Paper LXXII: A Geometric Framework for Standard Model Parameters from Six-Dimensional Spacetime

The Discriminant Theorem and Numerical Patterns

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Abstract

We present a geometric framework based on six-dimensional spacetime with signature (3,3) that yields expressions for Standard Model parameters. The central result is the **Discriminant Theorem**: requiring the compactification torus to have Complex Multiplication with discriminant $\Delta = D - 1 = 5$ uniquely selects the quadratic field $\mathbb{Q}(\sqrt{5})$, whose fundamental unit is the golden ratio φ. This determines the torus modular parameter $\tau = i/\varphi$. From this geometric structure, we identify expressions for several SM parameters with sub-percent precision. We carefully classify our results into three categories: (1) rigorously derived quantities, (2) geometrically motivated expressions, and (3) observed numerical patterns requiring further investigation. The framework makes falsifiable predictions and contains no free parameters beyond the electroweak scale v.

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1. Introduction

The Standard Model of particle physics contains approximately 25 free parameters whose values are determined experimentally but not explained theoretically. A fundamental theory should predict these values from deeper principles.

We explore whether six-dimensional spacetime with signature (3,3), compactified on a torus T^2, can provide geometric expressions for SM parameters. Our approach emphasizes intellectual honesty: we clearly distinguish between what is rigorously derived, what is geometrically motivated, and what remains numerical observation.

Structure of this paper:

- Section 2: The Discriminant Theorem (rigorous mathematical result)
- Section 3: Geometrically motivated expressions
- Section 4: Observed numerical patterns
- Section 5: Classification and status of results
- Section 6: Discussion and falsifiability

Notation: Throughout, $\varphi = (1+\sqrt{5})/2 \approx 1.618$ denotes the golden ratio, and $v = 246.22$ GeV is the electroweak vacuum expectation value (taken as input).

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2. The Discriminant Theorem

This section contains our central rigorous result.

2.1 Why D = 6?

We require spacetime to satisfy:

1. **Causality:** At least one temporal dimension
2. **Stable compactification:** A torus T^2 requires at least two compactifiable dimensions
3. **Signature symmetry:** N_space = N_time (equal spatial and temporal dimensions)
4. **Real modular solutions:** The equation $y + 1/y = \sqrt{(D-1)}$ must have real solutions

Theorem 2.1: The minimal dimension satisfying all requirements is $D = 6$.

Proof: Condition 4 requires $D - 1 \geq 4$, hence $D \geq 5$. Condition 3 requires D even. Therefore $D \geq 6$. The value $D = 6$ with signature (3,3) satisfies all conditions. ■

2.2 The Discriminant Principle

For the torus T^2 to have Complex Multiplication (CM), its modular parameter τ must generate a quadratic imaginary field. We propose:

Principle: The discriminant of the CM field equals $\Delta = D - 1$.

For $D = 6$, this gives $\Delta = 5$.

2.3 Uniqueness of Q(√5)

Theorem 2.2 (Discriminant Theorem): The quadratic field $Q(\sqrt{d})$ with discriminant $\Delta = 5$ is unique: it is $Q(\sqrt{5})$.

Proof: The discriminant of $Q(\sqrt{d})$ is:

$$\Delta = \begin{cases} d & \text{if } d \equiv 1 \pmod 4 \\ 4d & \text{if } d \equiv 2,3 \pmod 4 \end{cases}$$

For $\Delta = 5$: Since $5 \equiv 1 \pmod 4$, we need $d = 5$. No other value of d gives $\Delta = 5$.

Verification of small discriminants:

d	d mod 4	Δ
2	2	8
3	3	12
5	1	**5** ✓
6	2	24
7	3	28
13	1	13

Therefore $\mathbb{Q}(\sqrt{5})$ is the unique solution. ■

2.4 Determination of τ

The fundamental unit of $\mathbb{Q}(\sqrt{5})$ is $\varphi = (1+\sqrt{5})/2$, satisfying $\varphi^2 = \varphi + 1$.

For a CM torus with purely imaginary $\tau = iy$, we have $y \in \{\varphi, 1/\varphi\}$.

Physical selection: In signature (3,3), T-duality $\tau \leftrightarrow -1/\tau$ is broken because both torus cycles are temporal. A temporal hierarchy (visible time > compact times) requires $\mathrm{Im}(\tau) < 1$.

Result: $\tau = i/\varphi$, giving $\mathrm{Im}(\tau) = 1/\varphi \approx 0.618$.

Verification: $\varphi + 1/\varphi = \sqrt{5} = \sqrt{(D-1)}$ ✓

2.5 Summary of Rigorous Results

From the Discriminant Theorem, we obtain without ambiguity:

Quantity	Value	Status
D	6	**Derived**
Signature	(3,3)	**Derived**
CM Field	$\mathbb{Q}(\sqrt{5})$	**Theorem**
Fundamental unit	$\varphi = 1.618...$	**Derived**
Modular parameter	$\tau = i/\varphi$	**Derived**

3. Geometrically Motivated Expressions

The following expressions use only the geometric quantities (D, N_space, N_time, Im(τ)) but involve interpretive choices that are not uniquely determined.

3.1 Weak Mixing Angle

Expression:

$$\sin^2\theta_W = \frac{N_{time} \cdot \mathrm{Im}(\tau) - 1}{D \cdot \mathrm{Im}(\tau)} = \frac{3/\phi - 1}{6/\phi} = \frac{3-\phi}{6}$$

Numerical result: 0.2303 vs observed 0.23121 — **Error: 0.38%**

Geometric interpretation: The formula represents the "temporal weight" (N_time × Im(τ)) minus a "zero mode" contribution (−1), normalized by the total weight (D × Im(τ)).

Caveat: The subtraction of 1 (interpreted as the visible time mode) is motivated but not uniquely derived. Alternative normalizations could be considered.

Status: Geometrically motivated expression

3.2 Top Quark Mass

Expression:

$$m_t = \frac{v}{\sqrt{2}}$$

corresponding to Yukawa coupling y_t = 1.

Numerical result: 174.1 GeV vs observed 172.69 GeV — **Error: 0.82%**

Interpretation: The value y_t = 1 is the simplest non-trivial Yukawa coupling, suggesting the top quark is "maximally coupled" to the Higgs.

Caveat: While y_t = 1 is natural, values like y_t = φ or y_t = 1/φ would also be geometrically meaningful.

Status: Geometrically motivated (naturalness argument)

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4. Observed Numerical Patterns

The following expressions reproduce SM parameters with high precision but involve combinations whose origin is not fully understood. We present them as **observed patterns** requiring further investigation.

4.1 Fine Structure Constant

Observed pattern:

$$\alpha^{-1} = e^{N_{space}} \cdot \phi^4 - \frac{1}{\phi} = e^3\phi^4 - \frac{1}{\phi}$$

Numerical result: 137.050 vs observed 137.036 — **Error: 0.01%**

Tentative interpretation:

- $e^3 = \exp(N\_space)$: exponential factor from 3 spatial dimensions
- $\varphi^4 = 1/\mathrm{Im}(\tau)^4$: fourth power of inverse modular parameter
- $-1/\varphi = -\mathrm{Im}(\tau)$: volume correction

Open questions:

- Why does e (Euler's number) appear rather than π or another constant?
- Why the fourth power of φ?
- What physical mechanism produces this combination?

Status: Numerical pattern with tentative interpretation

4.2 Higgs Mass

Observed pattern:

$$m_H = \frac{v\phi}{\pi} = \frac{v}{\pi \cdot \mathrm{Im}(\tau)}$$

Numerical result: 126.8 GeV vs observed 125.25 GeV — **Error: 1.25%**

Tentative interpretation: The factor π may arise from the periodicity of the torus. The 1.25% discrepancy is consistent with expected radiative corrections.

Open question: Why π in the denominator?

Status: Numerical pattern

4.3 CKM CP-Violating Phase

Observed pattern:

$$\delta_{CKM} = \frac{\pi}{\phi^2} = \pi \cdot \mathrm{Im}(\tau)^2$$

Numerical result: 68.75° vs observed 68.8° — **Error: 0.07%**

Tentative interpretation: The phase may arise from interference between paths on the torus, with $\mathrm{Im}(\tau)^2$ measuring the "area" in modular units.

Open question: Why does π appear as a coefficient?

Status: Numerical pattern

4.4 Summary Table of Patterns

Parameter	Formula	Predicted	Observed	Error
α^{-1}	$e^3\varphi^4 - 1/\varphi$	137.050	137.036	0.01%
m_H	vφ/π	126.8 GeV	125.25 GeV	1.25%
δ_CKM	π/φ^2	68.75°	68.8°	0.07%

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5. Further Numerical Observations

The following relations have been observed but are more speculative. They may represent genuine structure or numerical coincidence.

5.1 Koide Angle

The Koide formula for charged lepton masses uses an angle θ_0.

Observed relation:

$$\theta_0 = \frac{4\pi}{5} - \arctan\left(\frac{1}{5}\right)$$

Numerical result: 132.69° vs fitted value 132.73° — **Error: 0.03%**

Interpretation: 4π/5 is the external angle of a regular pentagon (connected to 5-fold symmetry and φ). The correction arctan(1/5) involves the discriminant Δ = 5.

Status: Numerical observation (found by searching for matching formulas)

5.2 Koide Mass Scale

Observed relation:

$$m_0 = \frac{v \sin^4 \theta_W}{\pi^2 \phi^3}$$

Numerical result: 312.4 MeV vs fitted 313.8 MeV — **Error: 0.44%**

Status: Numerical observation

5.3 Proton Mass

Observed relation:

$$m_p = \frac{v(3-\phi)^2}{12\pi^2\phi^3} = \frac{v \cdot 36 \sin^4 \theta_W}{12\pi^2\phi^3}$$

Numerical result: 937.3 MeV vs observed 938.27 MeV — **Error: 0.10%**

Tentative interpretation: The factor $(3-\varphi)^2 = 36\sin^4\theta_W$ connects to the electroweak sector. The denominator $12 = 4 \times N_c$ may relate to QCD with $N_c = 3$ colors.

Status: Numerical observation

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6. Classification of Results

We classify all results by their epistemic status:

Level A: Rigorously Derived (Mathematical Theorems)

Result	Derivation Method	Confidence
D = 6	Minimality argument	**100%**
$\mathbb{Q}(\sqrt{5})$	Discriminant Theorem	**100%**
φ as fundamental unit	Number theory	**100%**
$\tau = i/\varphi$	CM + hierarchy	**95%**

Level B: Geometrically Motivated (Physical Arguments)

Result	Basis	Confidence
$\sin^2\theta_W = (3-\varphi)/6$	Temporal weight formula	**80%**
$m_t = v/\sqrt{2}$	Natural Yukawa $y_t = 1$	**75%**

Level C: Numerical Patterns (Observed, Interpretation Tentative)

Result	Status	Confidence
$\alpha^{-1} = e^3\varphi^4 - 1/\varphi$	Pattern with structure	**60%**
$m_H = v\varphi/\pi$	Pattern	**55%**
$\delta_{CKM} = \pi/\varphi^2$	Pattern	**55%**

Level D: Numerical Observations (Possibly Coincidental)

Result	Status	Confidence
$\theta_0 = 4\pi/5 - \arctan(1/5)$	Found by search	**40%**
m_0 formula	Constructed	**40%**
m_p formula	Constructed	**40%**

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7. Discussion

7.1 What This Framework Achieves

1. **Unique geometric structure:** The Discriminant Theorem provides a rigorous mathematical selection of $Q(\sqrt{5})$ and hence φ and $\tau = i/\varphi$.
2. **Expressions for multiple parameters:** From one geometric structure, we obtain expressions for at least 8 SM-related quantities.
3. **High numerical precision:** Average error across all formulas is ~0.4% with no free parameters (given v as input).
4. **Internal consistency:** All expressions use the same fundamental quantities (D, φ, τ, π, e).

7.2 What Remains to Be Understood

1. **Why $\Delta = D - 1$?** This principle is motivated by dimensional counting but not derived from first principles.
2. **Origin of π and e in formulas:** These constants appear in Levels C-D without clear derivation.
3. **The "−1" in $\sin^2\theta_W$:** The zero-mode subtraction needs rigorous justification.
4. **Quark masses and CKM angles:** Beyond m_t and δ_{CKM}, other quark parameters are not addressed.

7.3 Falsifiability

The framework makes specific predictions:

Prediction	Value	Testable?
$\sin^2\theta_W$(tree)	0.2303	Yes (precision EW)
m_H/v	$\varphi/\pi = 0.515$	Yes (lattice QCD)
δ_{CKM}	68.75°	Yes (B-physics)

Any significant deviation from these values would falsify the geometric interpretation.

7.4 Comparison with Other Approaches

Unlike string theory (which has a landscape of solutions), this framework:

- Selects a **unique** compactification geometry
- Uses **signature (3,3)** rather than (1,9) or (1,5)
- Derives φ from **number theory** rather than postulating it

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8. Conclusion

We have presented a geometric framework based on six-dimensional spacetime (3,3) that produces expressions for Standard Model parameters. The core contribution is the **Discriminant Theorem**, which rigorously establishes that requiring $\Delta = D - 1 = 5$ uniquely selects $Q(\sqrt{5})$, giving φ and $\tau = i/\varphi$.

From this geometric foundation, we identify expressions of varying epistemic status:

- **Derived:** D, $Q(\sqrt{5})$, τ
- **Motivated:** $\sin^2\theta_W$, m_t

- **Observed patterns:** α^{-1}, m_H, δ_CKM
- **Numerical observations:** θ_0, m_0, m_p

We emphasize that Levels C and D require further theoretical development to understand why these particular combinations appear. The numerical precision (0.4% average) is remarkable but does not by itself constitute derivation.

The framework's strength lies in its **unique selection** of the geometric structure and its **falsifiable predictions**. Future work should focus on deriving the Level C patterns from the torus geometry, particularly understanding the role of π and e in the formulas.

Appendix A: Mathematical Background

A.1 Quadratic Fields and Discriminants

A quadratic field Q(√d) with d squarefree has discriminant:

$$\Delta = \begin{cases} d & d \equiv 1 \pmod 4 \\ 4d & d \equiv 2, 3 \pmod 4 \end{cases}$$

A.2 The Golden Ratio

$$\phi = \frac{1 + \sqrt{5}}{2} = 1.6180339887...$$

Key identities:

- $\varphi^2 = \varphi + 1$
- $1/\varphi = \varphi - 1$
- $\varphi + 1/\varphi = \sqrt{5}$
- $\varphi^4 = 3\varphi + 2$

A.3 Numerical Constants

$$\pi = 3.14159265...$$

$$e = 2.71828182...$$

$$v = 246.22 \text{ GeV}$$

Appendix B: Verification Code

```python
import numpy as np

phi = (1 + np.sqrt(5)) / 2
pi = np.pi
e = np.e
v = 246.22  # GeV

# Level A: Derived
print(f"φ = {phi:.6f}")
print(f"τ = i/{phi:.6f}")

# Level B: Motivated
sin2_W = (3 - phi) / 6
m_t = v / np.sqrt(2)
print(f"sin²θ_W = {sin2_W:.6f} (obs: 0.23121)")
print(f"m_t = {m_t:.2f} GeV (obs: 172.69)")

# Level C: Patterns
alpha_inv = phi**4 * e**3 - 1/phi
m_H = v * phi / pi
delta_CKM = np.degrees(pi / phi**2)
print(f"α⁻¹ = {alpha_inv:.4f} (obs: 137.036)")
print(f"m_H = {m_H:.2f} GeV (obs: 125.25)")
print(f"δ_CKM = {delta_CKM:.2f}° (obs: 68.8°)")

# Level D: Observations
theta_0 = np.degrees(4*pi/5 - np.arctan(0.2))
m_0 = v * 1000 * sin2_W**2 / (pi**2 * phi**3)
m_p = v * 1000 * (3-phi)**2 / (12 * pi**2 * phi**3)
print(f"θ₀ = {theta_0:.2f}° (obs: 132.73°)")
print(f"m₀ = {m_0:.1f} MeV (obs: 313.8)")
print(f"m_p = {m_p:.1f} MeV (obs: 938.27)")
```

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References

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"Non facciamo le cose a metà — but we do them honestly."

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