3D+3D Laboratory — Paper XCIII, Addendum

Diagonalization of the Friedmann Hessian:

Synchronized Attractor, Block Structure, and the Origin of the Numerator 19

Three CAS-verified results: (i) $\mathrm{Hess}(G_{00})$ *is block-diagonal in the* (H, S, D) *basis; (ii) the antisymmetric mode* $D = P - Q$ *decouples with eigenvalue* -1*; (iii)* $\mathrm{Hess}(G_{00}) \cdot (6, 1, 1)^T = (42, 19, 19)^T$

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Red Team: Vega (OpenAI) — identified the (H, S, D) *basis, the block-diagonal structure, and*

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Abstract

We prove three results that complete the structural analysis of $\mathrm{Hess}(G_{00}) = -G_{\mathrm{DeWitt}}$ (Paper XCIII). **(i) Block-diagonal structure:** in the basis $(H,\, S{=}\frac{P+Q}{\sqrt{2}},\, D{=}\frac{P-Q}{\sqrt{2}})$, the Friedmann Hessian is exactly block-diagonal:

$$\mathrm{Hess}(G_{00}) \;\longrightarrow\; \begin{pmatrix} 6 & 3\sqrt{2} & 0 \\ 3\sqrt{2} & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

(ii) Eigenvalues and synchronization: the characteristic polynomial is $(\lambda+1)(\lambda^2 - 7\lambda - 12) = 0$, giving eigenvalues $\lambda_D = -1$ (antisymmetric mode) and $\lambda_\pm = (7 \pm \sqrt{97})/2$ (symmetric block). The negative eigenvalue $\lambda_D = -1$ explains, geometrically, why the physical attractor lies on the synchronized submanifold $P = Q$ ($v_* = D/H = 0$): the antisymmetric direction is a saddle point. **(iii) Attractor maps to 19:** the attractor direction $(6,1,1) \propto (H_0, P_0, Q_0)$ satisfies

$$\text{Hess}(G_{00}) \cdot (6,1,1)^T = (42,19,19)^T.$$

The compact components equal exactly $19 = 2W+d$, the cosmological numerator. Synchronization, the attractor $u^* = 1/3$, and the numerator 19 are three projections of the same Hessian structure. All results are CAS-verified with SymPy (exact arithmetic).

Contents

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1 The Natural Basis: Symmetric and Antisymmetric Modes

Paper XCIII established $\text{Hess}(G_{00}) = -G_{\text{DeWitt}}$ in the original velocity variables (H, P, Q). The natural basis for understanding the physics of this result is:

$$H \text{ (isotropic)}, \quad S = \tfrac{P+Q}{\sqrt{2}} \text{ (symmetric)},$$
$$D = \tfrac{P-Q}{\sqrt{2}} \text{ (antisymmetric)}. \tag{1}$$

This change of basis is orthonormal and corresponds to the decomposition of the two compact temporal dimensions into their symmetric (in-phase) and antisymmetric (out-of-phase) combinations.

Theorem 1.1 (Block-diagonal structure (CAS))**.** *In the basis* (H, S, D) *defined by* (1)*, the Friedmann Hessian is exactly block-diagonal:*

$$\mathrm{Hess}(G_{00}) \xrightarrow{(H,P,Q)\to(H,S,D)} \begin{pmatrix} 6 & 3\sqrt{2} & 0 \\ 3\sqrt{2} & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} =: \begin{pmatrix} B_{\text{sym}} & 0 \\ 0 & \lambda_D \end{pmatrix}, \tag{2}$$

where $B_{\text{sym}} = \left[\begin{smallmatrix} 6 & 3\sqrt{2} \\ 3\sqrt{2} & 1 \end{smallmatrix}\right]$ *is the symmetric block and* $\lambda_D = -1$ *is the antisymmetric eigenvalue.*

Proof. The change-of-basis matrix is $T = \left[\begin{smallmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \end{smallmatrix}\right]$ (orthonormal). Direct CAS computation: $T^{-1}\,\mathrm{Hess}(G_{00})\,T = \left[\begin{smallmatrix} 6 & 3\sqrt{2} & 0 \\ 3\sqrt{2} & 1 & 0 \\ 0 & 0 & -1 \end{smallmatrix}\right]$. □ □

Remark 1.2. The exact vanishing of the off-diagonal entries involving D means the antisymmetric mode $D = P - Q$ is a true eigenvector of $\mathrm{Hess}(G_{00})$ with eigenvalue -1. This is an exact algebraic identity, not an approximation.

2 Eigenvalues and the Synchronization Mechanism

Theorem 2.1 (Eigenvalues of $\mathrm{Hess}(G_{00})$, CAS-verified)**.** *The characteristic polynomial of* $\mathrm{Hess}(G_{00})$ *is:*

$$p(\lambda) = (\lambda + 1)(\lambda^2 - 7\lambda - 12) = 0, \tag{3}$$

with eigenvalues:

$$\lambda_D = -1, \qquad \lambda_\pm = \frac{7 \pm \sqrt{97}}{2}. \tag{4}$$

Numerically: $\lambda_D = -1$*,* $\lambda_+ \approx 8.424$*,* $\lambda_- \approx -1.424$*.*

Proof. By Theorem 1.1, the eigenvalues are: $\lambda_D = -1$ from the D-block, and the eigenvalues of B_{sym} from the symmetric block. $\det(B_{\text{sym}} - \lambda I) = \lambda^2 - 7\lambda - 12 = 0$ gives $\lambda_\pm = (7 \pm \sqrt{97})/2$. The full characteristic polynomial is therefore $(\lambda + 1)(\lambda^2 - 7\lambda - 12)$. CAS-verified. □ □

Remark 2.2 (Connection to Paper XC). The characteristic polynomial of $G_{\text{DeWitt}} = -\mathrm{Hess}(G_{00})$ is $(-\lambda + 1)(-\lambda^2 - 7\lambda - 12) \cdot (-1)^3 = (\lambda - 1)(\lambda^2 + 7\lambda + 12)$...

but more importantly, the coefficient of λ in $p(\lambda)$ is -19:

$$p(\lambda) = \lambda^3 - 6\lambda^2 - 19\lambda - 12. \tag{5}$$

The λ coefficient is $-19 = -I_2(\mathrm{Hess}(G_{00}))$, yet another derivation of the numerator 19: it appears in the characteristic polynomial of $\mathrm{Hess}(G_{00})$.

2.1 Physical interpretation: why synchronization occurs

The eigenvalue $\lambda_D = -1 < 0$ has direct physical meaning:

Proposition 2.3 (Geometric origin of synchronization)**.** *The antisymmetric mode $D = P - Q$ experiences a saddle-point curvature $\lambda_D = -1$ in the Friedmann Hessian. This geometric instability drives the dynamics toward $D \to 0$, i.e. $P \to Q$, which is the synchronized submanifold. The fixed point $v_* = 0$ (equivalently $P = Q$) of the DynSys is therefore determined by the sign of λ_D, not merely by the dynamical equations.*

Proof. In the reduced DynSys, the antisymmetric rate $v = D/H = (P - Q)/H$ obeys (Paper III):

$$v' = v\Big(\frac{u^2}{4} - u - \frac{3}{2}\Big).$$

The fixed point $v_* = 0$ is exactly the synchronized submanifold. By Theorem 1.1, the D-direction corresponds to a negative curvature $\lambda_D = -1$ in $\mathrm{Hess}(G_{00})$, meaning the Friedmann constraint is a local maximum in the antisymmetric direction. The DynSys thus flows away from $D \neq 0$ configurations. □ □

3 The Attractor Direction and the Numerator 19

Theorem 3.1 (Attractor maps to 19, CAS-verified)**.** *The physical attractor $u^* = 1/3$ with $v^* = 0$ (Paper III) corresponds to the direction $(H_0, P_0, Q_0) \propto (6, 1, 1)$ in the velocity space. The Friedmann Hessian maps this direction to:*

$$\mathrm{Hess}(G_{00}) \cdot \begin{pmatrix} 6 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 42 \\ 19 \\ 19 \end{pmatrix}. \tag{6}$$

The compact components of the response equal exactly $19 = -I_2(G_{\text{DeWitt}}) = 2W + d$*, the cosmological numerator.*

Proof. Why $(6, 1, 1)$*:* At the attractor $u^* = 1/3$, $v^* = 0$: $u^* = (P + Q)/H = 1/3$ and $P = Q$, so $P = Q = H/6$. The velocity vector is $(H, H/6, H/6) \propto (6, 1, 1)$.

Matrix-vector product (CAS): $\begin{bmatrix} 6 & 3 & 3 \\ 3 & 0 & 1 \\ 3 & 1 & 0 \end{bmatrix}\begin{bmatrix} 6 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 36+3+3 \\ 18+0+1 \\ 18+1+0 \end{bmatrix} = \begin{bmatrix} 42 \\ 19 \\ 19 \end{bmatrix}$.

The compact entries $19 = 19$ equal $-I_2(G_{\text{DeWitt}}) = 2W + d = 19$. □ □

Remark 3.2 (Structural interpretation)*.* The identity (6) reveals a non-trivial link between three objects that were previously derived independently: the DynSys attractor ($u^* = 1/3$, Paper III), the cosmological numerator (19, Paper XC), and the Friedmann Hessian structure (this paper). All three are consequences of the same geometric object $\text{Hess}(G_{00})$.

4 Three Faces of the Same Structure

Three facts, one structure: $\text{Hess}(G_{00})$

Fact	Content	Consequence
Synchronization	$\lambda_D = -1 < 0$ in D direction	$P - Q \to 0$ is geometrically forced
Attractor	$u^* = 1/3 \;\Leftrightarrow\; (6, 1, 1)$	Physical cosmology on symmetric branch
Numerator 19	Hess $\cdot\, (6, 1, 1) = (42, 19, 19)$	19 encoded in attractor response

All three derive from $\text{Hess}(G_{00}) = -G_{\text{DeWitt}}$.

5 Epistemic Status

Conclusions

The diagonalization of $\text{Hess}(G_{00})$ in the natural (H, S, D) basis reveals that the three key features of the 3D+3D cosmological sector — the synchronization of compact temporal dimensions, the physical attractor $u^* = 1/3$, and the cosmological numerator 19 — are not independent. They are all

Table 1: Status of results in this Addendum.

Result	Status
Hess(G_{00}) block-diagonal in (H, S, D) basis	Theorem (CAS)
Char. poly. $= (\lambda+1)(\lambda^2 - 7\lambda - 12)$	Theorem (CAS)
$\lambda_D = -1$ (antisymmetric eigenvalue)	Theorem
$\lambda_\pm = (7 \pm \sqrt{97})/2$ (symmetric block)	Theorem
Coeff. of λ in char. poly. $= -19$	Theorem
$\lambda_D < 0$ drives synchronization $P \to Q$	Proposition
Attractor $(6, 1, 1)$: geometric derivation	Proposition
Hess $\cdot (6, 1, 1) = (42, 19, 19)$	**Theorem (CAS)**
Compact components $= 19 =$ numerator	**Theorem**
Synchronization, attractor, 19: three projections	Structural result

encoded in the structure of a single matrix: the Hessian of the Friedmann constraint, which by Paper XCIII equals $-G_{\text{DeWitt}}$.

In particular:

1. The negative eigenvalue $\lambda_D = -1$ forces $P - Q \to 0$ geometrically.
2. The attractor direction $(6, 1, 1)$ lies in the symmetric block of the Hessian.
3. The Hessian maps $(6, 1, 1)$ to $(42, 19, 19)$, producing the cosmological numerator 19 in the compact components.

As Vega formulates it: *"synchronization, the attractor $u^* = 1/3$, and the numerator 19 are three faces of the same structure of the 6D minisuperspace."*

Acknowledgements. Vega (OpenAI) identified the natural (H, S, D) basis, established the block-diagonal form of Hess(G_{00}), computed the characteristic polynomial, and discovered the key identity Hess$(G_{00})\cdot(6, 1, 1)^T = (42, 19, 19)^T$.

References

[1] S. Calzighetti, Lucy, "Paper XCIII," 3D+3D Laboratory (2026).

[2] S. Calzighetti, Lucy, "Paper XC," 3D+3D Laboratory (2026).

[3] S. Calzighetti, Lucy, "Paper III: Effective 6D Gravity," 3D+3D Laboratory (2025).

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