N-Body Validation of the 3D+3D Geometric Fifth Force: Scale-Dependent Power Spectrum Enhancement and Degeneracy Breaking

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Abstract

We present the first N-body cosmological simulation of the 3D+3D discrete spacetime framework, implemented as a modification of the Gadget-4 TreePM code. The 3D+3D theory proposes six-dimensional spacetime with signature (−,+,+,+,−,−) where compactified temporal dimensions generate scalar Q-fields that modify gravity through a scale-dependent fifth force mu(k,a). We run three matched simulations — LambdaCDM baseline, full 3D+3D (with fifth force), and background-only control — each with 256^3 = 16,777,216 dark matter particles in a 200 Mpc/h periodic box from z = 99 to z = 0. At z = 0, we measure: (i) a scale-dependent power spectrum enhancement of +2.6% at k < 0.05 h/Mpc declining to +1.1% at k > 0.5 h/Mpc, with screening signal Delta = 0.015; (ii) linear growth factor ratio D(3D3D)/D(LambdaCDM) = 1.0115; (iii) f*sigma_8 enhancement of +2.9%, within the sensitivity window of Euclid and DESI; (iv) growth index Delta(gamma_eff) = −0.042 at z ~ 1. A three-way decomposition demonstrates that 100% of the observed effect originates from the fifth force mu(k,a), with zero contribution from background cosmology modifications. The 3D+3D fingerprint — simultaneously positive Delta(f*sigma_8), negative power spectrum slope with scale, and negative Delta(gamma) — occupies a unique region in the model degeneracy plane, distinguishable from f(R) gravity, massive neutrinos, w0wa-CDM quintessence, and baryonic feedback. This constitutes a falsifiable prediction for upcoming large-scale structure surveys.

Keywords: modified gravity, N-body simulation, extra dimensions, dark matter alternative, power spectrum, growth rate, Gadget-4, scale-dependent screening

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1. Introduction

The standard cosmological model (LambdaCDM) successfully describes the large-scale structure of the universe but relies on two unexplained components: a cosmological constant Lambda responsible for ~68.5% of the energy budget, and cold dark matter (CDM) constituting ~26.5%. Despite decades of experimental searches, no dark matter particle has been directly detected, motivating exploration of alternative gravitational frameworks.

The 3D+3D discrete spacetime theory (Calzighetti & Lucy, 2025–2026) proposes a six-dimensional spacetime with metric signature (−,+,+,+,−,−), where two additional temporal dimensions (tau_2, tau_3) are compactified on a 2-torus T^2 with canonical parameters L_2 = 9.5 ly, L_3 = 6.0 ly, T_2 = 30 yr, T_3 = 19 yr (Clarification

Note, Calzighetti & Lucy 2026). The Kaluza-Klein reduction of the 6D Einstein-Hilbert action produces scalar Q-fields (Q_2, Q_3) that couple to matter gravitationally, generating an effective fifth force that enhances gravitational binding at galactic and cosmic scales while being screened at solar system scales through Vainshtein-type mechanisms.

Previous work in this series has demonstrated consistency of the 3D+3D framework with galactic rotation curves (175 SPARC galaxies, RMS ~ 15 km/s), SLACS gravitational lensing (4-sigma detection of screening at M_{crit}), cosmic web structure at the characteristic scale $\lambda_{13} = 0.856$ Mpc, and NANOGrav pulsar timing arrays. The framework derives all 42 Standard Model parameters from 6D geometry with average 1.2% error. However, all previous cosmological tests have relied on semi-analytic or perturbative calculations.

In this paper, we present the first fully nonlinear N-body simulation of the 3D+3D framework, implemented as a modification of the public Gadget-4 code (Springel et al. 2021). This allows direct measurement of the matter power spectrum P(k), growth rate f(z), and derived observables $f\sigma_8(z)$ and the growth index γ_{eff}, providing quantitative predictions testable by Euclid (Laureijs et al. 2011) and DESI (DESI Collaboration 2016).

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2. Theoretical Framework

2.1 The 3D+3D Metric and Q-Field Origin

The six-dimensional spacetime has the fundamental metric:

> $$ds^2_6 = -c^2 dt^2 + dx^2 + dy^2 + dz^2 - c^2 d(\tau_2)^2 - c^2 d(\tau_3)^2 \quad (1)$$

with periodic identification $\tau_a \sim \tau_a + T_a$. The Kaluza-Klein decomposition of the 6D graviton field yields, in the zero-mode sector, the 4D graviton plus two scalar fields Q_2 and Q_3 (the breathing modes of the compact dimensions). These Q-fields satisfy coupled Klein-Gordon equations with masses $m_a = \hbar/(L_a * c)$, where the compactification scales are $L_2 = 9.5$ ly and $L_3 = 6.0$ ly, giving ultralight masses $m_2 \sim 1.47 \times 10^{-24}$ eV and $m_3 \sim 2.32 \times 10^{-24}$ eV.

2.2 Effective Gravitational Modification

The Q-fields mediate a scale-dependent modification to the Poisson equation in Fourier space:

> $$\nabla^2 \Phi = 4\pi G * [1 + \mu(k,a)] * \rho \quad (2)$$

where the modification function $\mu(k,a)$ encapsulates the fifth force contribution. From the 3D+3D theory, this takes the factorized form:

> $$\mu(k,a) = \mu_0 * S(a) * G(k) \quad (3)$$

where the three factors are:

Amplitude: $\mu_0 = 0.05$, the maximum gravitational enhancement at $z = 0$ on large scales. This is derived from the Q-field coupling strength to matter, constrained by SPARC rotation curve fits ($\beta \sim 3.0$) and normalized to cosmological scales.

Temporal activation: S(a) = [a^3 / (a^3 + a_t^3)] / [1/(1 + a_t^3)] with a_t = 0.45 (z_t ~ 1.2). This sigmoid ensures that the Q-field effect grows from zero at early times (when the compact dimensions are effectively frozen) to full strength at late times, consistent with the temporal breathing mode activation. The cubic power arises from the volume scaling of the compact torus in an expanding universe.

Scale-dependent screening (Lorentzian): G(k) = 1/(1 + (k/k_mu)^2) with k_mu = 0.20 h/Mpc. This Lorentzian profile arises from the Yukawa-type spatial dependence of the Q-field Green's function: the Q-field Compton wavelength lambda_Q = 1/m_eff sets the transition scale. At k << k_mu (large scales), mu -> mu_0 * S(a) (full enhancement). At k >> k_mu (small scales), mu -> 0 (Newtonian gravity recovered). This is the Fourier-space manifestation of the Vainshtein screening mechanism.

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3. Numerical Implementation in Gadget-4

3.1 Code Modification Strategy

We implement the 3D+3D fifth force in the Gadget-4 cosmological N-body code (Springel et al. 2021), which uses a hybrid TreePM algorithm: long-range forces are computed via the Particle-Mesh (PM) method with FFT, while short-range forces use a hierarchical oct-tree. Our modification applies exclusively to the PM solver, multiplying the Fourier-space gravitational potential by (1 + mu(k,a)) in the function `compute_potential_kspace()`. The tree component remains unmodified (Newtonian), providing an effective screening at small scales that mirrors the physical Vainshtein mechanism.

3.2 Source Code Modification

The modification consists of an inline physics header defining the namespace `ThreePlus3D_PM` with a single function `mu_effective(k, a, mu0, k_mu)` implementing Equation (3). This is inserted into `pm_periodic.cc` at both the slab-decomposition and column-decomposition branches of `compute_potential_kspace()`. The key code block is:

```cpp
double k_phys = sqrt(k2_mode) * k_fund * 1000.0;  // h/kpc -> h/Mpc
double mu_val = ThreePlus3D_PM::mu_effective(k_phys, a, 0.05, 0.20);
fft_of_rhogrid[ip] *= complex<fft_real>(1.0 + mu_val, 0.0);
```

The scale factor `a` is passed from `pmforce_periodic()` through the modified function signature `compute_potential_kspace(double a)`. Unit conversion from Gadget-4 internal units (kpc/h) to physical units (h/Mpc) is performed by the factor 1000.0.

3.3 Compilation Flags

Three compile-time flags control the 3D+3D physics: `THREEPLUS3D_GEOMETRIC_DE` (geometric dark energy background, not yet implemented in H(a)), `THREEPLUS3D_MU_GRAVITY` (fifth force in PM solver, active), and `THREEPLUS3D_VERBOSE` (diagnostic output). The build system uses Meson with MPI support (8 tasks) and double precision FFT (`DOUBLEPRECISION_FFTW`).

3.4 Simulation Suite

We run three matched simulations sharing identical initial conditions (seed = 12345):

Run	mu_0	Omega_Q background	Purpose
LambdaCDM	0 (off)	Standard Lambda	Baseline reference
3D+3D v3 (full)	0.05	Standard Lambda*	Full fifth force
Background-only	0 (off)	Standard Lambda*	Isolate background

Table 1. Simulation suite. *The geometric dark energy Omega_Q(a) modification to H(a) is defined in Config.sh but not yet implemented in the Hubble function code. All three runs use identical H(a) = H_0 * [Omega_m / a^3 + (1 − Omega_m − Omega_Lambda) / a^2 + Omega_Lambda]^(1/2). This is explicitly verified by the background-only control run yielding P(k) ratios of exactly 1.000000 at all scales. The entire measured signal therefore originates exclusively from mu(k,a) in the PM solver.

3.5 Simulation Parameters

Parameter	Value	Description
N_particles	256^3 = 16,777,216	DM only (Type 1)
BoxSize	200 Mpc/h	Periodic cube
PMGRID	256	PM mesh resolution
Omega_m	0.315	Planck 2018
Omega_Lambda	0.685	Planck 2018
h	0.674	Planck 2018
sigma_8	0.811	Input normalization
n_s	0.9649	Planck 2018
z_init	99	Starting redshift
z_final	0	End redshift
N_snapshots	13	Output times
Softening	50 kpc/h (comoving)	Gravitational softening
k_fundamental	0.0314 h/Mpc	2*pi / L_box
k_Nyquist	4.02 h/Mpc	pi * N / L_box

Table 2. Simulation parameters. Cosmological parameters from Planck 2018 (Planck Collaboration, 2020).

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4. Analysis Methodology

4.1 Power Spectrum Estimation

The matter power spectrum P(k) is estimated from each snapshot using a standard pipeline: (1) Cloud-in-Cell (CIC) mass assignment to a 256^3 grid (Hockney & Eastwood 1981); (2) FFT of the density contrast field delta(x) = rho/rho_bar − 1; (3) CIC window function deconvolution via division by W_CIC(k) = [product_i sinc(pi * k_i / (2 * k_Ny))]^2 (Jing 2005); (4) Poisson shot noise subtraction P_shot = V_box / N_part = 0.476 (Mpc/h)^3; (5) Isotropic averaging in 80 logarithmic k-bins from k_fund to k_Ny/2.

4.2 Linear Growth Factor

The linear growth factor D(a) is extracted from the power spectrum restricted to purely linear modes k < 0.10 h/Mpc, where D(a) is proportional to sqrt(P_lin(k,a)). This avoids contamination from nonlinear mode coupling at high k. The growth rate f(z) = d(ln D)/d(ln a) is computed via central finite differences between adjacent snapshots.

4.3 RSD Observable f*sigma_8(z)

The combination f(z)*sigma_8(z) is the primary observable for redshift-space distortion (RSD) surveys. We compute sigma_8(z) from the full power spectrum integrated with a top-hat filter at R = 8 Mpc/h, and calibrate using the measured sigma_8(z=0) values: sigma_8(LambdaCDM) = 0.914 and sigma_8(3D+3D) = 0.922.

4.4 Growth Index gamma_eff

The growth index gamma is defined through f(z) = Omega_m(a)^gamma, where Omega_m(a) = Omega_m * a^(−3) / E^2(a). We compute gamma_eff = ln(f) / ln(Omega_m(a)) only at z < 1, where ln(Omega_m(a)) is sufficiently far from zero to avoid numerical divergence. At z >> 1, Omega_m approaches 1 and gamma_eff becomes mathematically ill-defined.

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5. Results

5.1 Power Spectrum Ratio P(k) at z = 0

Table 3 presents the power spectrum ratio P(3D3D,k)/P(LambdaCDM,k) measured in three k-bands at z = 0. The ratio exceeds unity at all scales, with a clear scale dependence: the enhancement is largest at low k (large scales) and smallest at high k (small scales), consistent with the Lorentzian screening profile G(k) = 1/(1 + (k/k_mu)^2).

k-band [h/Mpc]	P_3D3D / P_LambdaCDM	BG only / LambdaCDM	mu only (v3/BG)	Screening
k < 0.05	1.026088	1.000000	1.026088	—
k = 0.1–0.2	1.018171	1.000000	1.018171	—
k > 0.5	1.010876	1.000000	1.010876	—
Delta (low−high)	0.015212	0.000000	0.015212	1.52%

Table 3. Three-way power spectrum decomposition at z = 0. The background-only run yields P(k) ratios of exactly 1.000000 at all scales, demonstrating that the entire measured signal (2.6% at k < 0.05 declining to 1.1% at k > 0.5) originates exclusively from the fifth force mu(k,a). The screening signal Delta = ratio(k<0.05) − ratio(k>0.5) = 0.015 confirms scale-dependent screening.

5.2 Redshift Evolution of P(k) Ratio

Table 4 shows the evolution of the P(k) ratio across selected snapshots. The enhancement grows monotonically with the scale factor a, tracking the sigmoid activation S(a). At all redshifts, the ratio is larger at low k than at high k, confirming persistent scale-dependent screening.

z	a	S(a)	k < 0.05	k = 0.1–0.2	k > 0.5	Screening
99.0	0.010	1.2 x 10^(−5)	1.000000	1.000000	—	—
49.5	0.020	9.3 x 10^(−5)	1.000001	1.000001	—	—
18.8	0.051	0.00154	1.000013	1.000009	1.000006	7 x 10^(−6)
9.0	0.100	0.0118	1.000104	1.000071	1.000015	8.9 x 10^(−5)
4.0	0.198	0.0858	1.000793	1.000541	1.000095	7.0 x 10^(−4)
2.0	0.328	0.304	1.00329	1.00227	1.00073	0.00256
1.0	0.505	0.639	1.00941	1.00658	1.00320	0.00620
0.5	0.673	0.840	1.01606	1.01127	1.00630	0.00977
0.0	1.000	1.000	1.02609	1.01817	1.01088	0.01521

Table 4. Redshift evolution of P(k) ratio in three bands. The ratio grows monotonically with S(a) at all scales, with persistent scale-dependent screening. The transition from undetectable (z > 20) to percent-level (z < 2) enhancement tracks the sigmoid activation of Q-field breathing modes.

5.3 Linear Growth Factor and f*sigma_8(z)

Using only linear modes (k < 0.10 h/Mpc) to construct D_lin(a), we find:

z	a	D_3D / D_Lambda	f_3D	f_Lambda	Delta_f %	f*sigma8_3D	f*sigma8_Lambda
9.00	0.100	1.000047	0.999	0.999	0.0	0.115	0.115
4.05	0.198	1.000359	0.994	0.994	0.0	0.226	0.226
2.05	0.328	1.001482	0.976	0.972	+0.4	0.364	0.363
0.98	0.505	1.004204	0.878	0.869	+1.0	0.493	0.487
0.49	0.673	1.007139	0.802	0.791	+1.4	0.573	0.563
0.00	1.000	1.011521	0.548	0.538	+1.9	0.506	0.492

Table 5. Linear growth factor ratio, growth rate f(z), and f*sigma_8(z) from linear modes (k < 0.10 h/Mpc). The 3D+3D growth rate systematically exceeds LambdaCDM at z < 2, reaching +1.9% at z = 0. f*sigma_8(3D+3D) / f*sigma_8(LambdaCDM) = 1.029 at z = 0.

5.4 Growth Index gamma_eff

Table 6 presents the effective growth index at z < 1, where the computation is mathematically well-defined. General Relativity predicts gamma ~ 0.55 for LambdaCDM (Linder 2005). Our measured LambdaCDM values at z ~ 1 (gamma = 0.570) are consistent with this prediction, validating the analysis pipeline. The 3D+3D framework produces systematically lower gamma_eff by Delta_gamma ~ −0.02 to −0.04, consistent with enhanced gravity (mu > 0 implies faster growth implies higher f for given Omega_m).

z	gamma_LambdaCDM	gamma_3D3D	Delta_gamma
0.98	0.5696	0.5277	−0.042
0.49	0.4600	0.4341	−0.026
0.29	0.5736	0.5501	−0.024
0.11	0.5660	0.5466	−0.019

Table 6. Effective growth index at z < 1 (linear regime). The systematic offset Delta_gamma ~ −0.02 to −0.04 is the signature of the fifth force enhancing the growth rate beyond the GR prediction.

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6. Three-Way Decomposition: Fifth Force vs. Background

A critical systematic test is the decomposition of the total signal into its physical components. We achieve this through three matched runs:

Total effect: P(v3) / P(LambdaCDM) = background modification + fifth force mu(k,a)

Background only: P(bgonly) / P(LambdaCDM) = background modification alone

Pure fifth force: P(v3) / P(bgonly) = mu(k,a) contribution only

The result (Table 3) is unambiguous: P(bgonly)/P(LambdaCDM) = 1.000000 at all scales and all redshifts, to within numerical precision. This means that the Hubble function H(a) is identical between the background-only and LambdaCDM runs, as independently confirmed by inspection of the Gadget-4 source code (the `THREEPLUS3D_GEOMETRIC_DE` flag is defined in Config.sh but the corresponding modification to the Hubble function is not yet implemented in the compiled code).

Consequently, 100% of the measured signal — the 2.6% enhancement at k < 0.05, the 1.1% enhancement at k > 0.5, the screening signal Delta = 0.015, and the entire f*sigma_8 and gamma_eff deviations — originates exclusively from the scale-dependent fifth force mu(k,a) operating in the PM solver. This is a clean result: the physics is fully attributable to a single mechanism.

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7. Degeneracy Breaking Analysis

A fundamental challenge for modified gravity theories is breaking degeneracies with other cosmological models that produce similar observational signatures. We compare the 3D+3D fingerprint against five competing scenarios in the (Delta_P(k) slope, Delta_f*sigma_8, Delta_gamma) parameter space:

Model	Delta_P(k<.05)	Delta_P(.1–.2)	Delta_P(k>.5)	Delta_f*sigma8	Delta_gamma	Slope
3D+3D (this work)	**+0.026**	**+0.018**	**+0.011**	**+0.029**	**−0.042**	**−0.015**
f(R), abs(f_R0)=10^(−5)	+0.005	+0.080	+0.150	+0.060	−0.080	+0.145
f(R), abs(f_R0)=10^(−6)	+0.002	+0.025	+0.060	+0.030	−0.040	+0.058
Sum(m_nu) = 0.1 eV	−0.005	−0.015	−0.035	−0.025	+0.008	−0.030
Sum(m_nu) = 0.3 eV	−0.015	−0.045	−0.100	−0.070	+0.020	−0.085
w0wa-CDM (w=−0.9)	+0.015	+0.015	+0.015	+0.020	−0.015	0.000
Baryonic feedback	0.000	−0.020	−0.100	−0.005	+0.002	−0.100

Table 7. Degeneracy breaking: 3D+3D geometric fingerprint vs. competing models at z = 0. Slope = Delta_P(k>0.5) − Delta_P(k<0.05). Literature values from: Hu & Sawicki (2007) for f(R); Lesgourgues & Pastor (2006) for massive neutrinos; Chevallier & Polarski (2001) for w0wa-CDM; van Daalen et al. (2011) for baryonic feedback.

The 3D+3D framework occupies a unique region characterized by:

(1) Negative P(k) slope (enhancement decreasing from large to small scales): This is the Lorentzian screening signature. In contrast, f(R) models have positive slope (chameleon screening enhances small scales more), and w0wa-CDM has zero slope (scale-independent).

(2) Positive Delta_f*sigma_8 (faster growth): This distinguishes 3D+3D from massive neutrinos (which suppress growth) and baryonic feedback (which has negligible effect on growth).

(3) Negative Delta_gamma (lower growth index): Combined with the negative slope in Delta_P, this combination is unique to the 3D+3D framework. The f(R) model has negative Delta_gamma but positive Delta_P slope — the opposite scale dependence.

The combination of negative slope + positive Delta_f*sigma_8 + negative Delta_gamma is a three-dimensional fingerprint that no competing model replicates. This enables robust identification with Euclid and DESI data.

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8. Verification and Systematic Controls

8.1 Code Verification

Seven-point verification of the simulation: (1) MD5 checksum of the compiled executable matches the patched source (`2544c94d`); (2) Source code inspection confirms `mu_effective` is active with correct unit conversion (*1000.0); (3) Process memory verification via `/proc/PID/exe` confirms the running binary matches the compiled executable; (4) Output directory points to the correct path; (5) Particle positions diverge between 3D+3D and LambdaCDM from snap_002 onward (0.44 kpc/h mean divergence at z = 18.8), growing monotonically to z = 0; (6) Log output confirms `THREEPLUS3D_MU_GRAVITY` active with mu_0 = 0.0500, k_mu = 0.2000; (7) All 13 snapshots completed for all three runs.

8.2 N-Body Chaos vs. Physical Signal

N-body simulations are chaotic systems where particle positions diverge exponentially between runs with different floating-point round-off (Thiebaut et al. 2008). Simple position divergence is therefore not proof of modified physics. The definitive test is the power spectrum ratio P(k), which is statistically stable against chaos. Our P(k) ratio shows: (a) correct scale dependence (larger at low k, smaller at high k, matching the Lorentzian G(k)); (b) monotonic growth with S(a); (c) amplitude consistent with mu_max = 0.05 (the 2.6% at P(k) is physically expected given the PM-only implementation and nonlinear damping); (d) no explosive or pathological behavior at any redshift. These four characteristics distinguish physical signal from numerical noise.

8.3 Resolution Effects

With 256^3 particles and PMGRID = 256, our fundamental mode is k_fund = 0.0314 h/Mpc and the Nyquist frequency is k_Ny = 4.02 h/Mpc. We restrict analysis to k < k_Ny/2 = 2.01 h/Mpc and use CIC deconvolution following Jing (2005). The low-k bin (k < 0.05 h/Mpc) contains only ~9 independent modes, limiting statistical precision. A planned upgrade to 640^3 particles with 48 GB RAM will increase mode counts by 15.6x, enabling publication-quality constraints.

8.4 Limitations and Future Work

(1) The background cosmology modification Omega_Q(a) is not yet implemented in H(a). A future version will modify the Hubble function to test the full 3D+3D cosmological model. (2) The PM-only implementation of mu(k,a) provides effective screening at the tree-PM split scale, but a full tree-level modification would test the screening mechanism more rigorously. (3) Baryonic physics, neutrino masses, and redshift-space distortions are not included in this dark-matter-only simulation. (4) The simulation volume (200 Mpc/h)^3 is smaller than Euclid or DESI survey volumes; comparison with data will require larger boxes or emulators.

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9. Summary of Key Results

Observable	3D+3D	LambdaCDM	Deviation
P(k<0.05) / P_LambdaCDM at z=0	1.0261	1.0000	+2.61%
P(k=0.1–0.2) / P_LambdaCDM at z=0	1.0182	1.0000	+1.82%
P(k>0.5) / P_LambdaCDM at z=0	1.0109	1.0000	+1.09%
Screening Delta(z=0)	0.0152	0.0000	1.52%
sigma_8(z=0)	0.922	0.914	+0.92%
f*sigma_8(z=0)	0.506	0.492	+2.88%
D_lin ratio (z=0)	1.01152	1.0000	+1.15%
gamma_eff (z ~ 1)	0.528	0.570	−0.042
Background contribution	0.000%	—	100% from mu(k,a)

Table 8. Summary of all measured quantities. The 3D+3D framework produces a coherent, physically consistent signature: enhanced growth on all scales, with Lorentzian scale-dependent screening, 100% attributable to the fifth force mu(k,a).

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10. Falsifiable Predictions for Euclid and DESI

Based on the simulation results, we register the following quantitative predictions:

Prediction 1 (RSD): The growth rate f*sigma_8(z) at z ~ 0.5–1.0 should exceed the LambdaCDM prediction by 2–3%, corresponding to Delta(f*sigma_8) ~ +0.013 +/- 0.005 at z = 0.5. This is within the projected sensitivity of DESI (sigma(f*sigma_8) ~ 0.005–0.01 per redshift bin) and Euclid (sigma(f*sigma_8) ~ 0.008–0.015).

Prediction 2 (Power Spectrum Slope): The ratio P_observed / P_LambdaCDM should decrease monotonically with k from +2–3% at k < 0.05 h/Mpc to +1–1.5% at k > 0.5 h/Mpc, with transition centered at k_mu ~ 0.20

h/Mpc. This Lorentzian profile is the primary distinguishing feature from f(R) gravity (which predicts increasing ratio with k) and massive neutrinos (which predict decreasing ratio but with negative Delta_f*sigma_8).

Prediction 3 (Growth Index): gamma_eff at z ~ 0.5–1.0 should be lower than the GR prediction (gamma ~ 0.55) by Delta_gamma ~ −0.02 to −0.04, measurable with combined Euclid+DESI constraints on gamma.

Falsification criterion: If Delta(f*sigma_8) < 0 or P(k) ratio increases with k, or Delta_gamma > 0, the 3D+3D fifth force model as implemented here is ruled out.

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11. Conclusions

We have presented the first N-body cosmological simulation of the 3D+3D discrete spacetime framework. The key findings are:

The scale-dependent fifth force mu(k,a) = mu_0 * S(a) / (1 + (k/k_mu)^2) produces a physically coherent and numerically stable modification to the matter power spectrum, with a unique observational fingerprint distinguishable from all major competing models (f(R), massive neutrinos, w0wa-CDM, baryonic feedback). The three-way simulation decomposition demonstrates that 100% of the signal originates from the fifth force mechanism, with zero contribution from background cosmology modifications (which are not yet implemented in the code). The +2.9% enhancement in f*sigma_8 and the characteristic Lorentzian P(k) slope provide concrete, falsifiable predictions for Euclid and DESI within their projected sensitivity windows.

These results represent the first step in a comprehensive N-body validation program. Planned upgrades include: 640^3 particle simulations for publication-quality statistics, implementation of the geometric dark energy Omega_Q(a) modification to H(a), and comparison with upcoming survey data.

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