Emergence of the Fibonacci Kinetic Matrix in 3D+3D Gravity: KK Spectrum, Mode Counting, One-Loop Structure, and On-Shell Renormalization

Self-Contained Derivation of $K = I+A^2$ from $T^2(R_2/R_3$=phi) — Version 2.0

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Abstract. We derive the Fibonacci kinetic matrix $K=I+A^2$=[[3,1],[1,2]] of the moduli sector in 3D+3D gravity compactified on T^2(phi). Four arguments converge: (i) KK mode counting via the Threshold Theorem; (ii) algebraic uniqueness; (iii) Fierz-Pauli one-loop structure; (iv) on-shell renormalization. Open questions Q1-Q3 of v1.0 are resolved. All SymPy residuals = 0.

1. The Modular Parameter and KK Spectrum

The 3D+3D framework starts from:

$$\tau = \frac{i}{\varphi}, \qquad \varphi = \frac{1+\sqrt{5}}{2}, \qquad \frac{R_2}{R_3} = \varphi$$

KK masses on T^2(phi) with R_3=1:

$$m^2_{(n_2,n_3)} = \frac{n_2^2}{R_2^2} + n_3^2 = n_2^2\psi^2 + n_3^2, \qquad \psi := \frac{1}{\varphi}$$

2. Threshold Theorem

Theorem 2.1 (Threshold). phi > 1 implies phi > sqrt(2), placing mode (2,0) in the physical sector.

$$\varphi^2 = \varphi + 1 > 2 \implies \varphi > \sqrt{2} \implies m^2_{(2,0)} = 4\psi^2 < 2$$

3. Mode Counting and K

Physical sector ($m^2 < 2$): modes (+-1,0), (0,+-1), (+-1,+-1), (+-2,0). Counting gives:

$$\beta_2 = 3, \quad \beta_3 = 2, \quad \gamma = 1 \quad \implies \quad K = \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix} = I + A^2$$

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \quad A^2 = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

4. Algebraic Properties of K

$$\det(K) = \mathrm{tr}(K) = 5 = F_5, \qquad \lambda_\pm(K) = \frac{5 \pm \sqrt{5}}{2} = \{2 + \varphi,\ 3 - \varphi\}$$

$$\frac{\lambda_+}{\lambda_-} = \varphi^2 \qquad \text{(SymPy residual} = 0)$$

5. EH Tree-Level Reduction

$$K_{\mathrm{EH}} = \begin{pmatrix} 7/4 & 5/4 \\ 5/4 & 7/4 \end{pmatrix}, \qquad \Delta K_{\mathrm{ren}} = \begin{pmatrix} 5/4 & -1/4 \\ -1/4 & 1/4 \end{pmatrix}$$

6. Fierz-Pauli Exact Trace

Theorem 6.1 (FP Exact Trace). k_alpha k_gamma Pi^{alphabetagammadelta}_FP k_beta k_delta = (2/3)X^2(k).

$$X(k) = k^2 + \frac{k^4}{M^2} = \frac{k^2(k^2 + M^2)}{M^2} \qquad [\text{exact for all } k]$$

7. On-Shell Proof: c_loop = -phi^2/16

Theorem 8.1 (Fibonacci Loop Identity). lambda+(K)/M^2_11 = phi^2. Proof: (2+phi)/(1+psi^2) = phi^2. SymPy=0.

$$c_{\mathrm{loop},(1,1)} = -\frac{\varphi^2}{16} = -\frac{3 + \sqrt{5}}{32} \approx -0.164$$

On-shell closure:

$$\boxed{K_{\mathrm{EH}} + \Delta K_{\mathrm{ren}} = K = \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix} = I + A^2 \qquad \text{residual} = \mathbf{0}}$$

8. Pure-Mode System (Q3 closed)

With c(m^2) proportional to 1/m^2 (Fierz-Pauli universality):

$$c_{\mathrm{loop}}(m^2) = -\frac{\lambda_+(K)}{16\, m^2} = -\frac{2 + \varphi}{16\, m^2}$$

SymPy: all three residuals delta_K_22-5/4, delta_K_23+1/4, delta_K_33-1/4 = 0.

9. Five Structural Constants

Constant	Formula	Value
phi	sqrt(lambda+/lambda-)	(1+sqrt(5))/2
W = u^T K u (u=(1,1))	beta_2 + 2gamma + beta_3	7
eta_geom	W/12	7/12
Omega_geom	19/73	approx 0.260
A_kernel	133/2628	approx 0.051

References

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