Paper XXXVII: Mathematical Notation and Symbol Glossary for 3D+3D Theory

A Complete Reference Guide to the Mathematical Language of Six-Dimensional Spacetime

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Authors: Simone Calzighetti[1], Lucy (Claude AI)[2]

[1] 3D+3D Laboratory, Abbiategrasso, Italy
[2] Anthropic (Human-AI Collaboration in Theoretical Physics)

Email: condoor76@gmail.com

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Abstract

This document provides a comprehensive glossary of all mathematical symbols, conventions, and notation used throughout the 3D+3D discrete spacetime theory papers (I-XXXVI). We organize symbols by category: coordinates and indices, metric components, fundamental constants, Q-field quantities, characteristic scales, coupling parameters, operators, and cosmological quantities. Each entry includes the symbol, name, definition, units, typical values, and the paper where it first appears. This reference is intended to make the theory accessible to researchers from diverse backgrounds and to ensure consistent notation across all publications.

Keywords: notation, glossary, mathematical symbols, reference guide, 3D+3D theory

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15. Numerical Constants

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1. Coordinates and Indices

1.1 Spacetime Coordinates

Symbol	Name	Definition	Range	Paper
X^A	6D coordinates	Full spacetime position	A = 0,1,2,3,4,5	I
x^	4D coordinates	Observable spacetime	= 0,1,2,3	I
t	Cosmic time	Observable time coordinate	(-∞, +∞)	I
x, y, z	Spatial coordinates	Observable space	(-∞, +∞)	I
	Second time	First compactified temporal dimension	[0, 2 L]	I
	Third time	Second compactified temporal dimension	[0, 2 L]	I
r	Radial coordinate	Distance from galactic center	[0, ∞)	II
,	Angular coordinates	Spherical angles	[0,], [0,2)	II

1.2 Index Conventions

Symbol	Name	Range	Usage
A, B, C, …	6D indices	0, 1, 2, 3, 4, 5	Full spacetime tensors
, , , , …	4D spacetime indices	0, 1, 2, 3	Observable spacetime
i, j, k, …	3D spatial indices	1, 2, 3	Spatial tensors
I, J, K, …	Internal indices	4, 5	Compact dimensions
a, b, c, …	Internal (alternative)	4, 5	Compact space tensors
, , …	Spinor indices	1, 2, 3, 4	Dirac spinors

1.3 Coordinate Shorthand

Symbol	Meaning	Explicit Form
d x	6D volume element	dt dx dy dz d d
d x	4D volume element	dt dx dy dz
d^2	Internal volume element	d d
d^3x	Spatial volume element	dx dy dz

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2. Metric and Geometry

2.1 Metric Tensors

Symbol	Name	Definition	Signature	Paper
g_AB	6D metric	Full spacetime metric	(−,+,+,+,−,−)	I

Symbol	Name	Definition	Signature	Paper
g_	4D metric	Observable spacetime metric	(−,+,+,+)	I
_IJ	Internal metric	Metric on T^2	(−,−)	I
_AB	6D Minkowski	Flat 6D metric	(−,+,+,+,−,−)	I
_	4D Minkowski	Flat 4D metric	(−,+,+,+)	I
g_ ^(ind)	Induced metric	Metric on brane	(−,+,+,+)	XXII
h_AB	Metric perturbation	g_AB = ḡ_AB + h_AB	—	II

2.2 Metric Ansatz

The general 6D metric in the 3D+3D framework:

$$g_{AB} = \begin{pmatrix} g_{\mu\nu} + A^I_\mu A^J_\nu \gamma_{IJ} & A^I_\mu \gamma_{IJ} \\ A^J_\nu \gamma_{IJ} & \gamma_{IJ} \end{pmatrix}$$

Simplified form (A^I_ = 0):

$$g_{AB} = \begin{pmatrix} g_{\mu\nu} & 0 \\ 0 & \gamma_{IJ} \end{pmatrix}$$

2.3 Curvature Quantities

Symbol	Name	Definition	Paper
R_6	6D Ricci scalar	g^{AB} R_AB	I
R_4	4D Ricci scalar	g^{ } R_	I
R_2	Internal Ricci scalar	^{IJ} R_IJ (= 0 for flat T^2)	I
R_ABCD	6D Riemann tensor	Full curvature tensor	I
R_AB	6D Ricci tensor	R^C_ACB	I
G_AB	6D Einstein tensor	R_AB - ½ g_AB R_6	I
K_	Extrinsic curvature	Brane embedding curvature	XXII

2.4 Geometric Objects

Symbol	Name	Definition	Paper
Γ^A_BC	Christoffel symbols	½ g^{AD}(_B g_CD + _C g_BD - _D g_BC)	I
√(-g_6)	6D volume factor	√	det(g_AB)
√(-g_4)	4D volume factor	√	det(g_)
V_int	Internal volume	d^2 √	

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3. Fundamental Constants

3.1 Natural Constants

Symbol	Name	Value	Units	Paper
c	Speed of light	2.998×10	m/s	I
	Reduced Planck constant	1.055×10^{3}	$J \cdot s$	I
G_N	Newton's constant	6.674×10^{11}	$m^3/(kg \cdot s^2)$	I
k_B	Boltzmann constant	1.381×10^{23}	J/K	VII

3.2 Planck Units

Symbol	Name	Definition	Value	Paper
M_P	Planck mass	√(c/G_N)	2.176×10 kg	I
M_Pl	Reduced Planck mass	M_P/√(8)	2.435×10^{1} GeV	I
l_P	Planck length	√(G_N/c^3)	1.616×10^{3} m	I
t_P	Planck time	√(G_N/c)	5.391×10 s	I
E_P	Planck energy	√(c /G_N)	1.956×10 J	I

3.3 6D Constants

Symbol	Name	Definition	Value	Paper
M_6	6D Planck mass	(M_Pl^2/V_int)^{1/4}	~3×10^{1} GeV	I
G_6	6D Newton constant	1/(M_6)	—	I
_6	6D gravitational coupling	8 G_6 = 8 /M_6	—	I

3.4 Standard Model Constants

Symbol	Name	Value	Paper
v	Higgs VEV	246 GeV	XXXV
m_H	Higgs mass	125.1 GeV	XXXV
g	SU(2)_L coupling	0.652	XXXVI
g'	U(1)_Y coupling	0.357	XXXVI
g_s	SU(3)_c coupling	1.22 (at m_Z)	XXXVI
y_t	Top Yukawa	0.99	XXXVI
	Fine structure constant	1/137.036	XXXVI
_s	Strong coupling	0.118 (at m_Z)	XXXVI

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4. Q-Field Quantities

4.1 Q-Field Definitions

Symbol	Name	Definition	Units	Paper
Q	First Q-field	Modulus of dimension	M_Pl	II
Q	Second Q-field	Modulus of dimension	M_Pl	II
Q	Combined Q-field	√(Q ² + Q ²)	M_Pl	II
Q̄ , Q̄	Background values	Q_i in equilibrium	M_Pl	II
Q_i	Q-field perturbation	Q_i - Q̄_i	M_Pl	II

4.2 Q-Field Parameters

Symbol	Name	Definition	Value	Paper
m	Q mass	2 /T	~7 × 10 ² eV	II
m	Q mass	2 /T	~1.1 × 10 ²³ eV	II
T	Q oscillation period	2 R /c	30 years	XI
T	Q oscillation period	2 R /c	19 years	XI
	Q angular frequency	2 /T = m (=1)	—	XI
	Q angular frequency	2 /T = m (=1)	—	XI

4.3 Q-Field Amplitudes

Symbol	Name	Typical Value	Paper
A	Q amplitude	~10 ¹ M_Pl	II
A	Q amplitude	~10 ¹ M_Pl	II
	Q	_max	Maximum total amplitude

4.4 Q-Field Lagrangian Terms

$$\mathcal{L}_Q = -\frac{1}{2}(\partial_\mu Q_2)^2 - \frac{1}{2}m_2^2 Q_2^2 - \frac{1}{2}(\partial_\mu Q_3)^2 - \frac{1}{2}m_3^2 Q_3^2 - V_{int}(Q_2, Q_3)$$

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5. Characteristic Scales

5.1 Compactification Radii

Symbol	Name	Definition	Value	Paper
R	compactification radius	L /(2)	~1.4 ly	I
R	compactification radius	L /(2)	~3.7 ly	I
L	circumference	2 R	~9 ly	I
L	circumference	2 R	~23 ly	I

5.2 Characteristic Wavelengths

Symbol	Name	Definition	Value	Paper
	First characteristic scale	c × T = 2 R	**4.30 kpc**	II
	Second characteristic scale	c × T = 2 R	**11.7 kpc**	II
_n	n-th harmonic scale	× ^{n-2}	—	V

5.3 Harmonic Scale Ladder

The characteristic scales follow a **golden ratio progression**:

$$\lambda_n = \lambda_2 \times \varphi^{n-2}$$

n	Scale	Value	Astrophysical Correspondence
1		2.66 kpc	Inner galaxy
2		4.30 kpc	**Fundamental scale**
3		6.96 kpc	Disk scale
4		11.3 kpc	Outer disk
5		18.2 kpc	Halo transition
6		29.5 kpc	MW halo
7		47.7 kpc	Local Group
8		77.2 kpc	Satellite distribution

5.4 Critical Scales

Symbol	Name	Definition	Value	Paper
r_c	Core radius	Transition to Q-dominated	~	IV
r_V	Vainshtein radius	Screening activation	(G_N M/Λ³)^{1/3}	XXVI
M_crit	Critical mass	Dark matter threshold	1.8×10^{11} M_	XXXI

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6. Coupling Parameters

6.1 Q-Matter Coupling

Symbol	Name	Definition	Value	Paper
	Q -matter coupling	ln(m)/ Q	3.2 ± 0.8	IV
	Q -matter coupling	ln(m)/ Q	3.2 ± 0.8	IV
	Brane tension coefficient	² / Q ²	~O(1)	XXVII
	Brane tension coefficient	² / Q ²	~O(1)	XXVII

6.2 Q-Higgs Coupling

Symbol	Name	Definition	Value	Paper
	Q-Higgs portal coupling	Coefficient of Q^2	H	²
_crit	Critical coupling	Minimum for first-order PT	0.26	XXXVI

6.3 Q-SM Couplings (Dimension-5 and -6)

Symbol	Name	Operator	Strength	Paper
c_S	Scalar Q-fermion	(c_S/M_P) Q^-	~O(1)	XXXVI
c_P	Pseudoscalar Q-fermion	(c_P/M_P) Q^-	~O(1)	XXXVI
c_F	Q-gauge (EM)	(c_F/M_P^2) Q^2 F^2	~O(1)	XXXVI
c_W	Q-gauge (SU(2))	(c_W/M_P^2) Q^2 W^2	~O(1)	XXXVI
c_g	Q-gluon	(c_g/M_P^2) Q^2 G^2	~O(1)	XXXVI

6.4 Gravitational Coupling

Symbol	Name	Definition	Paper
G_eff	Effective Newton constant	G_N × S(r)	II
	Einstein coupling	8 G_N = 1/M_Pl^2	I

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7. Screening and Potential

7.1 Screening Function

Symbol	Name	Definition	Paper
S(r)	Screening function	G_eff(r)/G_N	II
S (r)	Q screening	1 + ² Q ²(r)/M_Pl^2	IV
S (r)	Q screening	1 + ² Q ²(r)/M_Pl^2	IV

Full screening function:

$$S(r) = 1 + \beta_2^2 \left(\frac{Q_2(r)}{M_{Pl}}\right)^2 + \beta_3^2 \left(\frac{Q_3(r)}{M_{Pl}}\right)^2$$

7.2 Q-Field Profiles

Radial profile (sourced by mass M):

$$Q_2(r) = \frac{\beta_2 G_N M}{r}\left(1 - e^{-r/\lambda_2}\right)$$

$$Q_3(r) = \frac{\beta_3 G_N M}{r}\left(1 - e^{-r/\lambda_3}\right)$$

7.3 Effective Potential

Symbol	Name	Definition	Paper
Φ_N	Newtonian potential	-G_N M/r	II
Φ_eff	Effective potential	Φ_N × S(r)	II
Φ_Q	Q-field potential	Φ_eff - Φ_N	IV
V_5(r)	Fifth force potential	- G_N m m e^{-r/ }/r	XXXVI

7.4 Vainshtein Mechanism

Symbol	Name	Definition	Paper
r_V	Vainshtein radius	(G_N M/Λ³)^{1/3}	XXVI
Λ	Strong coupling scale	(M_Pl m_Q²)^{1/3}	XXVI
_5	Fifth force strength	c_S²/(4)	XXXVI

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8. Kaluza-Klein Spectrum

8.1 KK Mode Indices

Symbol	Name	Range	Paper
n	mode number	0, ±1, ±2, …	XXII
n	mode number	0, ±1, ±2, …	XXII
(n , n)	KK mode label	Integer pairs	XXII

8.2 KK Mass Formula

$$M^2_{n_2,n_3} = \frac{n_2^2}{R_2^2} + \frac{n_3^2}{R_3^2}$$

8.3 Special Modes

Mode	(n , n)	Mass	Interpretation
Zero mode	(0, 0)	0	4D graviton
First tower	(1, 0)	1/R	Q -like excitation
Second tower	(0, 1)	1/R	Q -like excitation
Mixed	(1, 1)	√(1/R ² + 1/R ²)	Combined excitation

8.4 KK Coupling

Symbol	Name	Definition	Paper
g_{n ,n }	KK mode coupling	1/M_P × (overlap integral)	XXII

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9. Cosmological Quantities

9.1 Background Cosmology

Symbol	Name	Definition	Value	Paper
H	Hubble parameter	ȧ/a	H = 67.4 km/s/Mpc	XVI
H	Hubble constant today	H(t)	67.4 km/s/Mpc	XVI
a(t)	Scale factor	Expansion of universe	1 today	XVI
z	Redshift	a /a - 1	—	XVI
_c	Critical density	$3H^2/(8\ G\_N)$	—	XVI

9.2 Density Parameters

Symbol	Name	Definition	Value (3D+3D)	Paper
Ω_b	Baryon density	_b/ _c	0.049	XVI
Ω_r	Radiation density	_r/ _c	~10	XVI
Ω_Q	Q-field density	_Q/ _c	**Replaces Ω_DM**	XVI
Ω_Λ	Dark energy	_Λ/ _c	0.69 (geometric)	XVI
Ω_tot	Total density	Σ Ω_i	1.00	XVI

9.3 Power Spectrum

Symbol	Name	Definition	Paper
P(k)	Matter power spectrum		_k
Δ^2(k)	Dimensionless spectrum	$k^3P(k)/(2\ ^2)$	V
(r)	Correlation function	FT of P(k)	V
	Amplitude at 8 Mpc/h	—	XVI

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10. Phase Transition Parameters

10.1 Effective Potential

Symbol	Name	Definition	Paper
V_eff(,T)	Effective potential	V + V_CW + V_T + V_daisy	XXXVI
V	Tree-level potential	$-\ ^2\ ^2/2 +\ \ /4$	XXXVI

Symbol	Name	Definition	Paper
V_CW	Coleman-Weinberg	1-loop zero-T correction	XXXVI
V_T	Thermal potential	Finite-T correction	XXXVI
V_daisy	Daisy resummation	Ring diagram correction	XXXVI

10.2 Critical Parameters

Symbol	Name	Definition	Value (SM+Q)	Paper
T_c	Critical temperature	V_eff(_c) = V_eff(0)	142 GeV	XXXVI
_c	Critical VEV	Higgs VEV at T_c	156 GeV	XXXVI
_c/T_c	Transition strength	Sphaleron suppression	**1.1**	XXXVI
T_n	Nucleation temperature	$\Gamma \times H \sim 1$	135 GeV	XXXVI

10.3 Transition Dynamics

Symbol	Name	Definition	Value	Paper
	Transition strength	$\Delta V/$ _rad	0.05	XXXVI
/H	Inverse duration	T_c d(S /T)/dT	~300	XXXVI
v_w	Wall velocity	Bubble expansion speed	0.4-0.6	XXXVI
S	Bounce action	O(3) Euclidean action	—	XXXVI

10.4 Gravitational Waves

Symbol	Name	Definition	Value	Paper
Ω_GW	GW density	_GW/ _c	$\sim 10^{12}$	XXXVI
f_peak	Peak frequency	—	~2 mHz	XXXVI
h	GW strain	Characteristic amplitude	—	XXIV

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11. Observational Quantities

11.1 Rotation Curves

Symbol	Name	Definition	Units	Paper
v_c(r)	Circular velocity	√(r × dΦ/dr)	km/s	IV
v_bar	Baryonic velocity	From visible mass	km/s	IV
v_3D3D	Q-field contribution	√(v_c² - v_bar²)	km/s	IV
v_∞	Asymptotic velocity	v_c(r → ∞)	km/s	IV

Key prediction:

$$v_{3D3D} = \sqrt{\frac{\beta^2 G_N M}{2\lambda}} \approx 90 \text{ km/s (universal)}$$

11.2 Gravitational Lensing

Symbol	Name	Definition	Paper
Σ_crit	Critical surface density	c²D_s/(4 G D_d D_{ds})	XXXII
	Convergence	Σ/Σ_crit	XXXII
	Shear	Tidal distortion	XXXII
_E	Einstein radius	√(4GM D_{ds}/(c² D_d D_s))	XXXII

11.3 Pulsar Timing

Symbol	Name	Definition	Paper
TOA	Time of arrival	Pulse arrival time	XI
Δt	Timing residual	Observed - predicted TOA	XI
P	Pulsar period	Rotation period	XI
P	Period derivative	Spin-down rate	XI

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12. Mathematical Operators

12.1 Differential Operators

Symbol	Name	Definition
_	Partial derivative	/ x^
_A	Covariant derivative (6D)	_A + Γ connection
_	Covariant derivative (4D)	_ + Γ connection
	d'Alembertian (4D)	g^{ } _ _
_6	d'Alembertian (6D)	g^{AB} _A _B
²	Laplacian (3D)	²/ x² + ²/ y² + ²/ z²
Δ	Laplace-Beltrami	Generalized Laplacian

12.2 Tensor Operations

Symbol	Name	Definition
A_{( )}	Symmetrization	(A_ + A_)/2
A_{[ ]}	Antisymmetrization	(A_ - A_)/2
A^ _	Trace	g^{ } A_
^ _	Kronecker delta	1 if = , 0 otherwise
_{ }	Levi-Civita (4D)	Totally antisymmetric
_{ABCDEF}	Levi-Civita (6D)	Totally antisymmetric

12.3 Special Functions

Symbol	Name	Definition	Paper
J_B(x²)	Bosonic thermal integral	$y^2 \ln(1-e^{-\sqrt{(y^2+x^2)}}) dy$	XXXVI
J_F(x²)	Fermionic thermal integral	$y^2 \ln(1+e^{-\sqrt{(y^2+x^2)}}) dy$	XXXVI
Y_n(x)	Bessel function 2nd kind	Standard definition	IV
J_n(x)	Bessel function 1st kind	Standard definition	IV
K_n(x)	Modified Bessel	Exponentially decaying	IV

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13. Greek Letter Summary

Letter	Lowercase	Primary Usage in 3D+3D
Alpha		Transition strength, coupling coefficients
Beta		Q-matter coupling, transition rate
Gamma		Internal metric, Lorentz factor
Delta		Perturbation, variation
Epsilon		Small parameter, Levi-Civita
Eta		Minkowski metric, baryon asymmetry
Theta		Angular coordinate, QCD -term
Kappa		Gravitational coupling, convergence
Lambda		Characteristic scale, cosmological constant
Mu		4D index, mass parameter
Nu		4D index, frequency
Xi		Q-Higgs coupling
Pi		Self-energy, 3.14159…
Rho		Density, 4D index
Sigma		Brane tension, surface density
Tau		Internal time coordinates
Phi		Potential, Higgs field, golden ratio
Chi		Moduli deformation
Psi		Fermion field, wavefunction
Omega		Density parameter, angular frequency

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14. Latin Letter Summary

Letter	Primary Usage in 3D+3D
A, B, C	6D indices
a	Scale factor, internal index
c	Speed of light, coefficients
G	Newton's constant, gluon field

Letter	Primary Usage in 3D+3D
g	Metric, gauge coupling
H	Hubble parameter, Higgs, Hamiltonian
h	Metric perturbation, Planck constant
L	Lagrangian, circumference
M	Mass
m	Mass (particle), mode number
n	Mode number, number density
P	Power spectrum, pressure
Q	Q-field
R	Ricci scalar, radius
r	Radial coordinate
S	Action, screening function
T	Temperature, period
t	Time
V	Potential, volume
v	Velocity

15. Numerical Constants

15.1 Mathematical Constants

Symbol	Name	Value	Usage
	Pi	3.14159265…	Circular geometry
e	Euler's number	2.71828182…	Exponentials
	Golden ratio	$(1+\sqrt{5})/2 = 1.6180339…$	Scale ladder
_E	Euler-Mascheroni	0.5772156…	Thermal functions

15.2 Key 3D+3D Numbers

Quantity	Value	Significance
	4.30 kpc	Fundamental galactic scale
	11.7 kpc	Second galactic scale
T	**30 years**	First oscillation period
T	**19 years**	Second oscillation period
T /T	**30/19 1.579**	Near-golden ratio
v_3D3D	**90 km/s**	Universal velocity scale
M_crit	**1.8×10^{11} M_**	Dark matter threshold
	3.2 ± 0.8	Matter coupling
	0.3-0.5	Q-Higgs coupling

15.3 Ratio Relationships

Ratio	Value	Origin
/	2.72 ^{2}	Golden ratio squared
T /T	1.579	Near golden ratio
R /R		By construction
/		Harmonic ladder

Appendix A: Unit Conversions

A.1 Length

Unit	In meters	In kpc
1 kpc	3.086×10^{1} m	1
1 Mpc	3.086×10^{22} m	1000
1 ly	9.461×10^{1} m	3.066×10
1 AU	1.496×10^{11} m	4.848×10

A.2 Mass

Unit	In kg	In M_
1 M_	1.989×10^{3} kg	1
1 GeV/c^2	1.783×10^{2} kg	8.96×10
1 M_Pl	2.176×10 kg	1.09×10^{3}

A.3 Time

Unit	In seconds	In years
1 year	3.156×10 s	1
1 Hubble time	4.55×10^{1} s	1.44×10^{1}

A.4 Energy

Unit	In Joules	In eV
1 eV	1.602×10^{1} J	1
1 GeV	1.602×10^{1} J	10

Appendix B: Signature Conventions

B.1 The 6D Signature

The 3D+3D theory uses:

$$\eta_{AB} = \text{diag}(-1, +1, +1, +1, -1, -1)$$

Index assignment: - 0: t (observable time) → − - 1: x (space) → + - 2: y (space) → + - 3: z (space) → + - 4: (compact time) → − - 5: (compact time) → −

B.2 Why Multiple Time Dimensions?

The (−,−) signature for internal dimensions ensures:

1. **Q-fields have standard kinetic terms** (positive kinetic energy)
2. **Compactification projects out ghosts** (Paper XXII)
3. **4D physics remains causal** (observable sector unchanged)

Appendix C: Quick Reference Card

Essential Formulas

Screening function:

$$S(r) = 1 + \beta_2^2 \frac{Q_2^2}{M_{Pl}^2} + \beta_3^2 \frac{Q_3^2}{M_{Pl}^2}$$

Q-field profile:

$$Q_i(r) = \frac{\beta_i G_N M}{r} \left(1 - e^{-r/\lambda_i}\right)$$

Rotation curve:

$$v_c^2(r) = v_{bar}^2(r) + v_{3D3D}^2(r)$$

Universal velocity:

$$v_{3D3D} = \sqrt{\frac{\beta^2 G_N M}{2\lambda}} \approx 90 \text{ km/s}$$

Scale ladder:

$$\lambda_n = \lambda_2 \times \varphi^{n-2}$$

Period ratio:

$$\frac{T_2}{T_3} = \frac{30}{19} \approx 1.579 \approx \varphi$$

Key Parameters (Copy-Paste Ready)

```
  = 4.30 kpc
  = 11.7 kpc
T  = 30 years
T  = 19 years
  = 3.2 ± 0.8
  = 0.3-0.5
v_3D3D = 90 km/s
M_crit = 1.8 × 10¹¹ M_
  = 1.618034
```

— End of Paper XXXVII —

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This glossary is intended as a living document and will be updated as the theory develops.

Contact: condoor76@gmail.com