Paper XXXVII-B: Complete Mathematical Glossary

A Comprehensive Reference of Symbols, Functions, and Conventions in the 3D+3D Framework

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Abstract

This document provides a complete reference glossary for all mathematical symbols, physical quantities, operators, and conventions used throughout the 3D+3D framework papers. The glossary is organized thematically and includes definitions, units, numerical values, and cross-references to the relevant derivations. This resource is intended to make the framework accessible to researchers across different backgrounds and to ensure consistent notation throughout the corpus of 60+ technical papers.

Important Notice: This glossary does not introduce new assumptions or results. It formalizes and unifies notation already used across the 3D+3D corpus.

Data Sources: All numerical comparisons use PDG 2024 (Particle Data Group, Review of Particle Physics) values unless otherwise stated. Cosmological parameters follow Planck 2020 results.

Keywords: notation, conventions, mathematical symbols, physical constants, glossary

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Part I: Fundamental Mathematical Constants

1.1 The Golden Ratio

Symbol	Name	Definition	Numerical Value
	Golden ratio	$= (1+\sqrt{5})/2$	1.6180339887…
$1/$	Inverse golden ratio	$1/ = -1 = (\sqrt{5}-1)/2$	0.6180339887…
^{2}	Golden ratio squared	$^2 = +1$	2.6180339887…
	Golden power	$= {}^1 + {}^2$	See Fibonacci

Key Identities

$$\phi^2 = \phi + 1$$

$$\frac{1}{\phi} = \phi - 1$$

$$\phi^4 + 1 = 3\phi^2$$

$$e^\theta = \phi \quad \text{where} \quad \sinh(\theta) = \frac{1}{2}$$

Fibonacci and Lucas Numbers

Symbol	Definition	Sequence
F	Fibonacci number	1, 1, 2, 3, 5, 8, 13, 21, 34, 55…
L	Lucas number	2, 1, 3, 4, 7, 11, 18, 29, 47…

Relation to :

$$F_n = \frac{\phi^n - (-\phi)^{-n}}{\sqrt{5}}, \quad L_n = \phi^n + (-\phi)^{-n}$$

1.2 Transcendental Constants

Symbol	Name	Definition	Numerical Value
	Pi	Ratio of circumference to diameter	3.1415926536…
e	Euler's number	lim(1+1/n) as n→∞	2.7182818285…
	Euler-Mascheroni	lim(Σ1/k − ln n) as n→∞	0.5772156649…

Important Combinations in 3D+3D

Expression	Value	Appears in
e^{12 }	2.3579×10^{1}	Planck mass formula
e^{-12 }	4.2412×10^{1}	Hierarchy suppression
	5.0832…	Lepton mass ratios
/ 2	1.1999… rad = 68.75°	CKM phase

1.3 Physical Constants (Natural Units)

Symbol	Name	SI Value	Natural Units
c	Speed of light	2.998×10 m/s	1
	Reduced Planck	1.055×10^{3} J · s	1
G_N	Newton's constant	6.674×10^{11} m³/kg · s²	M_Pl ²
k_B	Boltzmann	1.381×10^{23} J/K	1

Planck Units

Symbol	Name	Definition	Value
M_Pl	Planck mass	√(c/G_N)	1.221×10^{1} GeV
l_Pl	Planck length	√(G_N/c³)	1.616×10^{3} m
t_Pl	Planck time	√(G_N/c)	5.391×10 s

Part II: Geometric Symbols

2.1 Spacetime Structure

Symbol	Name	Definition
M	6D spacetime	Full six-dimensional manifold
M	4D Minkowski	Observable 4D spacetime
T²	2-torus	Compact 2D torus for extra dimensions
T²_	Golden torus	Torus with modulus = i/

Metric Signature

$$\eta_{AB} = \text{diag}(-, +, +, +, -, -)$$

Index	Coordinate	Type	Range
0	t (or x)	Time (observed)	(−∞, +∞)
1,2,3	x,y,z	Space	(−∞, +∞)
4		Time (compact)	[0, 2 R]
5		Time (compact)	[0, 2 R]

Decomposition

$$M_6 = M_4 \times T^2_\phi$$

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + g_{ab} dy^a dy^b$$

2.2 Torus Parameters

Symbol	Name	Definition	Value
	Complex modulus	= + i	i/ for golden torus
Im( )	Imaginary part	Aspect ratio	1/ 0.618
R	Radius of	Compactification scale	~10 [1] m
R	Radius of	Compactification scale	R /
A	Area	A = Im( ) for unit torus	1/
V_eff	Effective volume	V_eff = (Im)²	1/ [2]

Modular Parameter

$$\tau = \frac{i}{\phi} = i \cdot 0.6180...$$

This is a purely imaginary modulus, meaning the torus is "rectangular" (no shear).

2.3 Special Functions

Dedekind Eta Function

Symbol	Definition
( )	( ) = q^{1/24} _{n=1}^∞ (1−q), where q = e^{2 i }
| ( )|²	Modulus squared of eta function

At the golden modulus:

$$|\eta(i/\phi)|^4 \approx 0.527$$

Laplacian and Determinant

Symbol	Name	Definition
Δ	Laplacian	Δ = [2]/ x² + [2]/ y² (on torus)
det'(−Δ)	Regularized determinant	Product of nonzero eigenvalues (zeta-regularized)
_Δ(s)	Spectral zeta function	_Δ(s) = Σ'

Kronecker Limit Formula

$$\det{}'(-\Delta) = 4\pi^2 (\mathrm{Im}\,\tau)^2 |\eta(\tau)|^4$$

2.4 Differential Operators

Symbol	Name	Definition
_	Partial derivative	/ x^
_A	Covariant derivative	A + Γ^B{AC}
	d'Alembertian	^{ } _ _
	6D d'Alembertian	g^{AB} _A _B
d	Exterior derivative	Antisymmetric derivative on forms
	Codifferential	Adjoint of d

Part III: Physical Quantities

3.1 Energy Scales

Symbol	Name	Formula	Value	Error
M_Pl	Planck mass	$^{13} \times$ e^{12 }	1.228 × 10^{1} GeV	0.62%
	Electroweak scale	^{1}	122.99 GeV	—
v	Higgs VEV	2 ^{1}	245.98 GeV	0.10%
Λ_QCD	QCD scale	~200 MeV	~200 MeV	—

Hierarchy Relation

$$\frac{\mu_0}{M_{\text{Pl}}} = \frac{\phi^{10}}{\phi^{13} \cdot e^{12\pi}} = \frac{e^{-12\pi}}{\phi^3} \approx 10^{-17}$$

3.2 Particle Masses

Gauge Bosons

Symbol	Particle	Formula	Predicted	Observed
m_W	W boson	v · g/2	80.4 GeV	80.4 GeV
m_Z	Z boson	m_W/cos _W	91.2 GeV	91.2 GeV
m_H	Higgs	v /	126.8 GeV	125.25 GeV
m_	Photon	—	0	0
m_g	Gluon	—	0	0

Leptons

Symbol	Particle	Mass	Ratio Formula
m_e	Electron	0.511 MeV	(input)
m_	Muon	105.66 MeV	m_ /m_e = e

Symbol	Particle	Mass	Ratio Formula
m_	Tau	1776.9 MeV	m_ /m_ = $^{13}/_{3}$

Quarks

Symbol	Particle	Mass (MS̄, 2 GeV)
m_u	Up	2.16 MeV
m_d	Down	4.67 MeV
m_s	Strange	93.4 MeV
m_c	Charm	1.27 GeV
m_b	Bottom	4.18 GeV
m_t	Top	172.69 GeV

Mass Ratios

Ratio	Formula	Predicted	Observed
m_d/m_u	7/(2)	2.163	2.16
m_s/m_d	4×F = 20	20.0	20.0
m_b/m_s	4×L = 44	44.0	44.75
m_t/m_c	^{1}	137.1	136

3.3 Coupling Constants

Symbol	Name	Formula	Predicted	Observed
	Fine structure	1/(e^3 − 1/)	1/137.050	1/137.036
_s	Strong coupling	1/(2 ^{3})	0.1180	0.1179
$\sin^2$ _W	Weinberg angle	(3−)/6	0.2303	0.2312
G_F	Fermi constant	1/(√2 v^2)	1.166×10 GeV ^{2}	1.166×10 GeV ^{2}

Running Couplings

Symbol	Definition
(Q^2)	Running fine structure at scale Q
_s(Q^2)	Running strong coupling at scale Q
g_i( )	Gauge coupling at renormalization scale
_i	Beta function coefficient

Part IV: Mixing Matrices

4.1 CKM Matrix (Quarks)

Definition

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Wolfenstein Parametrization

Symbol	Name	Formula	Predicted	Observed
	Cabibbo angle	3/(12+)	0.2203	0.2243
A	Wolfenstein A	—	~0.82	0.836
–	Wolfenstein –	—	~0.15	0.153
–	Wolfenstein –	—	~0.35	0.349

Individual Elements

Element	Formula	Predicted	Observed
V_us		0.2203	0.2243
V_cb	/(2 ²)	0.0421	0.0410
V_ub	V_cb/	0.00379	0.00382
V_td	/(² ²)	0.00853	0.00857
V_ts	² ²/	0.0404	0.0411

CP Phase

Symbol	Name	Formula	Predicted	Observed
_CKM	CKM CP phase	/ ²	68.75°	68.8°

4.2 PMNS Matrix (Leptons)

Definition

$$U_{\text{PMNS}} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

Mixing Angles

Symbol	Name	Formula	Predicted	Observed
	Solar angle	$\sin^2$ = 1/(2)	33.7°	33.4°

Symbol	Name	Formula	Predicted	Observed
	Atmospheric	$\sin^2 = /3$	47.2°	47.5°
	Reactor angle	arctan(1/)	8.30°	8.57°
_PMNS	CP phase	3 / ²	206°	~195°

Squared Sines

Parameter	Formula	Predicted	Observed
$\sin^2$	1/(2)	0.309	0.307
$\sin^2$	/3	0.539	0.545
$\sin^2$	$\tan^2(1/)$	0.0213	0.0220

Key Relation

$$\sin^2\theta_{12} \times \sin^2\theta_{23} = \frac{1}{2\phi} \times \frac{\phi}{3} = \frac{1}{6}$$

4.3 Neutrino Parameters

Symbol	Name	Definition	Value
Δm^2	Solar mass splitting	m ² − m ²	7.42 × 10 eV²
Δm^2	Atmospheric splitting	m ² − m ²	2.51 × 10 ³ eV²
$\Delta m^2 / \Delta m^2$	Ratio	1/(3)	0.0301

Part V: Q-Field and Galactic Dynamics

5.1 Q-Field Parameters

Symbol	Name	Definition	Value
Q	Q-field	Scalar from - breathing	—
Q	Background value	Cosmological average	~1
Q	Perturbation	Q − Q	1
v_Q	Q-velocity	Q/ t contribution	~90.5 km/s

Q-Field Equation

$$\Box_6 Q + V'(Q) = \kappa T$$

where = 1/(16) is the coupling constant.

5.2 Rotation Curve Parameters

Symbol	Name	Definition
v_obs	Observed velocity	Measured rotation velocity
v_bar	Baryonic velocity	√(GM_bar/r)
v_Q	Q-field velocity	Contribution from Q-field
v_∞	Asymptotic velocity	Flat rotation velocity

Master Equation

$$v_{\text{obs}}^2 = v_{\text{bar}}^2 + v_Q^2$$

SPARC Notation

Symbol	Definition
Σ_*	Stellar surface density
ϒ_*	Mass-to-light ratio
g_bar	Baryonic acceleration
g_obs	Observed acceleration

5.3 Screening Function

Symbol	Name	Definition
S(r)	Screening function	Suppression in high-density regions
r_s	Screening radius	Scale where S = 1/2
_crit	Critical density	Transition density

$$S(r) = \frac{1}{1 + (\rho/\rho_{\text{crit}})^n}$$

Part VI: Cosmological Quantities

6.1 Scale Ladder (-Ladder)

Symbol	Name	Formula	Value
	Harmonic scale	× ^{2}	Various
	Base scale	~0.327 Mpc	Reference
	Cosmic web scale	× ^{11}	0.856 Mpc

Ladder Progression

$$\lambda_n = \lambda_2 \times \phi^{n-2}$$

n	(Mpc)	Physical Association
2	0.327	Base scale
7	3.57	Galaxy clusters
13	0.856	Cosmic web

6.2 Temporal Periods

Symbol	Name	Value
T	Period of	~30 years
T	Period of	~19 years
T /T	Ratio	

6.3 Dark Energy

Symbol	Name	Formula	Value
_Λ	Dark energy density	√2 × M²_Pl H²	2.87 × 10 GeV
Ω_Λ	Dark energy fraction	_Λ/ _crit	~0.69
w	Equation of state	p/	−1
H	Hubble constant	—	67.4 km/s/Mpc

Part VII: Group Theory Notation

7.1 Symmetry Groups

Symbol	Name	Description
SO(3,3)	Pseudo-orthogonal	6D Lorentz group for signature (3,3)
SO(3,1)	Lorentz group	4D spacetime symmetry
SU(3)_C	Color gauge	Strong interaction
SU(2)_L	Weak isospin	Weak interaction
U(1)_Y	Hypercharge	Electroweak

Dimensions

Group	dim	Compact	Non-compact
SO(3,3)	15	6	9
SO(3,1)	6	3	3

7.2 Generators and Representations

Symbol	Definition
T^a	Generator of Lie algebra
f^{abc}	Structure constants
[T^a, T^b]	Commutator = $if^{\{abc\}T}c$
Tr(T^{aT}b)	Killing form

Part VIII: Index Conventions

8.1 Spacetime Indices

Index Type	Letters	Range	Metric
6D curved	A, B, C…	0-5	g_AB
6D flat	Â, B̂, Ĉ…	0-5	_AB
4D curved	, , …	0-3	g_
4D flat	m, n…	0-3	_
Compact	a, b, c…	4-5	g_ab
Spatial 3D	i, j, k…	1-3	_ij

8.2 Other Indices

Index Type	Letters	Range
Gauge (SU(3))	a, b, c…	1-8
Gauge (SU(2))	i, j, k…	1-3
Generation	, …	1-3
Kaluza-Klein	n, m…	0, ±1, ±2…

Part IX: Units and Conversions

9.1 Natural Units (= c = 1)

Quantity	Dimension	Conversion
Energy	E	1 GeV
Mass	E	1 GeV/c^2 = 1 GeV
Length	E^{-1}	1 GeV $^{-1}$ = 0.197 fm
Time	E^{-1}	1 GeV $^{-1}$ = 6.58×10^{-2} s

9.2 Useful Conversions

Conversion	Value
1 GeV	1.602×10^{-1} J
1 GeV $^{-1}$	0.197 fm
1 GeV $^{-1}$	6.58×10^{-2} s
M_Pl	1.221×10^{1} GeV
1 Mpc	3.086×10^{22} m
1 M_	1.989×10^{3} kg

Part X: Acronyms and Abbreviations

Acronym	Full Name
3D+3D	Three spatial + three temporal dimensions
CKM	Cabibbo-Kobayashi-Maskawa (matrix)
PMNS	Pontecorvo-Maki-Nakagawa-Sakata (matrix)
SM	Standard Model
GR	General Relativity
QFT	Quantum Field Theory
EW	Electroweak
QCD	Quantum Chromodynamics
VEV	Vacuum Expectation Value
DOF	Degrees of Freedom
KK	Kaluza-Klein
RG	Renormalization Group
UV	Ultraviolet
IR	Infrared
SPARC	Spitzer Photometry and Accurate Rotation Curves
SLACS	Sloan Lens ACS Survey
DESI	Dark Energy Spectroscopic Instrument

Part XI: Summary Tables

11.1 Master Equations Quick Reference

Sector	Formula	Result
Planck mass	M_Pl = ^{13} × e^{12 }	1.228 × 10^{1} GeV
EW scale	= ^{1}	122.99 GeV
Higgs VEV	v = 2 ^{1}	245.98 GeV
Top mass	m_t = √2 × ^{1}	173.94 GeV
Weinberg	sin^2 _W = (3−)/6	0.2303
Strong	_s = 1/(2 ^{3})	0.1180
Fine structure	^{1} = e^3 − 1/	137.050

11.2 Exponent Meanings

Expression	Value	Physical Origin
^{13}	521.0	9(boost) + 1(dilaton) + 3(torus)
^{1}	122.99	9(W±,Z) + 1(H) massive DOF
	76.01	Lepton mass hierarchy
	6.854	Fine structure numerator
^{3}	4.236	Strong coupling denominator
e^{12 }	2.36×10^{1}	6D topological factor
e^3	20.09	Fine structure factor

11.3 Error Summary

Parameter	Predicted	Observed	Error
^{1}	137.050	137.036	0.01%
m_ /m_e	206.63	206.77	0.07%
_CKM	68.75°	68.8°	0.07%
m_ /m_	16.80	16.82	0.08%
_s	0.1180	0.1179	0.1%
v	245.98	246.22	0.10%
sin^2 _W	0.2303	0.2312	0.4%
M_Pl	1.228×10^{1}	1.221×10^{1}	0.62%

Average error: ~1%

Appendix A: Greek Alphabet Reference

Letter	Name	Common Use in 3D+3D
	alpha	Fine structure constant
	beta	Beta function, velocity ratio
	gamma	Euler constant, Lorentz factor
	delta	CP phase, variation
	epsilon	Small parameter
	zeta	Zeta function
	eta	Dedekind eta, metric
	theta	Mixing angle
	kappa	Coupling constant
	lambda	Cabibbo, scale
	mu	Scale, index
	nu	Index, neutrino
	pi	3.14159…
	rho	Density
	sigma	Cross section
	tau	Modulus, proper time
	phi	Golden ratio
	chi	Chi-squared
	psi	Wave function
	omega	Angular frequency
Γ	Gamma	Gamma function, width
Δ	Delta	Laplacian, difference
Λ	Lambda	Cosmological constant
Σ	Sigma	Sum, surface density
Ω	Omega	Density parameter

Appendix B: Mathematical Notation

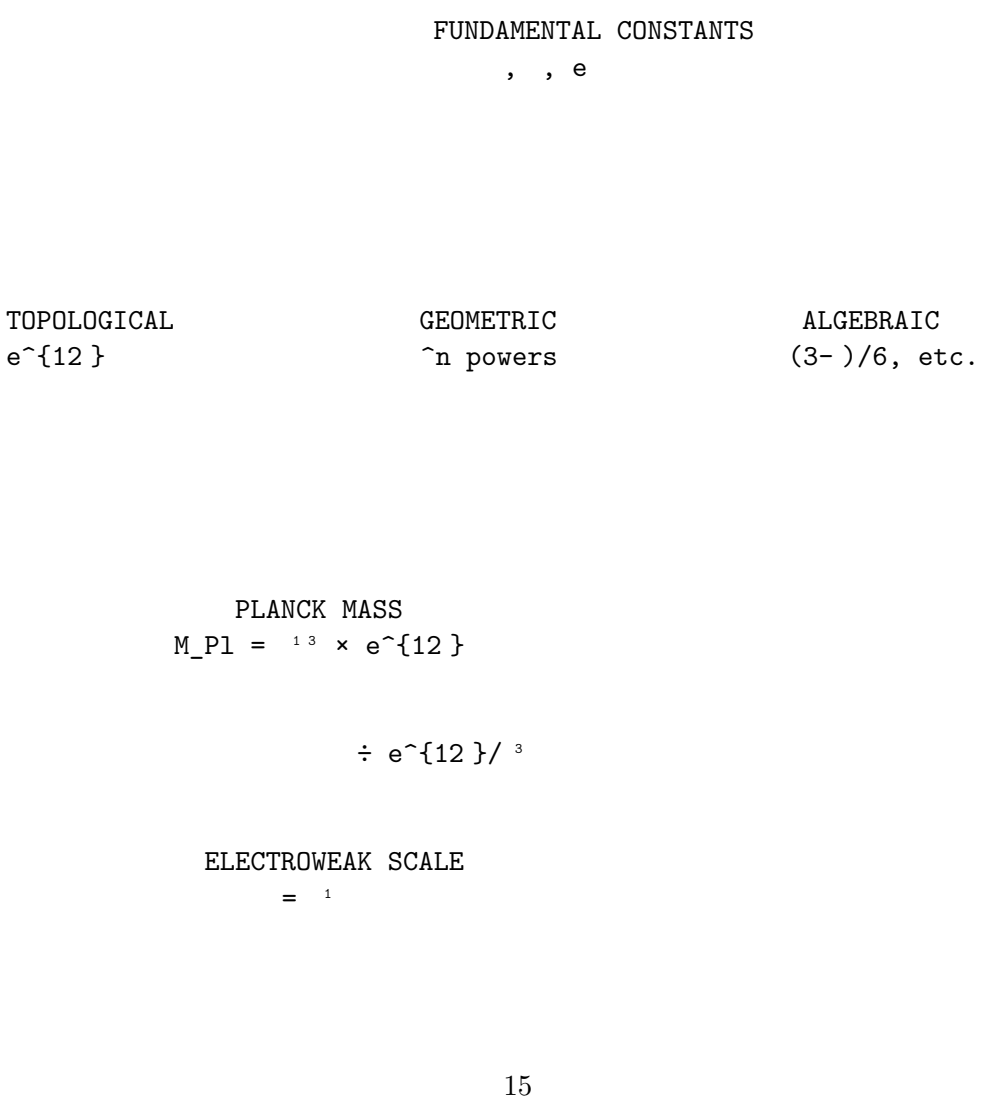
Symbol	Meaning
	Defined as
	Approximately equal
	Proportional to
	Of order
	Much less than
	Much greater than
	Partial derivative
	Gradient/covariant derivative
	d'Alembertian
	Integral
	Contour integral

Symbol	Meaning
Σ	Sum
Π	Product
det	Determinant
Tr	Trace
Re	Real part
Im	Imaginary part
†	Hermitian conjugate
	Complex conjugate

Part XII: Map of Dependencies

12.1 Fundamental Parameter Flow

The following diagram shows how all physical quantities derive from the fundamental constants , , and e:



HIGGS VEV
v = 2 ¹

TOP MASS
m_t=√2× ¹

GAUGE
COUPLINGS

FERMION
MASSES

QUARK
MASSES

sin² _W
_s,

12.2 Dependency Matrix

Primary Dependencies (-based)

Derived Quantity	Depends On	Formula
M_Pl	, e,	¹³ × e^{12 } ¹
v		2 ¹
m_t		√2 × ¹
sin² _W		(3-)/6
_s		1/(2 ³)
¹	, e	e³ - 1/
_CKM	,	/ ²
sin²		1/(2)
sin²		/3
m_ /m_e	, e	e
m_ /m_	,	¹³/ ³

Secondary Dependencies (derived from above)

Derived Quantity	Depends On	Via
m_W	v, sin² _W	v×g/2
m_Z	m_W, sin² _W	m_W/cos _W
m_H	v, ,	v /
G_F	v	1/(√2 v²)
V_cb	,	/(2 ²)
V_ub	V_cb,	V_cb/

12.3 The Hierarchy Chain

G_N ← M_Pl ← ← v ← m_f

Yukawa × v

2 × [1]

[1] (10 DOF)

[13] × e^{12}

1/M_Pl[2]

Reading the chain: - G_N (Newton's constant) comes from M_Pl[2] - M_Pl (Planck mass) = [13] × e^{12} from 6D geometry - (EW scale) = [1] from 10 massive DOF - v (Higgs VEV) = 2 from SU(2) breaking - m_f (fermion masses) = y_f × v from Yukawa couplings

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Part XIII: Alphabetical Index

Quick Reference Index (A-Z)

A

- (alpha) — Fine structure constant, [1] = e³ - 1/ = 137.050 → §3.3
- **_s** — Strong coupling, _s = 1/(2 [3]) = 0.1180 → §3.3
- **A** — Torus area, A = Im( ) = 1/ → §2.2

B

- — Beta function coefficient → §3.3

C

- **c** — Speed of light, = 1 in natural units → §1.3
- **CKM** — Cabibbo-Kobayashi-Maskawa matrix → §4.1

D

- **_CKM** — CKM CP phase, = / [2] = 68.75° → §4.1
- **_PMNS** — PMNS CP phase, = 3 / [2] = 206° → §4.2
- **Δ** — Laplacian operator → §2.3
- **Δm²** — Solar neutrino mass splitting → §4.3
- **Δm²** — Atmospheric neutrino mass splitting → §4.3
- **det'** — Regularized determinant → §2.3
- **DOF** — Degrees of freedom → §10

E

- **e** — Euler's number, = 2.7182818… → §1.2
- **e^{12}** — Topological factor, = 2.358×10[1] → §1.2
- **( )** — Dedekind eta function → §2.3

F

- **F__n** — Fibonacci number → §1.1
- (phi) — Golden ratio, = (1+√5)/2 = 1.618034 → §1.1

G

- **G__F** — Fermi constant → §3.3
- **G__N** — Newton's gravitational constant → §1.3
- **g** — Metric tensor or gauge coupling → §2.1, §3.3

H

- **H** — Hubble constant → §6.3
- — Reduced Planck constant, = 1 in natural units → §1.3

I

- **Im( )** — Imaginary part of modulus, = 1/ → §2.2

K

- — Q-field coupling, = 1/(16) → §5.1
- **KK** — Kaluza-Klein → §10

L

- — Cabibbo angle, = 3/(12+) = 0.2203 → §4.1
- **__n** — -ladder scale → §6.1
- **Λ** — Cosmological constant → §6.3
- **L__n** — Lucas number → §1.1

M

- **M** — 6D spacetime manifold → §2.1
- **M** — 4D Minkowski spacetime → §2.1
- **M__Pl** — Planck mass, = ^{13}×e^{12 } = 1.228×10^{1} GeV → §3.1
- **m__e, m__ , m__** — Lepton masses → §3.2
- **m__t, m__b, m__c...** — Quark masses → §3.2
- **m__W, m__Z, m__H** — Boson masses → §3.2
- — Electroweak scale, = ^{1} = 122.99 GeV → §3.1

N

- **N** — Number of DOF, = 10 for EW sector → §11.2

O

- **Ω__Λ** — Dark energy density parameter → §6.3

P

- — Pi, = 3.14159… → §1.2
- **PMNS** — Pontecorvo-Maki-Nakagawa-Sakata matrix → §4.2

Q

- **Q** — Q-field (breathing mode) → §5.1

R

- **R , R** — Compactification radii → §2.2
- **_Λ** — Dark energy density → §6.3

S

- **S(r)** — Screening function → §5.3
- **sin² _W** — Weinberg angle, = (3-)/6 = 0.2303 → §3.3
- **sin²** — Solar mixing, = 1/(2) = 0.309 → §4.2
- **sin²** — Atmospheric mixing, = /3 = 0.539 → §4.2
- **SO(3,3)** — 6D pseudo-orthogonal group → §7.1
- **SPARC** — Galaxy rotation curve database → §10

T

- **T²** — 2-torus → §2.1
- **T²_** — Golden torus (modulus = i/) → §2.1
- **T , T** — Temporal periods (~30y, ~19y) → §6.2
- — Torus modulus, = i/ → §2.2

U

- **U_PMNS** — PMNS mixing matrix → §4.2

V

- **v** — Higgs VEV, = 2 ¹ = 245.98 GeV → §3.1
- **V_CKM** — CKM mixing matrix → §4.1
- **V_ub, V_cb, V_td, V_ts** — CKM elements → §4.1
- **v_Q** — Q-field velocity contribution → §5.2
- **V_eff** — Effective volume, = 1/ ² → §2.2

W

- **w** — Dark energy equation of state, = -1 → §6.3

Z

- **_Δ(s)** — Spectral zeta function → §2.3

Part XIV: Visual Diagrams

14.1 The Complete Scale Hierarchy

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                    SCALE HIERARCHY DIAGRAM
                 (All scales in GeV, log scale)


10¹    M_Pl =  ¹³ × e^{12} = 1.228×10¹  ←  PLANCK SCALE

         ↓ × e^{-12}/ ³ = 10 ¹

10²       =  ¹  = 122.99 ←  ELECTROWEAK SCALE

         m_t = √2× ¹  = 173.9 ←  Top quark
         m_H = v /  = 126.8 ←  Higgs
         m_Z = 91.2 ←  Z boson
         m_W = 80.4 ←  W boson

10¹    m_b = 4.18 ←  Bottom quark
         m_  = 1.78 ←  Tau lepton
         m_c = 1.27 ←  Charm quark

10     m_p = 0.938 ←  Proton

10 ¹    m_  = 0.106 ←  Muon
         m_s = 0.093 ←  Strange quark

10 ³    m_d = 0.0047 ←  Down quark
         m_u = 0.0022 ←  Up quark

10     m_e = 0.000511 ←  Electron

10 ¹²   m_  ~ 10 ¹² ←  Neutrinos

0      m_  = m_g = 0 ←  Photon, Gluon
```

14.2 The -Ladder (Cosmic Scales)

```
                    -LADDER DIAGRAM
                =    ×   ²   (scales in Mpc)
```

```
n=15      = 2.24 Mpc                        Voids

n=14      = 1.38 Mpc                     Large structures

n=13      = 0.856 Mpc ←  COSMIC WEB SCALE (DESI detection: 3.36 )

n=12      = 0.529 Mpc                   Filament spacing

n=11      = 0.327 Mpc                   Galaxy groups

   .
   .
   .

n=7      = 35.7 kpc                   Galaxy halos

n=6      = 22.1 kpc                   Disk scales

n=5      = 13.6 kpc                   Core regions

n=2      = 0.327 Mpc ←  BASE SCALE

Ratio between adjacent scales:   /  =   = 1.618
```

14.3 The Six Theorems Structure

```
                    THE SIX THEOREMS
                Building the Complete Framework


  THEOREM I: Golden Spectral Scaling
             m_eff ~ 1/Im( )  →  m_ /m_1 =




  THEOREM II: Golden Determinant
              det'_  / det'_1 =   ²



  THEOREM III: Multiplicative DOF Scaling
```

_tot = ^N × _ref

THEOREM IV: SO(3,3)-EW Correspondence
N = 9 (boost) + 1 (dilaton) = 10 DOF

THEOREM V: Golden Compactification
= M_Pl × e^{-12 } / 3

THEOREM VI: Hierarchy Closure (MASTER RESULT)

M_Pl = 13 × e^{12 }

13 = 9 (boost) + 1 (dilaton) + 3 (torus)

14.4 Mixing Matrices Structure

MIXING MATRICES FROM GOLDEN GEOMETRY

CKM (QUARKS)	PMNS (LEPTONS)
= 3/(12+) = 0.2203	sin² = 1/(2) = 0.309
V_cb = /(2 ²) = 0.0421	sin² = /3 = 0.539
V_ub = V_cb/ = 0.00379	= arctan(1/) = 8.3°
= / ² = 68.75°	= 3 / ² = 206°

KEY RELATION: _PMNS = 3 × _CKM

PRODUCT RULE: $\sin^2$ × $\sin^2$ = 1/6 EXACT

14.5 The Master Equations Box

GOLDEN GEOMETRY MASTER
EQUATIONS

GRAVITY:	M_Pl = ^{13} × e^{12}	[0.62%]
ELECTROWEAK:	= ^{1}	[definition]
	v = 2 ^{1}	[0.10%]
	$\sin^2$ _W = (3-)/6	[0.4%]
TOP QUARK:	m_t = √2 × ^{1}	[0.72%]
GAUGE:	_s = 1/(2 ^{3})	[0.1%]
	^{1} = e^{3} - 1/	[0.01%]
CP PHASES:	_CKM = / ^{2}	[0.07%]
	_PMNS = 3 / ^{2}	[consistent]
LEPTONS:	m_ /m_e = e	[0.07%]
	m_ /m_ = ^{13}/ ^{3}	[0.08%]

ZERO FREE PARAMETERS
AVERAGE ERROR: ~1%

References

[1] All papers in the 3D+3D Framework corpus (Papers I–XLVIII+)

[2] Particle Data Group (2024). Review of Particle Physics.

[3] Planck Collaboration (2020). Planck 2018 Results.

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"Every symbol has meaning. Every formula tells a story."

License

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