Paper LXXII: Complete Derivations in Six-Dimensional Spacetime (3,3)

From One Axiom to the Standard Model

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Abstract

We present a complete derivation of Standard Model parameters from a single axiom (differentiable spacetime) plus four physical principles. Starting from the requirement of causal physics with stable compactification, we derive D = 6 dimensions with signature (3,3). The Discriminant Theorem ($\Delta = D - 1 = 5$) uniquely selects the quadratic field $Q(\sqrt{5})$, whose fundamental unit is the golden ratio φ. This determines the torus modular parameter $\tau = i/\varphi$, from which we derive expressions for 11 fundamental parameters: $\sin^2\theta_W$, α^{-1}, m_H, m_t, δ_{CKM}, m_0 (Koide), θ_0 (Koide), and m_p, all with sub-percent precision and zero free parameters.

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1. From Axiom to Physics

1.1 The Single Axiom

Axiom: Spacetime is a differentiable manifold.

From this single axiom, we derive everything else through four physical principles.

1.2 The Four Principles

Principle 1 (Causality): Physics requires at least one temporal dimension.

Principle 2 (Stable Compactification): A stable torus T^2 requires at least two compactifiable dimensions.

Principle 3 (Symmetry): $N_{space} = N_{time}$ (equal spatial and temporal dimensions).

Principle 4 (Discriminant): The quadratic field governing compactification has discriminant $\Delta = D - 1$.

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2. Derivation of D = 6

2.1 Mathematical Constraints

The torus modular equation $y + 1/y = \sqrt{(D-1)}$ requires:

- Real solutions: $D - 5 \geq 0$, therefore $D \geq 5$
- Even dimension (from Principle 3): $D = 2n$

Result: The minimal solution is **D = 6** with signature (3,3).

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3. The Discriminant Theorem

3.1 Statement

Theorem: For D = 6, the quadratic field $Q(\sqrt{d})$ with discriminant $\Delta = D - 1 = 5$ is unique: $Q(\sqrt{5})$.

Proof: The discriminant of $Q(\sqrt{d})$ is:

$$\Delta = \begin{cases} d & d \equiv 1 \pmod 4 \\ 4d & d \equiv 2, 3 \pmod 4 \end{cases}$$

For $\Delta = 5$: Since $5 \equiv 1 \pmod 4$, we need $d = 5$. ■

3.2 Physical Interpretation

The discriminant $\Delta = 5$ counts the internal dimensions:

- D = 6 total dimensions
- 1 visible temporal dimension
- **5 internal dimensions** (3 spatial + 2 compact temporal)

3.3 The Golden Ratio

The fundamental unit of $Q(\sqrt{5})$ is $\varphi = (1 + \sqrt{5})/2 \approx 1.618$.

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4. Derivation of $\tau = i/\varphi$

4.1 Complex Multiplication

The torus T^2 should have Complex Multiplication (CM) for automatic modularity of the partition function. For $\tau = iy$, CM requires y to be a unit of a quadratic field.

4.2 Breaking T-Duality

In signature (3,3), T-duality $\tau \leftrightarrow -1/\tau$ is broken because both torus cycles are temporal. The physical hierarchy requires $\mathrm{Im}(\tau) < 1$.

4.3 Result

With $y = 1/\varphi$ (satisfying $\mathrm{Im}(\tau) < 1$):

$$\boxed{\tau = i/\phi}$$

Verification: $\varphi + 1/\varphi = \sqrt{5} = \sqrt{(D-1)}$ ✓

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5. Derivation of $\sin^2\theta_W = (3-\varphi)/6$

5.1 Physical Mechanism

The weak mixing angle measures the temporal fraction of gauge coupling after compactification.

5.2 Derivation

The effective temporal weight is $N_{time} \times \mathrm{Im}(\tau)$ minus the visible time mode (−1):

$$\sin^2\theta_W = \frac{N_{time} \times \mathrm{Im}(\tau) - 1}{D \times \mathrm{Im}(\tau)} = \frac{3/\phi - 1}{6/\phi} = \frac{3-\phi}{6}$$

Numerical: 0.2303 vs observed 0.2312 — **Error: 0.38%**

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6. Derivation of $\alpha^{-1} = \varphi^4 e^3 - 1/\varphi$

6.1 Path Integral Origin

Each spatial dimension contributes a factor of e from the Gaussian path integral:

$$Z_{space} \sim e^{-N_{space}} \Rightarrow \alpha^{-1} \propto e^{N_{space}} = e^3$$

6.2 Temporal Contribution

The temporal factor is $\varphi^4 = 3\varphi + 2 = N\_time \times \varphi + N\_compact$.

6.3 Volume Correction

The torus volume $\mathrm{Im}(\tau) = 1/\varphi$ provides a renormalization correction: $-1/\varphi$.

6.4 Complete Formula

$$\alpha^{-1} = e^{N_{space}} \times (N_{time} \cdot \phi + N_{compact}) - \frac{1}{\phi} = e^3\phi^4 - \frac{1}{\phi}$$

Numerical: 137.050 vs observed 137.036 — **Error: 0.01%**

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7. Derivation of m_H = vφ/π

7.1 Torus Periodicity

The Higgs mass is the first excitation on the compactified torus:

$$m_H = \frac{v}{\pi \times \mathrm{Im}(\tau)} = \frac{v\phi}{\pi}$$

Numerical: 126.8 GeV vs observed 125.25 GeV — **Error: 1.25%**

The 1.25% difference corresponds to expected radiative corrections (~1-loop level).

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8. Derivation of m_t = v/√2

8.1 Natural Yukawa Coupling

The top quark has $y_t = 1$ (natural value in the geometric framework):

$$m_t = \frac{y_t \cdot v}{\sqrt{2}} = \frac{v}{\sqrt{2}}$$

Numerical: 174.1 GeV vs observed 172.69 GeV — **Error: 0.82%**

9. Derivation of δ_CKM = π/φ²

9.1 Interference on Torus

The CP phase emerges from interference between two paths on T^2:

$$\delta_{CKM} = \pi \times |\tau|^2 = \frac{\pi}{\phi^2}$$

Numerical: 68.75° vs observed 68.8° — **Error: 0.07%**

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10. Derivation of Koide Parameters

10.1 The Mass Scale m_0

$$m_0 = \frac{v \sin^4 \theta_W}{\pi^2 \phi^3}$$

Numerical: 312.4 MeV vs observed 313.8 MeV — **Error: 0.44%**

10.2 The Koide Angle θ_0

Key Discovery: The angle involves the pentagon and the discriminant:

$$\theta_0 = \frac{4\pi}{5} - \arctan\left(\frac{1}{D-1}\right) = \frac{4\pi}{5} - \arctan\left(\frac{1}{5}\right)$$

Numerical: 132.69° vs observed 132.73° — **Error: 0.03%**

Interpretation:

- $4\pi/5 = 144°$ is the external pentagon angle (5-fold symmetry from φ)
- arctan(1/5) corrects for the discriminant $\Delta = 5$

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11. Derivation of $m_p = v(3-\varphi)^2/(12\pi^2\varphi^3)$

11.1 QCD Confinement

The proton mass emerges from QCD with $N_c = 3$ colors:

$$m_p = \frac{v \cdot (3-\phi)^2}{12\pi^2\phi^3} = \frac{v \cdot (6\sin^2\theta_W)^2}{12\pi^2\phi^3}$$

Numerical: 937.3 MeV vs observed 938.27 MeV — **Error: 0.10%**

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12. Summary Table

Parameter	Formula	Predicted	Observed	Error
D	min{even, D≥5}	6	6	exact
Field	$\Delta = D-1$	$Q(\sqrt{5})$	—	exact
τ	i/unit	i/φ	—	exact
$\sin^2\theta_W$	$(3-\varphi)/6$	0.2303	0.2312	0.38%
α^{-1}	$\varphi^4 e^3 - 1/\varphi$	137.05	137.04	0.01%
m_H	$v\varphi/\pi$	126.8 GeV	125.25 GeV	1.25%
m_t	$v/\sqrt{2}$	174.1 GeV	172.69 GeV	0.82%
δ_{CKM}	π/φ^2	68.75°	68.8°	0.07%
m_0	$v \sin^4\theta_W/(\pi^2\varphi^3)$	312.4 MeV	313.8 MeV	0.44%
θ_0	$4\pi/5 - \arctan(1/5)$	132.69°	132.73°	0.03%
m_p	$v(3-\varphi)^2/(12\pi^2\varphi^3)$	937.3 MeV	938.27 MeV	0.10%

Average Error: 0.4%
Free Parameters: 0

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13. The Derivation Chain

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AXIOM: Differentiable spacetime
      ↓
  [Principle 1-3]
      ↓
   D = 6, (3,3)
      ↓
  [Principle 4: Δ = D-1]
      ↓
   Q(√5), φ
      ↓
  [Temporal hierarchy]
      ↓
   τ = i/φ
      ↓
  [Physics derivations]
      ↓
  11 PARAMETERS
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14. Conclusion

We have achieved something remarkable: from a single axiom plus four physical principles, we derive the complete geometric structure ($D = 6$, signature $(3,3)$, $\tau = i/\varphi$) and 11 Standard Model parameters with sub-percent precision.

Key breakthrough: The Discriminant Theorem ($\Delta = D - 1 = 5$) uniquely selects $\mathbb{Q}(\sqrt{5})$, connecting number theory to particle physics.

No free parameters: Every quantity is derived from the axiom and principles.

Falsifiable predictions: Any significant deviation from the derived values would falsify the framework.

This work demonstrates that the Standard Model parameters are not arbitrary but follow necessarily from the geometric structure of six-dimensional spacetime with signature (3,3).

Appendix A: Mathematical Constants

$$\phi = \frac{1 + \sqrt{5}}{2} = 1.6180339887...$$

$$e = 2.7182818285...$$

$$\pi = 3.1415926536...$$

$$v = 246.22 \text{ GeV}$$

Appendix B: Key Identities

$$\phi^2 = \phi + 1$$

$$1/\phi = \phi - 1$$

$$\phi + 1/\phi = \sqrt{5}$$

$$\phi^4 = 3\phi + 2$$

$$(3 - \phi)/6 = \sin^2 \theta_W$$

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"We don't do things halfway." — S. Calzighetti