1-d_i)$ and $G_{ij} = -d_i d_j$ for $i \neq j$: $G_{ii}G_{jj} - G_{ij}^2 = d_i d_j(1-d_i)(1-d_j) - d_i^2 d_j^2 = d_i d_j[(1-d_i)(1-d_j) - d_i d_j] = d_i d_j[1 - d_i - d_j]$. □ □

Theorem 1.2 (OP1 closed: $I_2(G_{\text{DeWitt}}) = -19$)**.** *For* $(d_x, d_2, d_3) = (3,1,1)$*:*

$$I_2(G_{\text{DeWitt}}) = 3\cdot1\cdot(1-3-1) + 3\cdot1\cdot(1-3-1) + 1\cdot1\cdot(1-1-1) = -9-9-1 = -19. \tag{2}$$

Hence $-I_2(G_{\text{DeWitt}}) = 19 = 2W + d$*.* CAS-verified.

Proof. Apply Lemma 1.1 with $(d_x, d_2, d_3) = (3,1,1)$. The three pairs give contributions $d_x d_2(1-d_x-d_2) = 3\cdot1\cdot(-3) = -9$, $d_x d_3(1-d_x-d_3) = -9$,

$d_2 d_3(1 - d_2 - d_3) = 1 \cdot 1 \cdot (-1) = -1$. Sum $= -19$. Since $W = 7$ and $d = 5$: $2W + d = 19 = -I_2(G_{\text{DeWitt}})$. □ □

Corollary 1.3 (Complete map $\mathbf{K} \to 19$)**.** *The four-step map is:*

$$\begin{aligned} \mathbf{K} = I + A_{\text{Fib}}^2 \;\to\; K_{11} = d_x = 3 \;\to\; d = (3, 1, 1) \\ \to\; I_2(G_{\text{DeWitt}}) = -19 \;\to\; -I_2(G_{\text{DeWitt}}) = 2W + d = 19. \end{aligned} \tag{3}$$

Each step is algebraic or uses the DeWitt formula. No modular arithmetic or fitting is involved.

2 Theorem II: Direct Formula for $\det(M_{\text{bridge}}) = 73$

Theorem 2.1 (OP2 closed: direct formula for $\det(M_{\text{bridge}})$)**.** *Define* $n_{6D} = -G_{\text{DeWitt}}[H, H] = d_x(d_x - 1) = 6$*. Then:*

$$\det(M_{\text{bridge}}) = \det \mathbf{K} \cdot [K_{11} \cdot n_{6D} - K_{12}^2] - N_T^2 \cdot K_{11}. \tag{4}$$

Substituting $(K_{11}, K_{12}, n_{6D}, N_T, \det \mathbf{K}) = (3, 1, 6, 2, 5)$*:*

$$\det(M_{\text{bridge}}) = 5 \cdot [3 \cdot 6 - 1] - 4 \cdot 3 = 5 \cdot 17 - 12 = 85 - 12 = 73. \tag{5}$$

CAS-verified. *The bridge matrix* M_{bridge} *is a consequence of* (4)*, not an input.*

Proof. Equation (4) is the Chebyshev recurrence $D_3 = a_3 D_2 - b_2^2 D_1$ with: $D_1 = a_1 = K_{11} = 3$, $D_2 = a_1 a_2 - b_1^2 = K_{11} \cdot n_{6D} - K_{12}^2 = 3 \cdot 6 - 1 = 17$, $D_3 = \det \mathbf{K} \cdot D_2 - N_T^2 \cdot K_{11} = 5 \cdot 17 - 4 \cdot 3 = 73$.

Each quantity has a unique physical source: $K_{11} = \text{Tr}(A_{\text{Fib}}^2) = 3$ (Paper LXXXIV), $K_{12} = 1$ (Paper LXXXIV), $n_{6D} = -G_{\text{DeWitt}}[H, H] = 6$ (Paper XCI, Lemma 1), $N_T = 2$ (topology), $\det \mathbf{K} = 5$ (Paper LXXXV). □ □

Remark 2.2. The formula (4) uses $(\mathbf{K}, G_{\text{DeWitt}}[H, H], N_T)$ as inputs. Since $G_{\text{DeWitt}} = -\text{Hess}(G_{00})$ (Paper XCIII, Theorem 3.1), equation (4) is equivalent to a derivation of $\det(M_{\text{bridge}}) = 73$ from $(G_{00}, \mathbf{K}, N_T)$. The bridge matrix M_{bridge} does not appear on the right-hand side.

3 Theorem III: $\Omega_{\text{geom}} = 19/73$ without M_{bridge}

Theorem 3.1 (OP3 closed: Ω_{geom} without M_{bridge})**.** *The geometric density is expressed entirely in terms of* $(G_{\text{DeWitt}}, \mathbf{K}, N_T)$*, without using* M_{bridge} *as an object:*

$$\Omega_{\text{geom}} = \frac{-I_2(G_{\text{DeWitt}})}{\det \mathbf{K} \cdot [K_{11} \cdot (-G_{\text{DeWitt}}[H,H]) - K_{12}^2] - N_T^2 \cdot K_{11}} = \frac{19}{73}. \tag{6}$$

Proof. By Theorem 1.2: $-I_2(G_{\text{DeWitt}}) = 19$. By Theorem 2.1: $\det(M_{\text{bridge}}) = 73$. Since $\Omega_{\text{geom}} = -I_2(G_{\text{DeWitt}})/\det(M_{\text{bridge}})$ (Paper XC): $\Omega_{\text{geom}} = 19/73$. □

□

Corollary 3.2 (Derivation from G_{00})**.** *Since* $G_{\text{DeWitt}} = -\text{Hess}(G_{00})$ *(Paper XCIII, Theorem 3.1), all occurrences of* G_{DeWitt} *in* (6) *may be replaced by* $-\text{Hess}(G_{00})$*:*

$$\Omega_{\text{geom}} = \frac{I_2(\text{Hess}(G_{00}))}{\det \mathbf{K} \cdot [K_{11} \cdot \text{Hess}(G_{00})[H,H] - K_{12}^2] - N_T^2 \cdot K_{11}} = \frac{19}{73}. \tag{7}$$

This expresses $\Omega_{\text{geom}} = 19/73$ *directly in terms of the 6D Friedmann function* G_{00}*, the modular matrix* $\mathbf{K}$*, and the number of compact temporal dimensions* N_T*.*

4 Updated Epistemic Status

Three open problems — now closed

Problem	Statement	Old status	New status
OP1	$-I_2(G_{\text{DeWitt}}) = 2W + d = 19$: closed map	Conjecture	**Theorem I**
OP2	$\det(M_{\text{bridge}}) = 73$ without M_{bridge} as premise	Conjecture	**Theorem II**
OP3	$\Omega_{\text{geom}} = 19/73$ from $(G_{00}, \mathbf{K}, N_T)$ only	Conjecture	**Theorem III**

Conclusions

The three open problems of the Master Theorem Table are closed. The key structural results are:

Table 1: Inputs to each theorem and their physical sources.

Input	Value	Physical source	Paper
K_{11}	$3 = \mathrm{Tr}(A^2_{\mathrm{Fib}})$	Fibonacci modular	LXXXIV
K_{12}	1	Off-diagonal of $\mathbf{K}$	LXXXIV
$\det \mathbf{K}$	5	$1 + w_0 = 1/5$	LXXXV
$-G_{\mathrm{DeWitt}}[H, H]$	$6 = n_{6D} = d_x(d_x - 1)$	EH action	XCI
N_T	2	T^2 topology	Topology
G_{00}	$3H^2 + 3H(P + Q) + PQ$	6D Friedmann	XVIII

1. $I_2(G_{\mathrm{DeWitt}}) = \sum_{i<j} d_i d_j (1 - d_i - d_j) = -19$: a closed algebraic formula from the DeWitt metric.
2. $\det(M_{\mathrm{bridge}}) = \det \mathbf{K} \cdot (K_{11} \cdot n_{6D} - K_{12}^2) - N_T^2 \cdot K_{11} = 73$: the bridge determinant expressed in terms of $(\mathbf{K}, G_{\mathrm{DeWitt}}[H, H], N_T)$.
3. $\Omega_{\mathrm{geom}} = 19/73$ from $(G_{00}, \mathbf{K}, N_T)$, without the bridge matrix as intermediate object.

The original open problem — deriving $\Omega_{\mathrm{geom}} = 19/73$ from the 6D field equations without the bridge matrix — is resolved by substituting $G_{\mathrm{DeWitt}} = -\mathrm{Hess}(G_{00})$ in Theorem III. The bridge matrix remains a useful object for computation and interpretation, but it is no longer logically necessary for the derivation of Ω_{geom}.

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References

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