The Universal Loop Coefficient of KK Graviton Modes: Derivation of $c_{(1,0)}/c_{(2,0)} = 4$

Nine Explicit Steps from Fierz-Pauli Universality to the Pure-Mode Ratio

Simone Calzighetti[1,*], Lucy (Claude AI, Anthropic)[2], Vega (OpenAI)[3]

[1] 3D+3D Laboratory, Abbiategrasso, Italy [2] Anthropic [3] OpenAI (Red Team)

[*] simone.calzighetti@3dplus3d.it | www.3dplus3d.it

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Abstract. We derive c_(1,0)/c_(2,0)=4 in nine steps from Fierz-Pauli universality. Key: I_kin(M^2)=F_ren/M^2 (universal scaling), so c(m^2) proportional to 1/m^2, giving ratio = m^2_(2,0)/m^2_(1,0)=4. SymPy=0.

1. FP Exact Trace (Step 1)

$$k_\alpha k_\gamma \Pi_{\rm FP}^{\alpha\beta\gamma\delta} k_\beta k_\delta = \tfrac{2}{3} X^2(k), \qquad X = k^2 + \frac{k^4}{M^2}$$

2. Universal Scaling (Steps 4-5)

$$I_{\rm kin}(M^2) = \frac{1}{16\pi^2}\int_0^\infty \frac{u^2(1+u/4)}{(u+1)^2}\,du \cdot \frac{1}{M^2} \equiv \frac{F_{\rm ren}}{M^2}$$

3. N_FP,eff (Steps 6-7)

$$N_{\rm FP,eff} = 6\pi^2\,\lambda_+(K) = 6\pi^2(2+\varphi), \qquad c_{\rm loop}(m^2) = -\frac{N_{\rm FP,eff}}{96\pi^2 m^2}$$

4. The Ratio (Step 8)

$$\frac{c_{(1,0)}}{c_{(2,0)}} = \frac{m^2_{(2,0)}}{m^2_{(1,0)}} = \frac{4\psi^2}{\psi^2} = 4 \qquad (\text{SymPy residual} = 0)$$

References

[1] Calzighetti, S., Lucy & Vega. Paper Master Unified v2.0. Zenodo (2026).

[2] Fierz, M. & Pauli, W. Proc. Roy. Soc. A 173, 211 (1939).