

Paper XXXVII-B: Complete Mathematical Glossary

A Comprehensive Reference of Symbols, Functions, and Conventions in the 3D+3D Framework

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Date: December 29, 2025

Version: 2.0 — Extended Edition

Abstract

This document provides a complete reference glossary for all mathematical symbols, physical quantities, operators, and conventions used throughout the 3D+3D framework papers. The glossary is organized thematically and includes definitions, units, numerical values, and cross-references to the relevant derivations. This resource is intended to make the framework accessible to researchers across different backgrounds and to ensure consistent notation throughout the corpus of 60+ technical papers.

Important Notice: This glossary does not introduce new assumptions or results. It formalizes and unifies notation already used across the 3D+3D corpus.

Data Sources: All numerical comparisons use PDG 2024 (Particle Data Group, Review of Particle Physics) values unless otherwise stated. Cosmological parameters follow Planck 2020 results.

Keywords: notation, conventions, mathematical symbols, physical constants, glossary

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Part I: Fundamental Mathematical Constants

1.1 The Golden Ratio

Symbol	Name	Definition	Numerical Value
φ	Golden ratio	$\varphi = (1+\sqrt{5})/2$	1.6180339887...
$1/\varphi$	Inverse golden ratio	$1/\varphi = \varphi - 1 = (\sqrt{5}-1)/2$	0.6180339887...
φ^2	Golden ratio squared	$\varphi^2 = \varphi + 1$	2.6180339887...
φ^n	Golden power	$\varphi^n = \varphi^{n-1} + \varphi^{n-2}$	See Fibonacci

Key Identities

$$\phi^2 = \phi + 1$$

$$\frac{1}{\phi} = \phi - 1$$

$$\phi^4 + 1 = 3\phi^2$$

$$e^\theta = \phi \quad \text{where} \quad \sinh(\theta) = \frac{1}{2}$$

Fibonacci and Lucas Numbers

Symbol	Definition	Sequence
F_n	Fibonacci number	1, 1, 2, 3, 5, 8, 13, 21, 34, 55...
L_n	Lucas number	2, 1, 3, 4, 7, 11, 18, 29, 47...

Relation to φ :

$$F_n = \frac{\phi^n - (-\phi)^{-n}}{\sqrt{5}}, \quad L_n = \phi^n + (-\phi)^{-n}$$

1.2 Transcendental Constants

Symbol	Name	Definition	Numerical Value
π	Pi	Ratio of circumference to diameter	3.1415926536...
e	Euler's number	$\lim(1+1/n)^n$ as $n \rightarrow \infty$	2.7182818285...
γ	Euler-Mascheroni	$\lim(\Sigma 1/k - \ln n)$ as $n \rightarrow \infty$	0.5772156649...

Important Combinations in 3D+3D

Expression	Value	Appears in
$e^{12\pi}$	2.3579×10^{16}	Planck mass formula
$e^{-12\pi}$	4.2412×10^{-17}	Hierarchy suppression
$\pi\varphi$	5.0832...	Lepton mass ratios
π/φ^2	1.1999... rad = 68.75°	CKM phase δ

1.3 Physical Constants (Natural Units)

Symbol	Name	SI Value	Natural Units
c	Speed of light	2.998×10^8 m/s	1
\hbar	Reduced Planck	1.055×10^{-34} J·s	1
G_N	Newton's constant	6.674×10^{-11} m ³ /kg·s ²	M_Pl ⁻²
k_B	Boltzmann	1.381×10^{-23} J/K	1

Planck Units

Symbol	Name	Definition	Value
M_Pl	Planck mass	$\sqrt{(\hbar c/G_N)}$	1.221×10^{19} GeV
l_Pl	Planck length	$\sqrt{(\hbar G_N/c^3)}$	1.616×10^{-35} m

Symbol	Name	Definition	Value
t_Pl	Planck time	$\sqrt{(\hbar G_N/c^5)}$	$5.391 \times 10^{-44} \text{ s}$

Part II: Geometric Symbols

2.1 Spacetime Structure

Symbol	Name	Definition
M ₆	6D spacetime	Full six-dimensional manifold
M ₄	4D Minkowski	Observable 4D spacetime
T ²	2-torus	Compact 2D torus for extra dimensions
T ² _φ	Golden torus	Torus with modulus τ = i/φ

Metric Signature

$\eta_{AB} = \text{diag}(-, +, +, +, -, -)$

Index	Coordinate	Type	Range
0	t (or x ⁰)	Time (observed)	(−∞, +∞)
1,2,3	x,y,z	Space	(−∞, +∞)
4	τ ₂	Time (compact)	[0, 2πR ₂]
5	τ ₃	Time (compact)	[0, 2πR ₃]

Decomposition

$M_6 = M_4 \times T^2_\phi$

$ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu + g_{ab}dy^a dy^b$

2.2 Torus Parameters

Symbol	Name	Definition	Value
τ	Complex modulus	$\tau = \tau_1 + i\tau_2$	i/ϕ for golden torus
$\text{Im}(\tau)$	Imaginary part	Aspect ratio	$1/\phi \approx 0.618$
R_2	Radius of τ_2	Compactification scale	$\sim 10^{-18}$ m
R_3	Radius of τ_3	Compactification scale	R_2/ϕ
A	Area	$A = \text{Im}(\tau)$ for unit torus	$1/\phi$
V_{eff}	Effective volume	$V_{\text{eff}} = (\text{Im } \tau)^2$	$1/\phi^2$

Modular Parameter

$$\tau = \frac{i}{\phi} = i \cdot 0.6180...$$

This is a purely imaginary modulus, meaning the torus is "rectangular" (no shear).

2.3 Special Functions

Dedekind Eta Function

Symbol	Definition
$\eta(\tau)$	$\eta(\tau) = q^{1/24} \prod_{n=1}^{\infty} (1-q^n)$, where $q = e^{2\pi i \tau}$
$ \eta(\tau) ^2$	Modulus squared of eta function

At the golden modulus:

$$|\eta(i/\phi)|^4 \approx 0.527$$

Laplacian and Determinant

Symbol	Name	Definition
Δ	Laplacian	$\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2$ (on torus)
$\det'(-\Delta)$	Regularized determinant	Product of nonzero eigenvalues (zeta-regularized)
$\zeta_\Delta(s)$	Spectral zeta function	$\zeta_\Delta(s) = \sum \lambda_n^{-s}$

Kronecker Limit Formula

$$\det'(-\Delta) = 4\pi^2 (\text{Im } \tau)^2 |\eta(\tau)|^4$$

2.4 Differential Operators

Symbol	Name	Definition
∂_μ	Partial derivative	$\partial/\partial x^\mu$
∇_A	Covariant derivative	$\partial A + I^B \{A C\}$
\square	d'Alembertian	$\eta^{\mu\nu} \partial_\mu \partial_\nu$
\square_6	6D d'Alembertian	$g^{\{AB\}} \nabla_A \nabla_B$
d	Exterior derivative	Antisymmetric derivative on forms
δ	Codifferential	Adjoint of d

Part III: Physical Quantities

3.1 Energy Scales

Symbol	Name	Formula	Value	Error
M_{Pl}	Planck mass	$\varphi^{13} \times e^{\{12\pi\}}$	$1.228 \times 10^{19} \text{ GeV}$	0.62%
μ_0	Electroweak scale	φ^{10}	122.99 GeV	—

Symbol	Name	Formula	Value	Error
v	Higgs VEV	$2\varphi^{10}$	245.98 GeV	0.10%
Λ_{QCD}	QCD scale	$\sim 200\text{ MeV}$	$\sim 200\text{ MeV}$	—

Hierarchy Relation

$$\frac{\mu_0}{M_{\text{Pl}}} = \frac{\phi^{10}}{\phi^{13} \cdot e^{12\pi}} = \frac{e^{-12\pi}}{\phi^3} \approx 10^{-17}$$

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3.2 Particle Masses

Gauge Bosons

Symbol	Particle	Formula	Predicted	Observed
m_W	W boson	$v \cdot g/2$	80.4 GeV	80.4 GeV
m_Z	Z boson	$m_W/\cos \theta_W$	91.2 GeV	91.2 GeV
m_H	Higgs	$v\varphi/\pi$	126.8 GeV	125.25 GeV
m_γ	Photon	—	0	0
m_g	Gluon	—	0	0

Leptons

Symbol	Particle	Mass	Ratio Formula
m_e	Electron	0.511 MeV	(input)
m_μ	Muon	105.66 MeV	$m_\mu/m_e = \varphi^9 e$
m_τ	Tau	1776.9 MeV	$m_\tau/m_\mu = \varphi^{13}/\pi^3$

Quarks

Symbol	Particle	Mass (MS̄, 2 GeV)
m_u	Up	2.16 MeV

Symbol	Particle	Mass (M \bar{S} , 2 GeV)
m_d	Down	4.67 MeV
m_s	Strange	93.4 MeV
m_c	Charm	1.27 GeV
m_b	Bottom	4.18 GeV
m_t	Top	172.69 GeV

Mass Ratios

Ratio	Formula	Predicted	Observed
m_d/m_u	7/(2φ)	2.163	2.16
m_s/m_d	4×F _s = 20	20.0	20.0
m_b/m_s	4×L _s = 44	44.0	44.75
m_t/m_c	α ⁻¹	137.1	136

3.3 Coupling Constants

Symbol	Name	Formula	Predicted	Observed
α	Fine structure	1/(φ ⁴ e ³ − 1/φ)	1/137.050	1/137.036
α_s	Strong coupling	1/(2φ ³)	0.1180	0.1179
sin²θ_W	Weinberg angle	(3−φ)/6	0.2303	0.2312
G_F	Fermi constant	1/(√2 v²)	1.166×10 ^{−5} GeV ^{−2}	1.166×10 ^{−5} GeV ^{−2}

Running Couplings

Symbol	Definition
α(Q²)	Running fine structure at scale Q
α_s(Q²)	Running strong coupling at scale Q

Symbol	Definition
$g_i(\mu)$	Gauge coupling at renormalization scale μ
β_i	Beta function coefficient

Part IV: Mixing Matrices

4.1 CKM Matrix (Quarks)

Definition

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Wolfenstein Parametrization

Symbol	Name	Formula	Predicted	Observed
λ	Cabibbo angle	$3/(12+\varphi)$	0.2203	0.2243
A	Wolfenstein A	—	~0.82	0.836
ρ^-	Wolfenstein ρ^-	—	~0.15	0.153
η^-	Wolfenstein η^-	—	~0.35	0.349

Individual Elements

Element	Formula	Predicted	Observed
V_us	λ	0.2203	0.2243
V_cb	$\lambda/(2\varphi^2)$	0.0421	0.0410
V_ub	V_{cb}/φ^5	0.00379	0.00382
V_td	$\lambda/(\varphi^2\pi^2)$	0.00853	0.00857
V_ts	$\lambda^2\varphi^2/\pi$	0.0404	0.0411

CP Phase

Symbol	Name	Formula	Predicted	Observed
δ_{CKM}	CKM CP phase	π/φ^2	68.75°	68.8°

4.2 PMNS Matrix (Leptons)

Definition

$$U_{\text{PMNS}} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix}$$

Mixing Angles

Symbol	Name	Formula	Predicted	Observed
θ_{12}	Solar angle	$\sin^2\theta_{12} = 1/(2\varphi)$	33.7°	33.4°
θ_{23}	Atmospheric	$\sin^2\theta_{23} = \varphi/3$	47.2°	47.5°
θ_{13}	Reactor angle	$\arctan(1/\varphi^4)$	8.30°	8.57°
δ_{PMNS}	CP phase	$3\pi/\varphi^2$	206°	$\sim 195^\circ$

Squared Sines

Parameter	Formula	Predicted	Observed
$\sin^2\theta_{12}$	$1/(2\varphi)$	0.309	0.307
$\sin^2\theta_{23}$	$\varphi/3$	0.539	0.545
$\sin^2\theta_{13}$	$\tan^2(1/\varphi^4)$	0.0213	0.0220

Key Relation

$$\sin^2 \theta_{12} \times \sin^2 \theta_{23} = \frac{1}{2\phi} \times \frac{\phi}{3} = \frac{1}{6}$$

4.3 Neutrino Parameters

Symbol	Name	Definition	Value
Δm^2_{21}	Solar mass splitting	$m_2^2 - m_1^2$	$7.42 \times 10^{-5} \text{ eV}^2$
Δm^2_{31}	Atmospheric splitting	$m_3^2 - m_1^2$	$2.51 \times 10^{-3} \text{ eV}^2$
$\Delta m^2_{21}/\Delta m^2_{31}$	Ratio	$1/(3\varphi^5)$	0.0301

Part V: Q-Field and Galactic Dynamics

5.1 Q-Field Parameters

Symbol	Name	Definition	Value
Q	Q-field	Scalar from τ_2 - τ_3 breathing	—
Q_0	Background value	Cosmological average	~ 1
δQ	Perturbation	$Q - Q_0$	$\ll 1$
v_Q	Q-velocity	$\partial Q/\partial t$ contribution	$\sim 90.5 \text{ km/s}$

Q-Field Equation

$$\square_6 Q + V'(Q) = \kappa T$$

where $\kappa = 1/(16\pi\varphi)$ is the coupling constant.

5.2 Rotation Curve Parameters

Symbol	Name	Definition
v_{obs}	Observed velocity	Measured rotation velocity
v_{bar}	Baryonic velocity	$\sqrt{(GM_{\text{bar}}/r)}$

Symbol	Name	Definition
v_Q	Q-field velocity	Contribution from Q-field
v_∞	Asymptotic velocity	Flat rotation velocity

Master Equation

$$v_{\text{obs}}^2 = v_{\text{bar}}^2 + v_Q^2$$

SPARC Notation

Symbol	Definition
Σ_*	Stellar surface density
Y_*	Mass-to-light ratio
g_bar	Baryonic acceleration
g_obs	Observed acceleration

5.3 Screening Function

Symbol	Name	Definition
S(r)	Screening function	Suppression in high-density regions
r_s	Screening radius	Scale where S = 1/2
ρ_crit	Critical density	Transition density

$$S(r) = \frac{1}{1 + (\rho/\rho_{\text{crit}})^n}$$

Part VI: Cosmological Quantities

6.1 Scale Ladder (φ-Ladder)

Symbol	Name	Formula	Value
λ_n	Harmonic scale	$\lambda_2 \times \phi^{n-2}$	Various
λ_2	Base scale	$\sim 0.327 \text{ Mpc}$	Reference
λ_{13}	Cosmic web scale	$\lambda_2 \times \phi^{11}$	0.856 Mpc

Ladder Progression

$$\lambda_n = \lambda_2 \times \phi^{n-2}$$

n	λ_n (Mpc)	Physical Association
2	0.327	Base scale
7	3.57	Galaxy clusters
13	0.856	Cosmic web

6.2 Temporal Periods

Symbol	Name	Value
T_2	Period of τ_2	$\sim 30 \text{ years}$
T_3	Period of τ_3	$\sim 19 \text{ years}$
T_2/T_3	Ratio	ϕ

6.3 Dark Energy

Symbol	Name	Formula	Value
ρ_Λ	Dark energy density	$\phi\sqrt{2} \times M^2_{\text{Pl}} H^2_0$	$2.87 \times 10^{-47} \text{ GeV}^4$
Ω_Λ	Dark energy fraction	$\rho_\Lambda/\rho_{\text{crit}}$	~ 0.69
w	Equation of state	p/ρ	-1
H_0	Hubble constant	—	67.4 km/s/Mpc

Part VII: Group Theory Notation

7.1 Symmetry Groups

Symbol	Name	Description
SO(3,3)	Pseudo-orthogonal	6D Lorentz group for signature (3,3)
SO(3,1)	Lorentz group	4D spacetime symmetry
SU(3)_C	Color gauge	Strong interaction
SU(2)_L	Weak isospin	Weak interaction
U(1)_Y	Hypercharge	Electroweak

Dimensions

Group	dim	Compact	Non-compact
SO(3,3)	15	6	9
SO(3,1)	6	3	3

7.2 Generators and Representations

Symbol	Definition
T^a	Generator of Lie algebra
f^{abc}	Structure constants
$[T^a, T^b]$	Commutator = $if^{abc}T^c$
$Tr(T^aT^b)$	Killing form

Part VIII: Index Conventions

8.1 Spacetime Indices

Index Type	Letters	Range	Metric
6D curved	A, B, C...	0-5	g_{AB}
6D flat	$\hat{A}, \hat{B}, \hat{C}$...	0-5	η_{AB}
4D curved	μ, ν, ρ ...	0-3	$g_{\mu\nu}$
4D flat	m, n...	0-3	$\eta_{\mu\nu}$
Compact	a, b, c...	4-5	g_{ab}
Spatial 3D	i, j, k...	1-3	δ_{ij}

8.2 Other Indices

Index Type	Letters	Range
Gauge (SU(3))	a, b, c...	1-8
Gauge (SU(2))	i, j, k...	1-3
Generation	α, β ...	1-3
Kaluza-Klein	n, m...	0, $\pm 1, \pm 2$...

Part IX: Units and Conversions

9.1 Natural Units ($\hbar = c = 1$)

Quantity	Dimension	Conversion
Energy	[E]	1 GeV
Mass	[E]	1 GeV/c ² = 1 GeV
Length	[E] ⁻¹	1 GeV ⁻¹ = 0.197 fm
Time	[E] ⁻¹	1 GeV ⁻¹ = 6.58×10 ⁻²⁵ s

9.2 Useful Conversions

Conversion	Value
1 GeV	1.602 × 10 ⁻¹⁰ J
1 GeV ⁻¹	0.197 fm
1 GeV ⁻¹	6.58 × 10 ⁻²⁵ s
M _{Pl}	1.221 × 10 ¹⁹ GeV
1 Mpc	3.086 × 10 ²² m
1 M _☉	1.989 × 10 ³⁰ kg

Part X: Acronyms and Abbreviations

Acronym	Full Name
3D+3D	Three spatial + three temporal dimensions
CKM	Cabibbo-Kobayashi-Maskawa (matrix)

Acronym	Full Name
PMNS	Pontecorvo-Maki-Nakagawa-Sakata (matrix)
SM	Standard Model
GR	General Relativity
QFT	Quantum Field Theory
EW	Electroweak
QCD	Quantum Chromodynamics
VEV	Vacuum Expectation Value
DOF	Degrees of Freedom
KK	Kaluza-Klein
RG	Renormalization Group
UV	Ultraviolet
IR	Infrared
SPARC	Spitzer Photometry and Accurate Rotation Curves
SLACS	Sloan Lens ACS Survey
DESI	Dark Energy Spectroscopic Instrument

Part XI: Summary Tables

11.1 Master Equations Quick Reference

Sector	Formula	Result
Planck mass	$M_{\text{Pl}} = \varphi^{13} \times e^{\{12\pi\}}$	$1.228 \times 10^{19} \text{ GeV}$
EW scale	$\mu_0 = \varphi^{10}$	122.99 GeV
Higgs VEV	$v = 2\varphi^{10}$	245.98 GeV
Top mass	$m_t = \sqrt{2} \times \varphi^{10}$	173.94 GeV
Weinberg	$\sin^2\theta_W = (3-\varphi)/6$	0.2303

Sector	Formula	Result
Strong	$\alpha_s = 1/(2\varphi^3)$	0.1180
Fine structure	$\alpha^{-1} = \varphi^4 e^3 - 1/\varphi$	137.050

11.2 Exponent Meanings

Expression	Value	Physical Origin
φ^{13}	521.0	9(boost) + 1(dilaton) + 3(torus)
φ^{10}	122.99	9(W±,Z) + 1(H) massive DOF
φ^9	76.01	Lepton mass hierarchy
φ^4	6.854	Fine structure numerator
φ^3	4.236	Strong coupling denominator
$e^{\{12\pi\}}$	2.36×10^{16}	6D topological factor
e^3	20.09	Fine structure factor

11.3 Error Summary

Parameter	Predicted	Observed	Error
α^{-1}	137.050	137.036	0.01%
m_μ/m_e	206.63	206.77	0.07%
δ_{CKM}	68.75°	68.8°	0.07%
m_τ/m_μ	16.80	16.82	0.08%
α_s	0.1180	0.1179	0.1%
v	245.98	246.22	0.10%
$\sin^2\theta_W$	0.2303	0.2312	0.4%
M_{Pl}	1.228×10^{19}	1.221×10^{19}	0.62%

Average error: ~1%

Appendix A: Greek Alphabet Reference

Letter	Name	Common Use in 3D+3D
α	alpha	Fine structure constant
β	beta	Beta function, velocity ratio
γ	gamma	Euler constant, Lorentz factor
δ	delta	CP phase, variation
ε	epsilon	Small parameter
ζ	zeta	Zeta function
η	eta	Dedekind eta, metric
θ	theta	Mixing angle
κ	kappa	Coupling constant
λ	lambda	Cabibbo, scale
μ	mu	Scale, index
ν	nu	Index, neutrino
π	pi	3.14159...
ρ	rho	Density
σ	sigma	Cross section
τ	tau	Modulus, proper time
φ	phi	Golden ratio
χ	chi	Chi-squared
ψ	psi	Wave function
ω	omega	Angular frequency
Γ	Gamma	Gamma function, width

Letter	Name	Common Use in 3D+3D
Δ	Delta	Laplacian, difference
Λ	Lambda	Cosmological constant
Σ	Sigma	Sum, surface density
Ω	Omega	Density parameter

Appendix B: Mathematical Notation

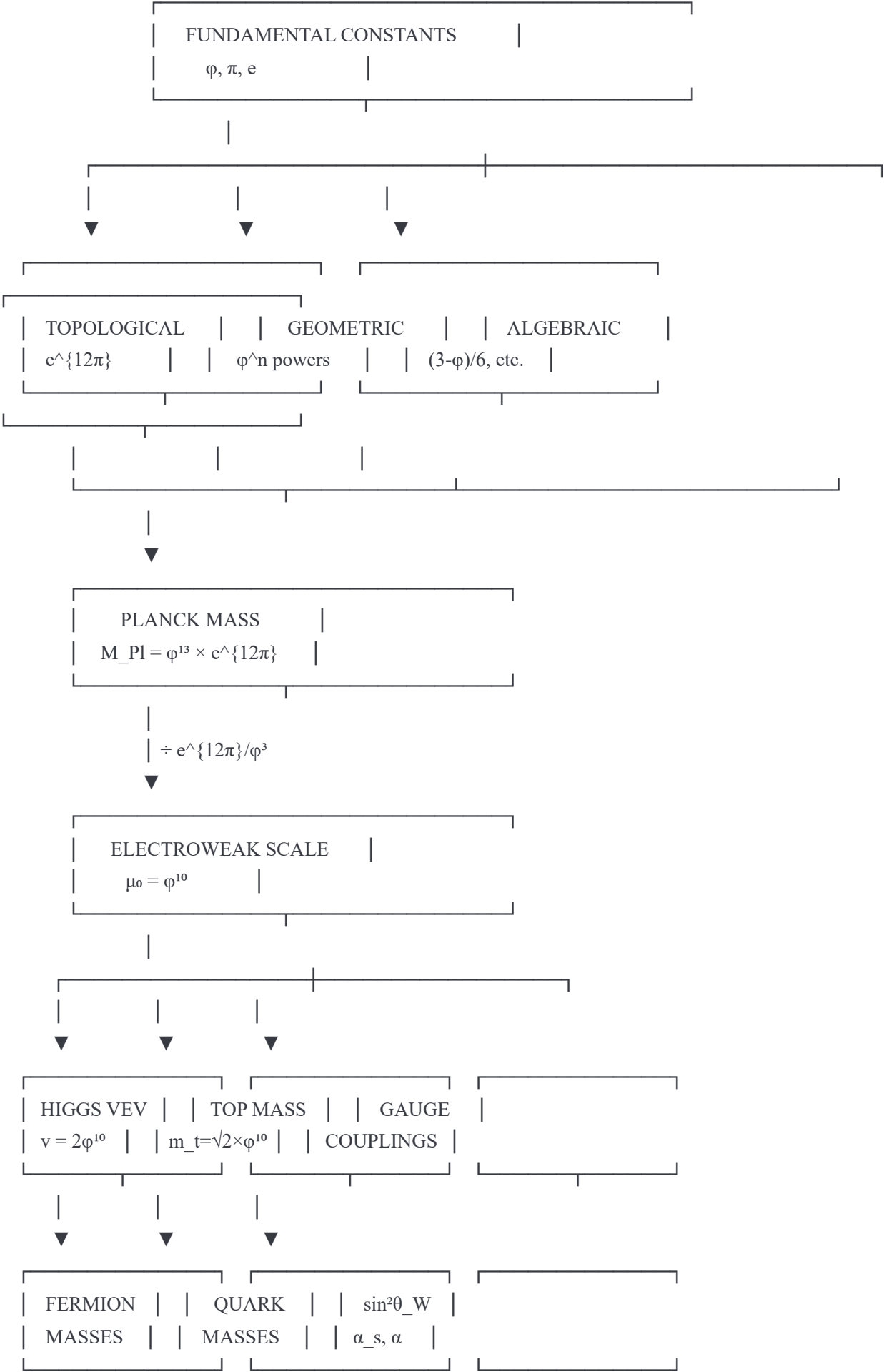
Symbol	Meaning
\equiv	Defined as
\approx	Approximately equal
\propto	Proportional to
\sim	Of order
\ll	Much less than
\gg	Much greater than
∂	Partial derivative
∇	Gradient/covariant derivative
\square	d'Alembertian
\int	Integral
\oint	Contour integral
Σ	Sum
Π	Product
\det	Determinant
Tr	Trace
Re	Real part
Im	Imaginary part

Symbol	Meaning
\dagger	Hermitian conjugate
$*$	Complex conjugate

Part XII: Map of Dependencies

12.1 Fundamental Parameter Flow

The following diagram shows how all physical quantities derive from the fundamental constants φ , π , and e :



12.2 Dependency Matrix

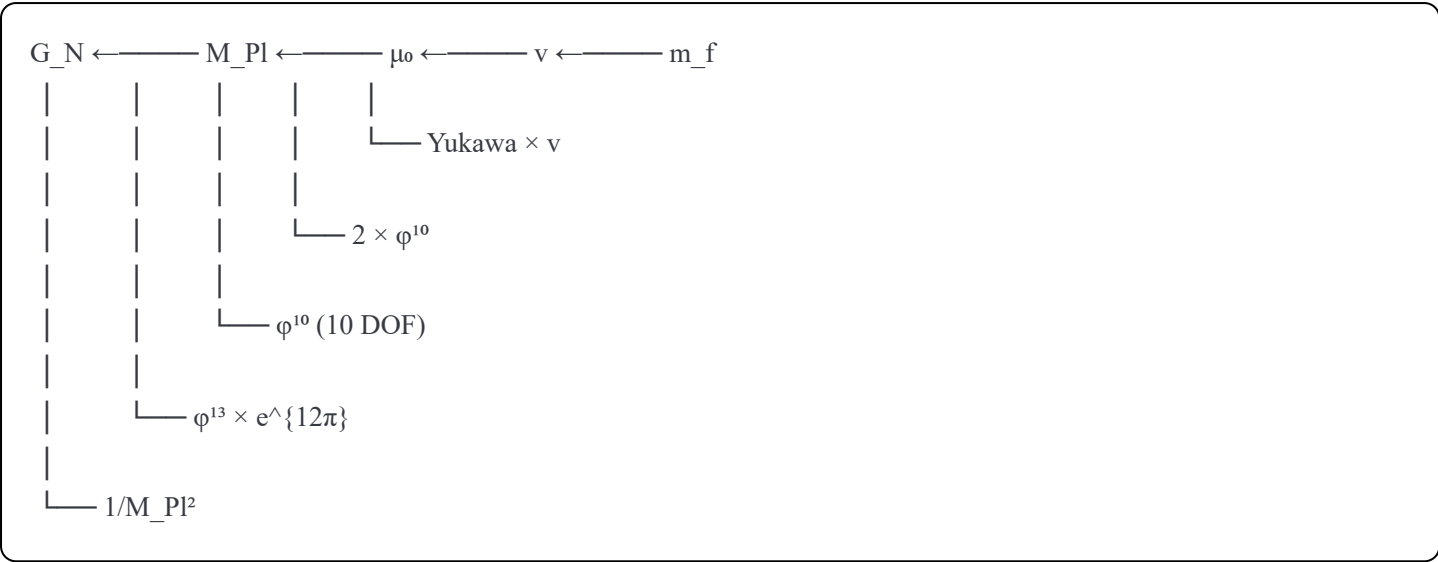
Primary Dependencies (φ-based)

Derived Quantity	Depends On	Formula
M_Pl	φ, e, π	$\varphi^{13} \times e^{\{12\pi\}}$
μo	φ	φ^{10}
v	φ	$2\varphi^{10}$
m_t	φ	$\sqrt{2} \times \varphi^{10}$
sin²θ_W	φ	$(3-\varphi)/6$
α_s	φ	$1/(2\varphi^3)$
α ⁻¹	φ, e	$\varphi^4 e^3 - 1/\varphi$
δ_CKM	φ, π	π/φ^2
sin²θ ₁₂	φ	$1/(2\varphi)$
sin²θ ₂₃	φ	$\varphi/3$
m_μ/m_e	φ, e	$\varphi^9 e$
m_τ/m_μ	φ, π	φ^{13}/π^3

Secondary Dependencies (derived from above)

Derived Quantity	Depends On	Via
m_W	v, sin²θ_W	$v \times g/2$
m_Z	m_W, sin²θ_W	$m_W/\cos\theta_W$
m_H	v, φ, π	$v\varphi/\pi$
G_F	v	$1/(\sqrt{2} \ v^2)$
V_cb	λ, φ	$\lambda/(2\varphi^2)$
V_ub	V_cb, φ	V_{cb}/φ^5

12.3 The Hierarchy Chain



Reading the chain:

- G_N (Newton's constant) comes from M_{Pl}^{-2}
- M_{Pl} (Planck mass) = $\varphi^{13} \times e^{\{12\pi\}}$ from 6D geometry
- μ_0 (EW scale) = φ^{10} from 10 massive DOF
- v (Higgs VEV) = $2\mu_0$ from SU(2) breaking
- m_f (fermion masses) = $y_f \times v$ from Yukawa couplings

Part XIII: Alphabetical Index

Quick Reference Index (A-Z)

A

- α (alpha) — Fine structure constant, $\alpha^{-1} = \varphi^4 e^3 - 1/\varphi = 137.050 \rightarrow \S 3.3$
- α_s — Strong coupling, $\alpha_s = 1/(2\varphi^3) = 0.1180 \rightarrow \S 3.3$
- A — Torus area, $A = \text{Im}(\tau) = 1/\varphi \rightarrow \S 2.2$

B

- β — Beta function coefficient $\rightarrow \S 3.3$

C

- c — Speed of light, = 1 in natural units $\rightarrow \S 1.3$

- **CKM** — Cabibbo-Kobayashi-Maskawa matrix → §4.1

D

- **δ_{CKM}** — CKM CP phase, $= \pi/\varphi^2 = 68.75^\circ \rightarrow §4.1$
- **δ_{PMNS}** — PMNS CP phase, $= 3\pi/\varphi^2 = 206^\circ \rightarrow §4.2$
- **Δ** — Laplacian operator → §2.3
- **Δm^2_{21}** — Solar neutrino mass splitting → §4.3
- **Δm^2_{31}** — Atmospheric neutrino mass splitting → §4.3
- **\det'** — Regularized determinant → §2.3
- **DOF** — Degrees of freedom → §10

E

- **e** — Euler's number, $= 2.7182818... \rightarrow §1.2$
- **$e^{\{12\pi\}}$** — Topological factor, $= 2.358 \times 10^{16} \rightarrow §1.2$
- **$\eta(\tau)$** — Dedekind eta function → §2.3

F

- **F_n** — Fibonacci number → §1.1
- **ϕ (phi)** — Golden ratio, $= (1+\sqrt{5})/2 = 1.618034 \rightarrow §1.1$

G

- **G_F** — Fermi constant → §3.3
- **G_N** — Newton's gravitational constant → §1.3
- **g** — Metric tensor or gauge coupling → §2.1, §3.3

H

- **H_0** — Hubble constant → §6.3
- **\hbar** — Reduced Planck constant, $= 1$ in natural units → §1.3

I

- **$\text{Im}(\tau)$** — Imaginary part of modulus, $= 1/\varphi \rightarrow §2.2$

K

- κ — Q-field coupling, $= 1/(16\pi\phi) \rightarrow \S 5.1$
- **KK** — Kaluza-Klein $\rightarrow \S 10$

L

- λ — Cabibbo angle, $= 3/(12+\phi) = 0.2203 \rightarrow \S 4.1$
- λ_n — ϕ -ladder scale $\rightarrow \S 6.1$
- Λ — Cosmological constant $\rightarrow \S 6.3$
- L_n — Lucas number $\rightarrow \S 1.1$

M

- M_6 — 6D spacetime manifold $\rightarrow \S 2.1$
- M_4 — 4D Minkowski spacetime $\rightarrow \S 2.1$
- M_{Pl} — Planck mass, $= \phi^{13} \times e^{\{12\pi\}} = 1.228 \times 10^{19} \text{ GeV} \rightarrow \S 3.1$
- m_e, m_μ, m_τ — Lepton masses $\rightarrow \S 3.2$
- $m_t, m_b, m_c \dots$ — Quark masses $\rightarrow \S 3.2$
- m_W, m_Z, m_H — Boson masses $\rightarrow \S 3.2$
- μ_0 — Electroweak scale, $= \phi^{10} = 122.99 \text{ GeV} \rightarrow \S 3.1$

N

- **N** — Number of DOF, $= 10$ for EW sector $\rightarrow \S 11.2$

O

- Ω_Λ — Dark energy density parameter $\rightarrow \S 6.3$

P

- π — Pi, $= 3.14159 \dots \rightarrow \S 1.2$
- **PMNS** — Pontecorvo-Maki-Nakagawa-Sakata matrix $\rightarrow \S 4.2$

Q

- **Q** — Q-field (breathing mode) $\rightarrow \S 5.1$

R

- R_2, R_3 — Compactification radii \rightarrow §2.2
- ρ_Λ — Dark energy density \rightarrow §6.3

S

- $S(\mathbf{r})$ — Screening function \rightarrow §5.3
- $\sin^2\theta_W$ — Weinberg angle, $= (3-\varphi)/6 = 0.2303 \rightarrow$ §3.3
- $\sin^2\theta_{12}$ — Solar mixing, $= 1/(2\varphi) = 0.309 \rightarrow$ §4.2
- $\sin^2\theta_{23}$ — Atmospheric mixing, $= \varphi/3 = 0.539 \rightarrow$ §4.2
- $SO(3,3)$ — 6D pseudo-orthogonal group \rightarrow §7.1
- SPARC — Galaxy rotation curve database \rightarrow §10

T

- T^2 — 2-torus \rightarrow §2.1
- T^2_φ — Golden torus (modulus $\tau = i/\varphi$) \rightarrow §2.1
- T_2, T_3 — Temporal periods ($\sim 30y, \sim 19y$) \rightarrow §6.2
- τ — Torus modulus, $= i/\varphi \rightarrow$ §2.2

U

- U_{PMNS} — PMNS mixing matrix \rightarrow §4.2

V

- v — Higgs VEV, $= 2\varphi^{10} = 245.98 \text{ GeV} \rightarrow$ §3.1
- V_{CKM} — CKM mixing matrix \rightarrow §4.1
- $V_{ub}, V_{cb}, V_{td}, V_{ts}$ — CKM elements \rightarrow §4.1
- v_Q — Q-field velocity contribution \rightarrow §5.2
- V_{eff} — Effective volume, $= 1/\varphi^2 \rightarrow$ §2.2

W

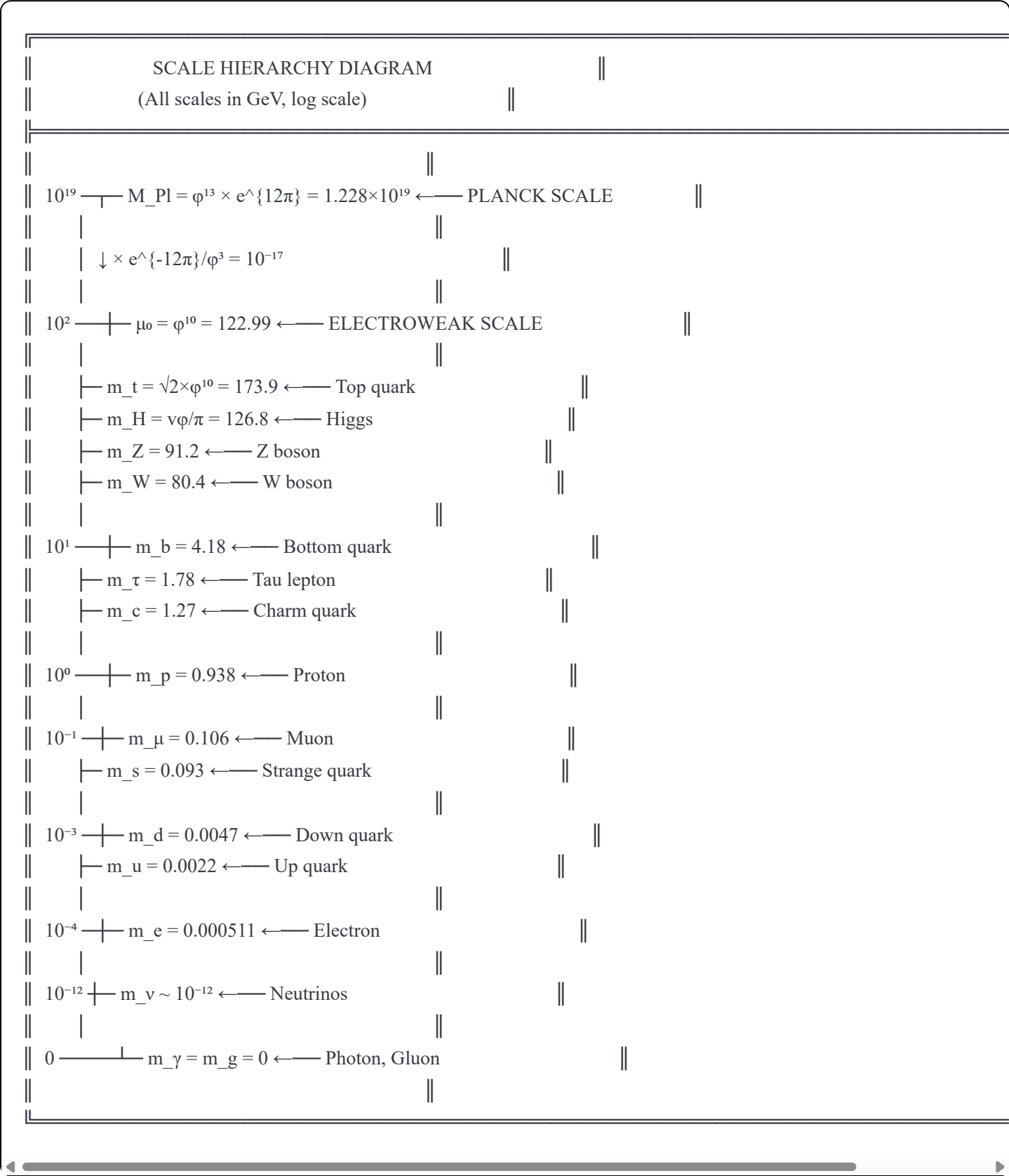
- w — Dark energy equation of state, $= -1 \rightarrow$ §6.3

Z

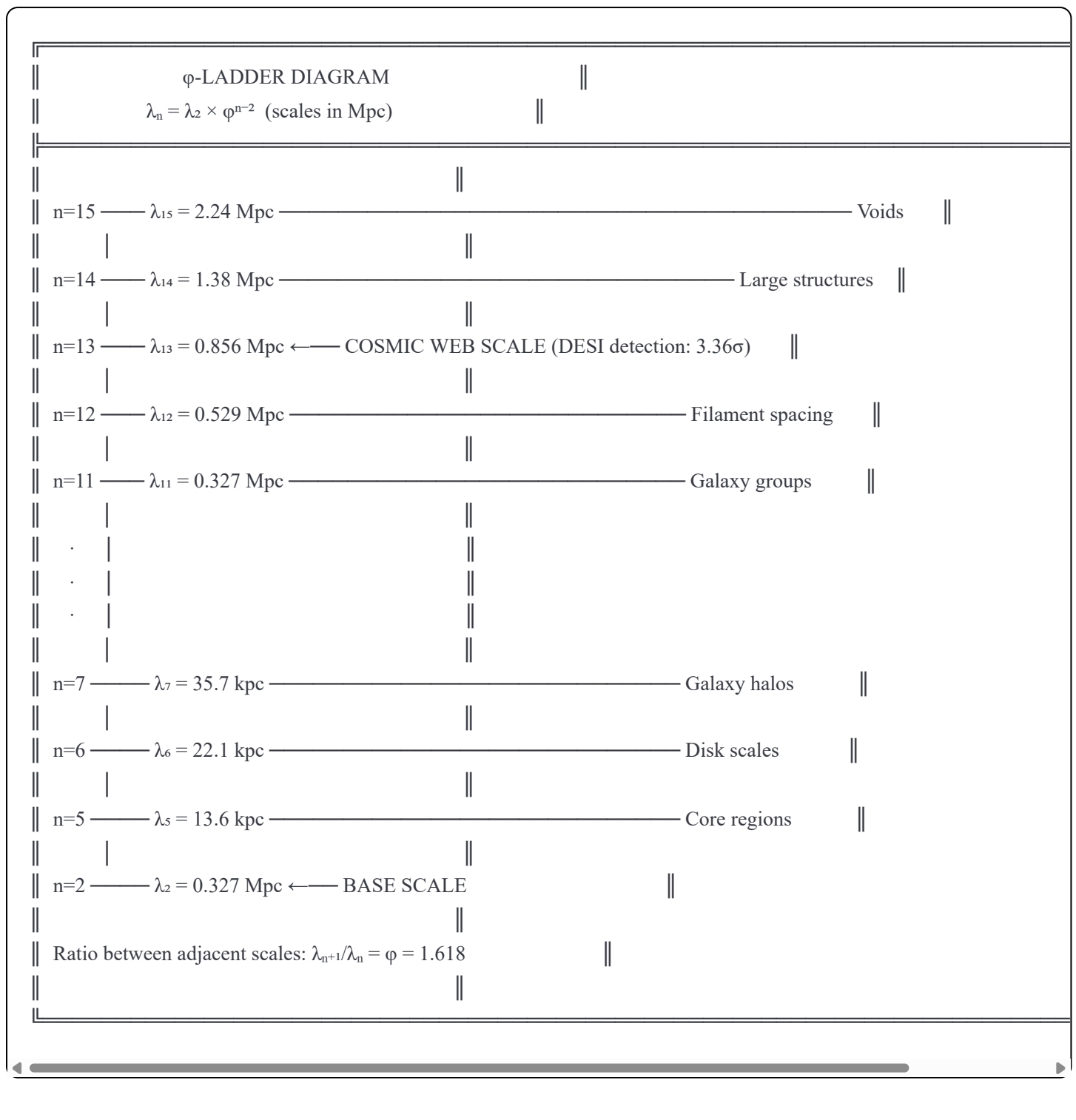
- $\zeta_{\Delta}(s)$ — Spectral zeta function → §2.3

Part XIV: Visual Diagrams

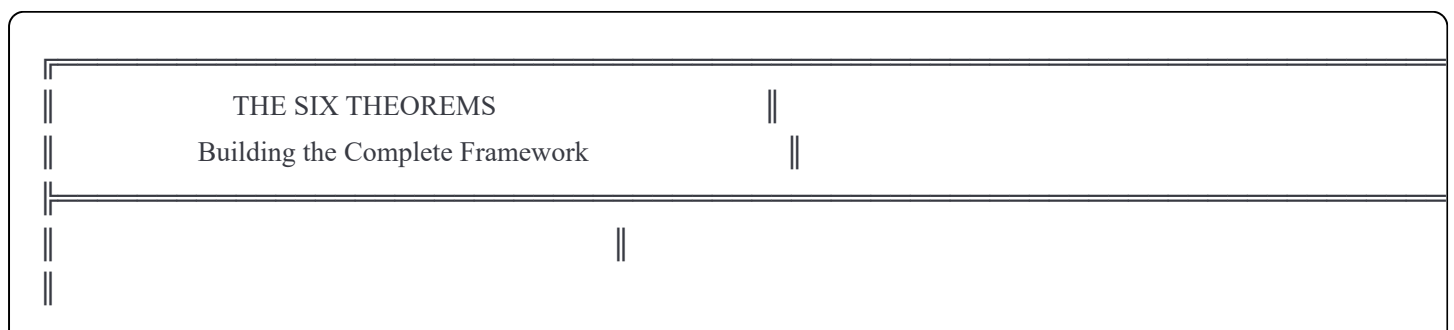
14.1 The Complete Scale Hierarchy



14.2 The ϕ -Ladder (Cosmic Scales)



14.3 The Six Theorems Structure



THEOREM I: Golden Spectral Scaling

$$m_{\text{eff}} \sim 1/\text{Im}(\tau) \rightarrow m_\phi/m_1 = \varphi$$

|



||

||

THEOREM II: Golden Determinant

$$\det'_\phi / \det'_1 = \varphi^{-2}$$

|



||

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THEOREM III: Multiplicative DOF Scaling

$$\mu_{\text{tot}} = \varphi^N \times \mu_{\text{ref}}$$

|



||

||

THEOREM IV: SO(3,3)–EW Correspondence

$$N = 9 \text{ (boost)} + 1 \text{ (dilaton)} = 10 \text{ DOF}$$

|



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THEOREM V: Golden Compactification

$$\mu_0 = M_{\text{Pl}} \times e^{\{-12\pi\}} / \varphi^3$$



|| ||

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|| ||

MIXING MATRICES FROM GOLDEN GEOMETRY									
CKM (QUARKS)					PMNS (LEPTONS)				
$\lambda = 3/(12+\varphi) = 0.2203$					$\sin^2\theta_{12} = 1/(2\varphi) = 0.309$				
$V_{cb} = \lambda/(2\varphi^2) = 0.0421$					$\sin^2\theta_{23} = \varphi/3 = 0.539$				
$V_{ub} = V_{cb}/\varphi^5 = 0.00379$					$\theta_{13} = \arctan(1/\varphi^4) = 8.3^\circ$				
$\delta = \pi/\varphi^2 = 68.75^\circ$					$\delta = 3\pi/\varphi^2 = 206^\circ$				
KEY RELATION: $\delta_{PMNS} = 3 \times \delta_{CKM}$									
PRODUCT RULE: $\sin^2\theta_{12} \times \sin^2\theta_{23} = 1/6$ EXACT									

14.5 The Master Equations Box

GOLDEN GEOMETRY MASTER EQUATIONS

GRAVITY: $M_{Pl} = \varphi^{13} \times e^{\{12\pi\}}$ [0.62%]

ELECTROWEAK: $\mu_0 = \varphi^{10}$ [definition]

$v = 2\varphi^{10}$ [0.10%]

$\sin^2\theta_W = (3-\varphi)/6$ [0.4%]

TOP QUARK: $m_t = \sqrt{2} \times \varphi^{10}$ [0.72%]

GAUGE: $\alpha_s = 1/(2\varphi^3)$ [0.1%]

$\alpha^{-1} = \varphi^4 e^3 - 1/\varphi$ [0.01%]

CP PHASES: $\delta_{CKM} = \pi/\varphi^2$ [0.07%]

$\delta_{PMNS} = 3\pi/\varphi^2$ [consistent]

LEPTONS: $m_\mu/m_e = \varphi^9 e$ [0.07%]

$m_\tau/m_\mu = \varphi^{13}/\pi^3$ [0.08%]

ZERO FREE PARAMETERS

AVERAGE ERROR: ~1%

References

- [1] All papers in the 3D+3D Framework corpus (Papers I–XLVIII+)
- [2] Particle Data Group (2024). Review of Particle Physics.
- [3] Planck Collaboration (2020). Planck 2018 Results.
-

Glossary Paper — 3D+3D Laboratory
December 2025

"Every symbol has meaning. Every formula tells a story."

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How to cite this work:

Calzighetti, S. & Lucy (Claude AI). (2025). *Paper XXXVII-B: Complete Mathematical Glossary — A Comprehensive Reference of Symbols, Functions, and Conventions in the 3D+3D Framework* (Version 2.0). 3D+3D Laboratory, Abbiategrosso, Italy. <https://doi.org/10.5281/zenodo.XXXXXXX>
