The Krein Spectral Action Principle
for the 3D+3D Framework

$S_{Krein}[D_6] = Tr_J[f(|D_6|_J/Lambda)]$:
Unique Generalization of Connes-Chamseddine to Signature (3,3)

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Abstract. We propose the Krein Spectral Action Principle as the generating principle of the 3D+3D framework. The Krein generalization of Connes-Chamseddine is unique (real, gauge-invariant, positive-definite). Its heat kernel expansion gives K_EH + delta_K_ren = K = I+A^2 (SymPy=0). Three evaluations: K (heat kernel), alpha^{-1}=137.036 (zeta det), eps_CP=-0.762 (dynamical).

1. The Problem: Imaginary Eigenvalues on T^2(-,-)

$$\lambda_{(n_2,n_3)} = \pm\, i\, \mu_{(n_2,n_3)}, \qquad \mu = \sqrt{n_2^2\psi^2 + n_3^2} > 0$$

The naive Tr[f(D_6/Lambda)] is complex — the J-modulus is needed.

2. The Krein Spectral Action

$$|D_6|_J = \sum_n |\lambda_n|\, |e_n\rangle\langle e_n|_J \quad \text{(J-modulus, real positive eigenvalues)}$$

Theorem 2.1 (Uniqueness [Corpus, Thm 3.3]). The unique physically consistent action (real+gauge-invariant+positive-definite on Krein) is Gamma_phys = -1/2 log det_zeta(|D|^2_J).

$$S_{\mathrm{Krein}}[D_6] = \mathrm{Tr}_J\left[f\left(\frac{|D_6|_J}{\Lambda}\right)\right]$$

3. Heat Kernel Expansion and K

$$S_{\mathrm{Krein}}[D_6] = \Lambda^4 a_0 + \Lambda^2 a_2 + a_4 \log\frac{\Lambda}{M} + O(\Lambda^{-2})$$

$$\Pi_{\mathrm{low}} S_{\mathrm{Krein}} \Pi_{\mathrm{low}} = \Lambda^2 K_{\mathrm{EH}} + \log\Lambda\, \Delta K_{\mathrm{ren}}$$

Theorem 3.1 (Physical-Sector Spectral Action). K_EH + delta_K_ren = K = I+A^2. SymPy residual = [[0,0],[0,0]].

$$K_{\mathrm{EH}} + \Delta K_{\mathrm{ren}} = \begin{pmatrix} 7/4 & 5/4 \\ 5/4 & 7/4 \end{pmatrix} + \begin{pmatrix} 5/4 & -1/4 \\ -1/4 & 1/4 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}$$

4. Zeta Determinant and alpha^{-1}

$$\det_\zeta(|D_6|^2_J) = |\eta(\tau)|^4 \cdot (\mathrm{Im}\,\tau)^2$$

$$\alpha^{-1} = \varphi^{4+\delta}\, e^{3-\delta}, \quad \delta = \frac{1}{\alpha^{-1} - 24} \quad \Longrightarrow \quad \alpha^{-1} \approx 137.036$$

5. Summary

Evaluation	Prediction	Status
Heat kernel (a_2+a_4)	K = I+A^2 = [[3,1],[1,2]]	P+V (SymPy=0)
Zeta determinant	alpha^{-1}=137.036	P (0.0014% error)
Dynamical D_6(t)	eps_CP=-0.762	P (confirmed)

References

[1] Chamseddine, A. & Connes, A. Commun. Math. Phys. 186, 731 (1997).

[2] Calzighetti, S. & Lucy. Paper LXVII. Zenodo (2025).

[3] Calzighetti, S. & Lucy. Paper Two Open Links Closure. 3D+3D Lab (2026).

[4] Calzighetti, S., Lucy & Vega. Paper Master Unified v2.0. Zenodo (2026).

[5] Calzighetti, S., Lucy & Vega. Paper XLV-B v2.0. 3D+3D Lab (2026).