

Paper: Natural Scale Delimiters in 3D+3D Discrete Spacetime Theory

The Geometric Origin of Physical Regime Boundaries

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Abstract

We demonstrate that the six-dimensional discrete spacetime framework (3D+3D) contains **natural scale delimiters** that emerge from the underlying geometry rather than being imposed phenomenologically. These delimiters separate physical regimes where different effective physics dominates: (i) Q-field dynamics at galactic and cosmological scales, (ii) Vainshtein screening at Solar System scales, and (iii) standard physics with geometrically-derived constants at compact object scales. We prove that these boundaries follow necessarily from three geometric quantities: the critical mass $M_{\text{crit}} = v^2_3 D_3 D \lambda_2 / G$, the Vainshtein radius $r_V = (GM / \Lambda^3 c^2)^{1/3}$, and the subcritical threshold $\psi_{\text{crit}} = v^2_3 D_3 D / c^2$. We argue that the existence of such derived boundaries constitutes a **theoretical strength** rather than a limitation: a physically correct theory must specify its domain of applicability through internal consistency rather than external imposition. The framework provides twelve orders of magnitude of multi-scale predictions from a single geometric structure, while correctly deferring to established physics (MHD, QED, QCD) where those theories have been empirically validated. This represents a paradigm for modified gravity theories: explain what needs explaining, screen where screening is required, and provide fundamental constants everywhere.

Keywords: Extra dimensions, modified gravity, scale transitions, screening mechanisms, philosophy of physics, natural boundaries

1. Introduction

1.1 The Problem of Scope in Modified Gravity

Every modification of General Relativity faces a fundamental challenge: it must explain anomalous phenomena

at large scales (galaxy rotation curves, cosmic acceleration) while remaining consistent with precision tests at small scales (Solar System, binary pulsars, laboratory experiments). This challenge has led to the development of various screening mechanisms—chameleon, symmetron, Vainshtein—that suppress modifications in high-density environments [1-3].

However, most implementations of these mechanisms involve phenomenological parameters that are tuned to achieve consistency. The screening scale, the transition sharpness, and the asymptotic behavior are typically adjusted to match observations rather than derived from first principles.

1.2 The 3D+3D Approach

The six-dimensional discrete spacetime framework (3D+3D) [4-7] proposes a different paradigm. The theory posits a spacetime with signature $(-, +, +, +, -, -)$, where two additional temporal dimensions (τ_2, τ_3) are compactified on a torus T^2 at galactic scales. From this single geometric structure emerge:

1. **Scale-dependent effects** through Q-field breathing modes
2. **Automatic screening** through Vainshtein mechanism from 6D Horndeski terms
3. **Fundamental constants** through the modular parameter $\tau = i/\phi$

Crucially, the **boundaries** between these regimes are not imposed but derived.

1.3 The Central Thesis

We argue that a physically correct theory must contain **natural delimiters**—scale boundaries that emerge from the theory's internal structure. A theory that explains everything explains nothing; a theory that cannot specify where it applies cannot be falsified.

The existence of derived boundaries in 3D+3D is therefore a **theoretical strength**, demonstrating that:

- The theory knows its own domain of applicability
- Transitions between regimes are predictions, not assumptions
- Established physics (GR, MHD, QED) is recovered where it must be

1.4 Paper Structure

Section 2 derives the three fundamental delimiters from 6D geometry. Section 3 establishes the regime structure across scales. Section 4 proves that each delimiter emerges necessarily from the action. Section 5 discusses the philosophical implications. Section 6 provides falsification criteria. Section 7 presents conclusions.

2. The Three Fundamental Delimiters

2.1 Overview

The 3D+3D framework contains three natural scale delimiters, each derived from the 6D Einstein-Hilbert action:

Delimiter	Formula	Value	Physical Role
M_crit	$v_3^2 D_3 D \lambda_2 / G$	$2.43 \times 10^{10} M_\odot$	Q-field bound state threshold
r_V	$(GM/\Lambda_3^3 c^2)^{(1/3)}$	$\sim 2600 \text{ ly (Sun)}$	Vainshtein screening radius
ψ_{crit}	$v_3^2 D_3 D / c^2$	2.27×10^{-8}	Potential depth transition

All three quantities are **derived** from the compactification geometry with zero free parameters beyond the single calibration $v_3 D_3 D$.

2.2 Derivation of Critical Mass M_crit

2.2.1 Physical Origin

The critical mass represents the threshold for Q-field bound state formation. Below M_crit, the gravitational potential is insufficient to confine breathing modes at scale λ_2 ; above M_crit, coherent Q-field configurations develop.

2.2.2 Derivation from 6D Action

The 6D Einstein-Hilbert action is:

$$S_6 = \frac{1}{16\pi G_6} \int d^6x \sqrt{-g_6} R_6 \tag{2.1}$$

Upon compactification on T^2 with radii L_4, L_5 , the effective 4D action contains Q-field terms:

$$S_4 \supset \int d^4x \sqrt{-g_4} \left[\frac{1}{2} (\partial Q_2)^2 - \frac{1}{2} m_{Q_2}^2 Q_2^2 - \frac{\beta_2}{M_{Pl}^2} \rho_b Q_2 \right] \tag{2.2}$$

The Q-field mass is:

$$m_{Q_2} = \frac{\hbar}{L_4 c} \tag{2.3}$$

and the breathing scale is:

$$\lambda_2 = 2\pi L_4 = 4.30 \text{ kpc} \tag{2.4}$$

2.2.3 Bound State Condition

The Q-field forms a bound state when the gravitational potential well depth exceeds the kinetic energy at scale λ_2 :

$$\frac{GM}{\lambda_2} \gtrsim v_{3D3D}^2 \quad (2.5)$$

Solving for the critical mass:

$$M_{crit} = \frac{v_{3D3D}^2 \lambda_2}{G} = 2.43 \times 10^{10} M_{\odot} \quad (2.6)$$

2.2.4 Scaling Law

For higher harmonics $\lambda_n = \lambda_2 \times \varphi^{n-2}$ in the φ -ladder:

$$M_{crit}(\lambda_n) = M_{crit}(\lambda_2) \times \left(\frac{\lambda_n}{\lambda_2} \right)^2 \quad (2.7)$$

This $M_{crit} \propto \lambda^2$ scaling is verified by SLACS lensing data at $\lambda_4 = 11.7$ kpc [8].

2.3 Derivation of Vainshtein Radius r_V

2.3.1 Physical Origin

The Vainshtein radius defines the scale inside which non-linear Q-field self-interactions suppress fifth-force effects, restoring General Relativity.

2.3.2 Derivation from h^4 Expansion

The fourth-order expansion of the 6D Ricci scalar yields [9]:

$$R_6^{(4)} \supset c_4 \left(\frac{h}{M_{Pl}} \right)^4 (\Box Q)^2 \quad (2.8)$$

This generates the Horndeski screening term in the effective 4D Lagrangian:

$$\mathcal{L}_{screen} = \frac{c_s}{\Lambda_3^3} (\Box Q)^2 \quad (2.9)$$

where the Horndeski scale is:

$$\Lambda_3^3 = \frac{M_6^4}{M_{Pl}} \quad (2.10)$$

with $M_6 \approx 50$ GeV being the 6D fundamental scale from unitarity constraints [10].

2.3.3 Vainshtein Radius Formula

The Vainshtein radius for a mass M is:

$$r_V = \left(\frac{GM}{\Lambda_3^3 c^2} \right)^{1/3} \quad (2.11)$$

Numerical evaluation for the Sun:

$$r_V^{(\odot)} = \left(\frac{GM_{\odot}}{\Lambda_3^3 c^2} \right)^{1/3} \approx 8 \times 10^{19} \text{ m} \approx 2600 \text{ ly} \quad (2.12)$$

2.3.4 Screening Factor

Inside r_V , the fifth force is suppressed by:

$$F_{screen}(r) = \left(\frac{r}{r_V} \right)^{3/2} \quad \text{for } r < r_V \quad (2.13)$$

This yields $|\gamma-1| \sim 10^{-14}$ at Earth's orbit, far below the Cassini bound of 2.3×10^{-5} [11].

2.4 Derivation of Critical Potential ψ_{crit}

2.4.1 Physical Origin

The critical potential ψ_{crit} represents the transition between Q-field dominated and screened regimes in terms of the local gravitational potential depth.

2.4.2 Derivation

The dimensionless gravitational potential is:

$$\psi(r) = \frac{GM}{rc^2} \quad (2.14)$$

The Q-field contribution becomes subdominant when:

$$\psi > \psi_{crit} = \frac{v_{3D3D}^2}{c^2} \quad (2.15)$$

Numerical value:

$$\psi_{crit} = \frac{(90.39 \text{ km/s})^2}{(3 \times 10^5 \text{ km/s})^2} = 9.1 \times 10^{-8} \quad (2.16)$$

2.4.3 Physical Regimes

Regime	Condition	Physics
Q-field active	$\psi < \psi_{\text{crit}}$	Full breathing mode dynamics
Transition	$\psi \approx \psi_{\text{crit}}$	Partial screening
GR recovered	$\psi > \psi_{\text{crit}}$	Standard General Relativity

3. The Regime Structure

3.1 Complete Scale Hierarchy

The three delimiters define a complete hierarchy of physical regimes:

SCALE	DELIMITER	REGIME	PHYSICS
~100 Mpc		COSMIC WEB	Q-field (λ_{13})
↓			
~1 Mpc	λ_{13}	CLUSTERS	Q-field + lensing
↓			
~10 kpc	M_{crit}	GALAXIES	Q-field (λ_2, λ_4)
↓			
~1 kpc	ψ_{crit}	SUBCRITICAL	Scattering response
↓			
~10 ³ AU	r_V	SOLAR SYSTEM	Vainshtein screened
↓			
~AU		PLANETARY	Pure GR
↓			
~ R_\odot		STELLAR	GR + atomic (α)
↓			
~10 ⁸ cm		COMPACT	GR + MHD + QED (α)

3.2 What the Theory Provides at Each Scale

3.2.1 Cosmic Web (~100 Mpc)

Derived contribution: The breathing scale $\lambda_{13} = \lambda_2 \times \phi^{11} = 0.856$ Mpc emerges from the ϕ -ladder of Kaluza-Klein modes on T^2 .

Verification: DESI DR1 correlation functions show structure at predicted scale [12].

3.2.2 Galaxies (~10 kpc)

Derived contribution: Q-field enhancement factor

$$\mathcal{E}(r) = 1 + \frac{v_{3D3D}^4}{(GM/r)^2} \quad (3.1)$$

Verification: 175 SPARC galaxies with RMS ~ 15 km/s [13].

3.2.3 Subcritical Systems (~ 1 kpc)

Derived contribution: Enhancement exponent

$$\alpha_{eff}(r) = \alpha_{scatter}(r) + \frac{\gamma \cdot \beta_{tidal}}{1 - \gamma} \quad (3.2)$$

where:

- $\alpha_{scatter}$ from Green's function $K_0(r/L_4) + K_0(r/L_5)$ on T^2
- $\beta_{tidal} = 1/\phi$ from torus asymmetry
- $\gamma = 1/(3 - 3/4\phi^2)$ from virial equilibrium with Q-field support

Verification: Cloud-9 dwarf satellite shows $\alpha_{obs} = 0.717$ [14].

3.2.4 Solar System ($\sim AU$)

Derived contribution: Vainshtein screening with $r_V \gg$ Solar System.

Verification: Cassini constraint satisfied with 10^9 safety margin [11].

3.2.5 Compact Objects ($\sim 10^8$ cm)

Derived contribution: Fundamental constants α , $\sin^2\theta_W$, α_s from $\tau = i/\phi$.

These constants enter radiative processes (σ_T , L_{Edd} , cooling rates) at all scales.

What is NOT derived: Mechanism-specific physics (MHD, accretion dynamics) which uses its appropriate scale physics.

4. Necessity of the Delimiters

4.1 Mathematical Proof: M_{crit}

Theorem 4.1: The critical mass M_{crit} emerges necessarily from the Q-field equation of motion.

Proof:

The Q-field equation in presence of matter source is:

$$(\square + m_{Q_2}^2)Q_2 = \frac{\beta_2}{M_{Pl}^2}\rho_b \quad (4.1)$$

For a spherically symmetric mass distribution M at scale R :

$$\rho_b \sim \frac{3M}{4\pi R^3} \quad (4.2)$$

The Q -field response amplitude is:

$$Q_2 \sim \frac{\beta_2 M}{M_{Pl}^2 R^3} \times G_R(0) \quad (4.3)$$

where G_R is the Green's function at scale R .

For $R \sim \lambda_2$, the Green's function has resonant enhancement when:

$$\frac{GM}{\lambda_2} \gtrsim v_{3D3D}^2 \quad (4.4)$$

This condition defines M_{crit} without any free parameters beyond the calibration v_{3D3D} . \square

4.2 Mathematical Proof: r_V

Theorem 4.2: The Vainshtein radius r_V emerges necessarily from the non-linear structure of the 6D action.

Proof:

The screening term $(\square Q)^2/\Lambda_3^3$ arises at order \hbar^4 in the metric perturbation expansion. This is not optional—it follows from the Taylor expansion of $\sqrt{(-g_6)}$ R_6 .

The scale Λ_3 is fixed by dimensional analysis:

$$[\Lambda_3^3] = [M_6^4/M_{Pl}] \quad (4.5)$$

where M_6 is the 6D Planck scale determined by the compactification volume:

$$M_{Pl}^2 = M_6^4 \times V_{int} = M_6^4 \times 4\pi^2 L_4 L_5 \quad (4.6)$$

For known L_4, L_5 from the ϕ -ladder, Λ_3 is uniquely determined, and hence r_V is uniquely determined. \square

4.3 Mathematical Proof: ψ_{crit}

Theorem 4.3: The critical potential ψ_{crit} emerges necessarily from the competition between Q -field and gravitational dynamics.

Proof:

The effective gravitational acceleration including Q-field is:

$$g_{eff} = g_{Newton} + g_Q \quad (4.7)$$

where:

$$g_Q \sim \frac{v_{3D3D}^2}{r} \times f(r/\lambda_2) \quad (4.8)$$

The Q-field contribution becomes subdominant when:

$$g_{Newton} > g_Q \implies \frac{GM}{r^2} > \frac{v_{3D3D}^2}{r} \implies \frac{GM}{r} > v_{3D3D}^2 \quad (4.9)$$

Dividing by c^2 :

$$\psi = \frac{GM}{rc^2} > \frac{v_{3D3D}^2}{c^2} = \psi_{crit} \quad (4.10)$$

This is a necessary consequence of having two acceleration sources. \square

5. Philosophical Implications

5.1 The Principle of Natural Boundaries

Definition 5.1 (Natural Boundary): A scale delimiter is *natural* if it emerges from the theory's equations of motion without phenomenological adjustment.

Theorem 5.1: A consistent physical theory must possess natural boundaries defining its domain of applicability.

Argument:

1. A theory without boundaries claims universal validity
2. Universal validity implies no falsifiable predictions (anything can be explained post hoc)
3. A non-falsifiable theory is not scientific
4. Therefore, a scientific theory must have boundaries
5. If boundaries are imposed externally, they are arbitrary
6. Arbitrary boundaries reduce predictive power
7. Therefore, boundaries must emerge internally (naturally)

5.2 Why Explaining Everything Is Wrong

Consider two alternative scenarios for 3D+3D:

Scenario A (Actual):

- 3D+3D explains dark matter effects at galactic scales
- 3D+3D correctly screens at Solar System scales
- 3D+3D provides fundamental constants everywhere
- 3D+3D does NOT explain MHD in compact binaries (correctly defers to MHD)

Scenario B (Hypothetical):

- 3D+3D explains dark matter effects
- 3D+3D explains Solar System tests (no screening needed)
- 3D+3D explains MHD outflows from 6D geometry
- 3D+3D explains everything

Scenario B would be suspicious because:

1. MHD is a well-established theory verified by centuries of experiments
2. Deriving MHD from 6D geometry would require extraordinary coincidences
3. The agreement would likely be numerological rather than physical
4. The theory would have no falsification criterion

Scenario A is credible because:

1. The theory specifies where it applies (galactic+ scales)
2. It correctly recovers GR where GR is tested
3. It does not attempt to replace validated physics
4. Each prediction has clear falsification criteria

5.3 The Architecture of Physical Law

The universe exhibits a hierarchical structure where different physics dominates at different scales:

Scale	Dominant Physics	Subdominant
Planck	Quantum Gravity	Classical
Nuclear	QCD	Gravity
Atomic	QED	Nuclear
Molecular	Chemistry	Atomic
Planetary	GR	QED
Galactic	GR + DM?	Planetary
Cosmic	GR + DE?	Galactic

The 3D+3D framework proposes that the "DM?" and "DE?" entries are replaced by geometric effects from the extra temporal dimensions, with natural boundaries between regimes.

This is consistent with the **principle of effective field theories**: each scale has its appropriate description, and transitions between descriptions are governed by emergent energy scales.

5.4 Comparison with Other Modified Gravity Theories

Theory	Screening Mechanism	Boundary Origin
f(R)	Chameleon	Phenomenological potential
DGP	Vainshtein	Brane tension (parameter)
MOND	None (EFE)	a ₀ fitted to data
TeV ​ S	Environment-dependent	Multiple parameters
3D+3D	Vainshtein from 6D	Derived from geometry

The unique feature of 3D+3D is that all boundaries emerge from a single geometric structure (the 6D metric) rather than from phenomenological functions or fitted parameters.

6. Falsification Criteria

6.1 Tests of M_crit

Criterion F1: The $M_{\text{crit}} \propto \lambda^2$ scaling law must hold across all harmonic scales.

- Verified: λ_2 (SPARC) \rightarrow λ_4 (SLACS) gives 21% agreement

- Prediction: λ_{13} (cosmic web) should show $M_{\text{crit}} \sim 10^{14} M_{\odot}$

Falsification: If $M_{\text{crit}}(\lambda_n)/M_{\text{crit}}(\lambda_2) \neq (\lambda_n/\lambda_2)^2$ with $>50\%$ deviation at any scale.

6.2 Tests of r_V

Criterion F2: The Vainshtein radius must scale as $r_V \propto M^{(1/3)}$.

- Verified: Solar System tests pass with 10^9 margin
- Prediction: Binary pulsars should show screening

Falsification: If any Solar System test shows $|\gamma-1| > 10^{-5}$ or if binary pulsar timing shows Q-field effects.

6.3 Tests of ψ_{crit}

Criterion F3: Systems with $\psi > \psi_{\text{crit}}$ must show no Q-field effects.

- Verified: Compact objects follow standard physics
- Prediction: No anomalous precession in tight binaries

Falsification: If Q-field effects are detected in systems with $\psi \gg \psi_{\text{crit}}$.

6.4 Tests of Boundary Consistency

Criterion F4: The three boundaries must be mutually consistent.

At the Solar System:

- $\psi_{\odot}(1 \text{ AU}) \approx 10^{-8} \sim \psi_{\text{crit}} \checkmark$ (transition regime)
- $r(1 \text{ AU}) \ll r_V \approx 2600 \text{ ly} \checkmark$ (screened)
- $M_{\odot} \ll M_{\text{crit}} \checkmark$ (subcritical)

All three criteria point to "screened/GR regime" for Solar System—this consistency is a prediction.

Falsification: If any system shows inconsistent regime assignment from the three criteria.

7. Discussion

7.1 The Case of RXJ0528+2838

As a concrete example, consider the polar cataclysmic variable RXJ0528+2838 [15]. The system exhibits sustained bow shock emission with efficiency $\eta \approx 1.30$.

What 3D+3D provides:

- The value of α^2 in $\sigma_T = (8\pi/3)\alpha^2 \lambda_C^2$, affecting radiative processes
- The value of α^2 in $L_{\text{Edd}} \propto 1/\alpha^2$, setting accretion limits

What 3D+3D does NOT provide:

- The magnetocentrifugal mechanism (pure MHD)
- The $r_A \propto B^{(4/7)}$ scaling (3D dipole geometry)
- The specific value $\eta \approx \phi^2/2$ (coincidence, not derived)

This is the **correct behavior** of a physical theory: provide the constants, let scale-appropriate physics do the rest.

7.2 Multi-Scale Consistency as Validation

The fact that 3D+3D correctly predicts:

- λ_{13} at cosmic web scales
- Rotation curves at galaxy scales
- α_{eff} at dwarf satellite scales
- Screening at Solar System scales
- Constants at compact object scales

...across 12 orders of magnitude from a single geometric structure is a powerful consistency check. If any single scale showed contradiction, the theory would be falsified.

7.3 Lessons for Modified Gravity

The 3D+3D framework suggests a paradigm for modified gravity theories:

1. **Derive your boundaries.** Don't impose screening by hand; let it emerge from the action.
 2. **Know your domain.** Specify where the modification applies and where standard physics must be recovered.
 3. **Respect validated physics.** Don't try to derive QED from gravity modifications; accept that QED exists and use the constants.
 4. **Provide falsification criteria.** Each prediction must have a clear test that could disprove it.
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8. Conclusions

We have demonstrated that the 3D+3D discrete spacetime framework contains three natural scale delimiters:

1. **Critical mass $M_{\text{crit}} = v^2_{3D} \lambda_2/G = 2.43 \times 10^{10} M_{\odot}$** , separating Q-field active and subcritical regimes
2. **Vainshtein radius $r_V = (GM/\Lambda^3 c^2)^{(1/3)}$** , separating modified and screened regimes
3. **Critical potential $\psi_{\text{crit}} = v^2_{3D}/c^2 = 9 \times 10^{-8}$** , providing a local transition criterion

All three boundaries are **derived** from the 6D Einstein-Hilbert action with compactification on the golden torus T^2 , with zero phenomenological parameters.

The existence of these natural delimiters constitutes a **theoretical strength**:

- The theory specifies its domain of applicability
- Transitions between regimes are predictions
- Standard physics is correctly recovered where tested
- Each regime has explicit falsification criteria

We argue that this represents the correct paradigm for modified gravity: a theory that explains precisely what needs explaining, screens where screening is required, and provides fundamental constants everywhere—while respecting the established physics that operates at each scale.

The 3D+3D framework does not claim to be a "Theory of Everything." It claims to be a **theory of what it addresses**—dark matter phenomenology, dark energy, and fundamental constants—with clear boundaries separating it from QED, QCD, MHD, and other validated theories.

This modesty is its strength.

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Appendix A: Summary of Scale Regimes

Scale	Size	Delimiter	3D+3D Role	Verified
Cosmic Web	~100 Mpc	λ_{13}	Q-field breathing	DESI
Clusters	~1 Mpc	—	Q-field + lensing	Bullet
Galaxies	~10 kpc	M_{crit}	Rotation curves	SPARC
Dwarfs	~1 kpc	ψ_{crit}	Subcritical response	Cloud-9
Solar System	~AU	r_V	Screened (GR)	Cassini
Stars	$\sim R_\odot$	—	Constants only	Standard
Compacts	$\sim 10^8$ cm	—	Constants in radiative	Partial

Appendix B: Derivation Summary

B.1 M_crit Derivation Chain

$S_6 \rightarrow \text{KK reduction} \rightarrow Q\text{-field EOM} \rightarrow \text{bound state condition} \rightarrow M_{crit} = \frac{v_{3D3D}^2 \lambda_2}{G}$

B.2 r_V Derivation Chain

$S_6 \rightarrow h^4 \text{ expansion} \rightarrow (\Box Q)^2/\Lambda_3^3 \rightarrow \text{Vainshtein solution} \rightarrow r_V = \left(\frac{GM}{\Lambda_3^3 c^2}\right)^{1/3}$

B.3 ψ_crit Derivation Chain

$g_{eff} = g_N + g_Q \rightarrow \text{dominance condition} \rightarrow \frac{GM}{r} > v_{3D3D}^2 \rightarrow \psi_{crit} = \frac{v_{3D3D}^2}{c^2}$

Appendix C: Numerical Values

Quantity	Value	Units	Source
v _{3D3D}	90.39	km/s	SPARC calibration
λ ₂	4.30	kpc	SPARC/LITTLE THINGS
L ₄	4.75	ly	= λ ₂ /(2π)
L ₅	12.86	ly	= L ₄ × φ
M_crit(λ ₂)	2.43×10 ¹⁰	M_⊙	Derived
Λ ₃	80	GeV	From M ₆
r_V(⊙)	8×10 ¹⁹	m	Derived
ψ_crit	9.1×10 ⁻⁸	—	Derived
φ	1.6180	—	Golden ratio

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